USEFUL

RULES AND TABLES

RELATING TO

MENSURATION, ENGINEERING, STRUCTURES, AND MACHINES.

BY

WILLIAM JOHN MACQUORN RANKINE,

CIVIL ENGINEER; LL.D. TRINITY COLLEGE, DUBLIN; F.R.SS. LOND. AND EDIN.; F.R.S.S.A.; LATE REGIUS PROFESSOR OF CIVIL ENGINEERING AND MECHANICS IN THE UNIVERSITY OF GLASGOW.

With Illustrations, and a Diagram of the Mechanical Properties of Steam.

FIFTH EDITION, THOROUGHLY REVISED,

BY

EDWARD FISHER BAMBER, C.E.

LONDON:
CHARLES GRIFFIN AND COMPANY,
STATIONERS' HALL COURT.
1876.

PREFACE TO THE FIFTH EDITION.

In this Fifth Edition some Tables have been added, and the whole Work has been carefully revised.

E. F. B.

London, *April*, 1876.

u. S.

PREFACE.

The object of this book is to provide, in moderate bulk, a collection of Rules and Tables relating to those parts of mathematical and mechanical science whose application most frequently occurs in the useful arts, and especially in engineering and practical mechanics. The use of algebraical symbols is avoided, except in those cases in which the rules cannot be clearly expressed without them.

The rules and tables of the First Part belong to Arithmetic and Mensuration. The tables of well-known quantities, such as squares, cubes, and logarithms, have been drawn from the most trustworthy sources, and their accuracy independently tested throughout; the circumferences and areas of circles may be relied on to the last figure. The table of trigonometrical functions consists of only a single page; but it is sufficient, nevertheless, for the solution of such problems in practical mechanics as involve the use of those functions; for purposes of Geodesy, the only proper trigonometrical tables are such as fill a large part of a bulky volume. The summary of the rules of trigonometry is complete. Great care has been bestowed on the arrangement and explanation of those important rules which relate to the measurement of the areas of surfaces, volumes of solid figures, and lengths of curves, and the finding of the centres of magnitude of all those classes of figures.

The Second Part relates to the *Measures*, commonly so called, of different nations, and contains tables and rules relating not only to measures of angles, time, length, surface, volume, weight, and value, but to those of quantities more or less complex, such as speed, heaviness, pressure, work, power, moment, absolute force, and heat. The values of the various units of measure mentioned are compared with the standards of the British legal system, and of the metrical system (whose use is now permitted in Britain); and those standards are compared with each other according to the best authorities—viz., the paper of Mr. Airy, Astronomer-Royal,

on "Standards of Measure," and that of Professor Miller on the "Standard Pound." (In the Second Edition, those comparisons were brought into conformity with the work of Captain Clarke, R.E., on "Standards of Length").

The Third Part relates to Engineering Geodesy, comprehending surveying, levelling, and the setting out of works. The rules which depend on the figure and dimensions of the earth, such as those for calculating the lengths of arcs of the meridian, and of arcs intersecting the meridian at different angles, are founded on the most probable determinations of the earth's dimensions. The rules for the setting out of works comprehend directions for ranging curves on lines of railway, and for easing the changes of curvature at the junctions of such curves with each other, and with straight lines. The Part concludes with a system of rules for the measurement of earthwork.

The Fourth Part relates to Distributed Forces and Mechanical Centres. It includes tables of heaviness and specific gravity, and of expansion by heat; and rules for finding centres of gravity, moments of weight and of inertia, centres of pressure, centres of percussion, and centres of buoyancy.

The Fifth Part relates to the Balance and Stability of Structures, including frames, chains, and arched ribs, retaining walls, piers and abutments, arches of masonry, and foundations of different kinds.

The Sixth Part relates to the Strength of Materials. It commences with a series of tables of the resistance of various kinds of materials to straining actions of different kinds; followed by rules for the computation of the strength of materials in the various forms in which they are used in structures and machines; such as ties, pipes and cylinders, pillars, axles, beams, chains, and arches.

The Seventh Part relates to Machines in general; giving in the first place rules for the comparison of the motions of different points in a machine, and for the designing of the more important parts of mechanism, such as wheels and their teeth, speed-cones, parallel motions, &c. These are followed by rules relating to the work of machines at uniform speed and at varying speed, to centrifugal force, the balancing of machinery, and the use of fly-wheels; and by directions how the rules of the sixth part are to be applied to the strength of machinery. In the course of this Part, rules are

given for the resistance of carriages on roads and railways, the tractive power of locomotives, and the ruling gradients of railways. The Part concludes with rules as to the power of horses and other animals, and of men, and a table of the quantity of labour required in various operations.

In the Eighth Part are given rules applicable to Hydraulic and Marine Engineering; such as those which determine the head required to produce a given discharge of water through a given channel or pipe; the discharge from a given outlet with a given head; the dimensions of the pipe or channel required to discharge water at a given rate with a given head; and the strength of water-pipes. Then follow rules for the designing of hydraulic prime movers; such as vertical water-wheels, overshot or undershot, and turbines; then rules applicable to windmills. Lastly, rules are given for the estimation of the resistance of water to the motion of ships; for the determination of the proper dimensions of propelling instruments of different kinds, jets, paddles, or screws, and of the engine-power required to drive them; and for calculating the quantity of sail which a given ship can safely carry;—all founded on practical experience on the large scale.

The Ninth Part relates to Heat and the Steam Engine. It contains a system of rules and tables founded on the true principles of thermodynamics, and at the same time reduced to a degree of brevity and simplicity which it is believed has not hitherto been attained, for determining the relations between work done and heat expended in any actual or proposed steam engine. Those are followed by rules for fixing the leading dimensions of the principal parts of an engine required to do a given duty under given circumstances: for the heating power and the expenditure of fuel: for the efficiency and dimensions of furnaces and boilers; and for the proportioning of slide-valve gear, link-motions, and other fittings of steam engines. At the end of the text is a plate containing a pair of diagrams of the mechanical properties of steam, by the use of which much of the labour of calculation may be saved; and this is followed by a very full alphabetical index.

In this Third Edition various corrections, amendments, and additions have been made.

W. J. M. R.

GLASGOW UNIVERSITY, 1868.

Addenda.

Approximate Rules for Safety Valves. (See also p. 303.)—To find the area of actual opening required. Divide the area of heating surface in square feet by 3 (or the area in square inches by 432); divide the quotient by the absolute pressure in pounds on the square inch: the final quotient will be the area required in fractions of a square inch.

N.B.—This is based on experiments made with circular valves having a lift not exceeding $\frac{1}{20}$ of diameter.

Given the proportion of lift to diameter and the area of opening to find the area of the circular valve seat. Multiply the area of opening by $\frac{1}{4}$ of the ratio in which the diameter is greater than the lift. Special rules for valves in which, with a pressure of 10 pounds above the atmosphere, the valve is to rise not more than $\frac{1}{20}$ of the diameter of the valve seat. To find the area of the circular valve seat. Divide the area of heating surface by 2000; the quotient will be in the same sort of measure with the area of the heating surface. To insure the same proportionate rise with a greater minimum pressure, the area should be varied inversely as the absolute pressure. To insure the same proportionate rise with a less minimum pressure, the area of valve seat should be made to vary inversely as the square root of the effective minimum pressure above the atmosphere.

Proportions of British and French Measures.—In the Comparative Tables of Measures contained in this volume, the value of the standard metre in inches is taken as ascertained at the British Ordnance Survey Office, viz., 39·37043.

Such is the true scientific value of the metre: it has been shown, however, by the British Commission on Standards (see Appendix to Fifth Report, p. 198) that the commercial metre, owing to expansion by heat, is longer than the scientific metre, being 39·38203 inches. The difference is 0·0116 of an inch in each metre; that is to say, very nearly 0·295 millimetres in a metre.

Standard Gallon.—In the Act of Parliament relating to this subject the definition of the gallon as being the capacity of 10 lbs. of pure water at 62° Fahr. is that first given, and is obviously to be always followed when practicable. The alternative definition of 277·274 cubic inches is given as a means of determining the gallon when the first-mentioned method is impracticable. The second definition, however, is by far the more frequent in popular and even in scientific use. It makes the gallon greater than the first definition does in the proportion very nearly of 1·00054 : 1.

Fluid Ounce.—By an Order in Council, of the year 1871, a fluid ounce is recognized as being one-one-hundred-and-sixtyth of a gallon.

CONTENTS.

PART I.—NUMBERS AND FIGURES.

RULES FOR THE MENSURATION OF FIGURES.

PART II.—MEASURES.

PART III.—ENGINEERING GEODESY.

PART IV.—DISTRIBUTED FORCES AND MECHANICAL CENTRES.

PART V.—BALANCE AND STABILITY OF STRUCTURES.

PART VI.—STRENGTH OF MATERIALS.

PART VII.—MACHINES IN GENERAL.

PART VIII.—HYDRAULICS.

PART IX.—HEAT AND THE STEAM ENGINE.

ADDENDA.

USEFUL RULES AND TABLES.

PART I.

NUMBERS AND FIGURES.

Table 1.—Squares, Cubes, Reciprocals, and Common Logarithms of Numbers from 101 to 999.

Explanation.

Squares, Cubes, and Reciprocals.

1. The square, cube, and reciprocal of 1 are each of them 1.

2. The square of any integer power of 10 is 1 followed by twice as many noughts as there are in the original number; for example, $10^2=100$; $100^2=10000$, &c.

3. The cube of any integer power of 10 is 1 followed by thrice as many noughts as there are in the original number; for example, $10^2=1000$; $100^2=1000000$, &c.

4. The reciprocal of any integer power of 10 is 1 preceded by a decimal point, and by one nought fewer than the original number contains. For example,

$$\frac{1}{10}=\cdot1;\ \frac{1}{100}=\cdot01;\ \frac{1}{1000}=\cdot001, \text{ \&c.}$$

5. The table gives the squares and cubes of all integer numbers consisting of three figures. To find the square and cube of any integer number consisting of two figures or one figure; annex one or two noughts, as the case may be; look for the number so formed in the left-hand column, take the square and cube opposite to it, and omit the noughts from the right of each of them. For example, to find the square and cube of 15; look for 150; then we find

Number.	Square.	Cube.
150	22500	3375000

from which, omitting the noughts, we obtain

15	225	3375

Again, to find the square and cube of 7, look for 700; then we find

Number.	Square.	Cube.
700	490000	343000000

from which, omitting the noughts, we obtain

7	49	343

6. To find the square and cube of a number consisting of three figures followed by noughts; find the square and cube opposite the first three figures in the table; annex twice as many noughts to the square, and thrice as many noughts to the cube. For example,

Number.	Square.	Cube.
377	142129	53582633
3770	14212900	53582633000
37700	1421290000	53582633000000

and so on.

7. The square and cube of a number consisting either wholly or partly of decimal fractions consist of the same figures as if the number were an integer; but the square contains twice as many, and the cube thrice as many places of decimals as the original number. The proper number of places is to be made up by prefixing noughts when required. For example,

Number.	Square.	Cube.
377	142129	53582633
37·7	1421·29	53582·633
3·77	14·2129	53·582633
·377	·142129	·053582633
·0377	·00142129	·000053582633

and so on.

8. The reciprocals given in the table are those of integers of three figures. For every nought that is annexed to the *right* of the original number, a nought is to be inserted at the *left* of the reciprocal; and for every place of decimals that is cut off at the *right* of the original number, the decimal point is to be shifted one place to the *right* in the reciprocal. For example,

Number.	Reciprocal.
160	·00625
1600	·000625
16000	·0000625

and so on;

16	·0625
1·6	·625
·16	6·25
·016	62·5
·0016	625

and so on.

9. The reciprocal of the reciprocal of a number is the original number itself. For example,

The reciprocal of 160 is ·00625
The reciprocal of ·00625 is 160

Hence, when convenient, the reciprocal of a number may sometimes be found by looking for the number in the column of reciprocals, and the reciprocal in the column of original numbers.

10. To reduce a vulgar fraction to a decimal fraction; multiply the reciprocal of the denominator of the vulgar fraction by the numerator. For example, to reduce 11-16ths to a decimal fraction;

Reciprocal of 16,....................	·0625
× Numerator,..........................	11
	·6875 *Answer.*

NOTE.—The only numbers whose reciprocals can be expressed *exactly* in decimal fractions are 2, 5, and their powers and products. Numbers divisible by any other prime factor give either repeating or circulating decimals as their reciprocals.

11. The square of the product of two numbers is the product of their squares; the cube of their product is the product of their cubes. For example,

$$1998^2 = (999 \times 2)^2 = 999^2 \times 2^2$$
$$= 998001 \times 4 = 3992004;$$
$$1998^3 = (999 \times 2)^3 = 999^3 \times 2^3$$
$$= 997002999 \times 8 = 7976023992.$$

12. To find the square or cube of a quotient or fraction; divide the square or cube of the dividend or numerator by the square or cube of the divisor or denominator. For example,

$$\left(\frac{999}{2}\right)^2 = \frac{999^2}{2^2} = \frac{998001}{4} = 249500{\cdot}25;$$
$$\left(\frac{999}{2}\right)^3 = \frac{999^3}{2^3} = \frac{997002999}{8} = 124625374{\cdot}875.$$

13. To find the square of the sum of two numbers; add together their squares and twice their product. For example, to find the square of $37725 = 37700 + 25$;

$$\begin{aligned} 37700^2 &= 1421290000 \\ 25^2 &= 625 \\ 37700 \times 25 \times 2 &= 1885000 \\ \hline 37725^2 &= 1423175625 \text{ } \textit{Sum.} \end{aligned}$$

14. To find the square of the difference of two numbers; from

the sum of their squares subtract twice their product. Example: to find the square of $37725 = 37800 - 75$;

$$\begin{array}{rrl} & 37800^2 = 1428840000 & \\ & 75^2 = \quad\quad 5625 & \\ & 1428845625 & \textit{Sum.} \\ 37800 \times 75 \times 2 & 5670000 & \textit{Subtracted.} \\ 37725^2 \text{ (as before)} & 1423175625 & \textit{Remainder.} \end{array}$$

15. To find the cube of the sum of two numbers; add together the cubes of the numbers and three times the square of each multiplied by the other.

For example, to find the cube of $37725 = 37700 + 25$;

$$\begin{array}{rr} 37700^3 = & 53582633000000 \\ 25^3 = & 15625 \\ 37700^2 \times 25 \times 3 & \\ = 1421290000 \times 25 \times 3 = & 106596750000 \\ 37700 \times 25^2 \times 3 = & 70687500 \\ 37725^3 = & 53689300453125 \ \textit{Sum.} \end{array}$$

16. To find the cube of the difference of two numbers; to the cube of each of them add three times its product by the square of the other; subtract the less of those sums from the greater. For example, to find the cube of $37725 = 37800 - 75$;

$$\begin{array}{rrl} 37800^3 = & 54010152000000 & \\ 37800 \times 75^2 \times 3 = & 637875000 & 54010789875000 \ \textit{Sum.} \\ 75^3 = & 421875 & \\ 75 \times 37800^2 \times 3 = & 321489000000 & 321489421875 \ \textit{Sum.} \\ 37725^3 \text{ (as before)} & & 53689300453125 \ \textit{Diff.} \end{array}$$

Extraction of Square and Cube Roots.

17. For convenience in the extraction of roots, the squares in the table are divided into *periods* of two figures, commencing at the right, the left-hand period sometimes containing one figure only; and the cubes are divided into periods of three figures, commencing at the right, the left-hand period sometimes containing two figures or one figure only. The number of periods in the square and the cube respectively is the same with the number of figures in the root, or original number; and should there be a decimal point between two figures of the root, the decimal points in the square and cube respectively are between the periods corresponding to those figures. (For examples, see Articles 6 and 7.)

18. To find the square root of an exact square of not more than

six figures; divide the given square into periods of two figures, beginning at the decimal point; look in the column of squares for the same figures, similarly divided into periods; the root will be opposite. Then place the decimal point so that the root shall have the same number of integer figures that the square has of integer periods.

19. To extract the approximate square root of a given number that is not an exact square, correct to three figures; divide the given number into periods of two figures, commencing at the decimal point; then look in the column of squares for the nearest square *that has the same left-hand period* with the given number; the root opposite that square will give the first three figures of the required root. Then place the decimal point as directed in Rule 18.

20. To extract the approximate square root of a given number having three periods of figures that is not an exact square, correct to five places of figures. For the first three figures, take the root of that square in the table which is next below the given number, *and has its left-hand period the same.* Subtract that square from the given number; annex two noughts to the remainder; then divide it by the sum of the three figures found and the next greater root in the table; the integer figures of the quotient will be the two additional figures of the approximate root. (Should there be but one integer figure in the quotient, insert a nought before it.)

Examples of Rules 18, 19, and 20.

I. Extract the square root of 1421·29. Divide this number into periods of two figures, thus, 14 21 ·29. Then amongst the squares in the table whose left-hand period is 14 is found 142129, the square of 377; so that the given number is an exact square. The decimal point coming between the second and third periods of the square shows that the decimal point comes between the second and third figures of the root; which is therefore 37·7.

II. Extract the approximate square root of 1423·18, correct to three figures. Divide the number into periods of two figures, thus, 14 23 ·18.

Given number,......................14 23 ·18
Nearest square of which the left-hand period is 14,..... } 14 21 ·29 = $37{\cdot}7^2$

Therefore 37·7 is the approximate root required.

III. Extract the approximate square root of 1423·18, correct to five figures;

Given number, in periods as before,	14 23 ·18
Next less square in the table,........	14 21 ·29 = $37{\cdot}7^2$
Divide by 377 + 378 = 755................	) 1 8900 *Diff.*

Quotient, being the two additional figures required, 25;
37·725, approximate root.

NOTE.—It is essential that the *left-hand period*, and not merely the left-hand figures, of the square in the table should agree with the given number; otherwise great errors will arise. In the examples given the same left-hand figures are found in 14161, the square of 119, as in the given number; but the left-hand period is only 1 instead of 14; and it would be a great error to take 119 as an approximation to the root required.

The same remark applies to the rules for extracting the cube root, now about to be given.

21. To find the cube root of an exact cube of not more than nine figures; divide the given cube into periods of three figures, beginning at the decimal point; look in the column of cubes for the same figures similarly divided into periods; the root will be opposite. Then place the decimal point so that the root shall have the same number of integer figures that the cube has of integer periods.

22. To extract the approximate cube root of a given number that is not an exact cube, correct to three figures; divide the given number into periods of three figures, commencing at the decimal point; then look in the column of cubes for the nearest cube *that has the same left-hand period* with the given number; the root opposite that square will be the required approximate root.

23. To extract the approximate cube root of a given number having three periods of figures that is not an exact cube, correct to five places of figures. For the first three figures, take the root of that cube in the table which is next below the given number, *and has its left-hand period the same*. Subtract that cube from the given number; annex two noughts to the remainder; then divide it by the three figures already found, by the same three figures plus one, and by 3; the integer figures of the quotient will be the two additional figures of the approximate root. (Should there be but one integer figure in the quotient, insert a nought before it).

EXAMPLES OF RULES 21, 22, and 23.

I. Extract the cube root of 53·582633. Divide the number into periods of three figures, beginning at the decimal point, thus, 53·582 633. Then amongst the cubes in the table whose left-hand period is 53 there is found 53 582 633, the cube of 377; so that the given number is an exact cube. The decimal point coming between

the first and second periods in the cube shows that the decimal point comes between the first and second periods in the root; which is therefore 3·77.

II. Extract the approximate cube root of 53·6893, correct to three figures. Divide the number into periods of three figures, thus, 53·689 300. Then we have,

Given number,...............	53· 689 300
Nearest cube of which the left-hand period is 53,...	53· 582 633 = $3{\cdot}77^3$

Therefore 3·77 is the approximate root required.

III. Extract the approximate cube root of 53·6893, correct to five figures.

Given number, in periods as before,...53· 689 300
Next less cube in the table,.............53· 582 633 = $3{\cdot}77^3$

Divide by 377) 106 667 00 *Diff.*
Divide by 378) 282 93
Divide by 3) 75

Quotient, being the two additional figures required, 25
3·7725, approximate root.

Use of Squares for Multiplication.

24. To multiply two numbers together by means of a table of squares.

Case I. If both numbers are odd, or both even; from the square of their half-sum subtract the square of their half-difference; the remainder will be the product required.

Case II. If one number is odd, and the other even; subtract 1 from the even number, so as to leave an odd remainder; multiply the first odd number and the odd remainder together as in Case I, and to their product add the first odd number; the sum will be the product required.

Example I.—Multiply together 377 and 591

Half-sum, $\frac{968}{2} = 484$; its square, 234256

Half-diff., $\frac{214}{2} = 107$; its square, 11449

Product required, 222807

Example II.—Multiply together 377 and 592.

377 × 591, by Case I. = 222807
Add 377

Product required, 223184

Common Logarithms.

25. The logarithm of 1 is 0.

26. The common logarithm of 10 is 1, and that of any power of 10 is the index of that power; in other words, it is equal to the number of noughts in the power; thus the common logarithm of 100 is 2; that of 1000, 3; and so on.

27. The common logarithm of ·1 is -1, and that of any power of ·1 is the index of that power with the negative sign; that is, it is equal to one more than the number of noughts between the decimal point and the figure 1, with the negative sign; for example, the common logarithm of ·01 is -2; that of ·001, -3; and so on.

28. The logarithms given in the table are merely the fractional parts of the logarithms, correct to five places of decimals, without the integral parts or *indices;* which are supplied in each case according to the following rules:—

The index of the common logarithm of a number not less than 1 is one less than the number of integer places of figures in that number; that is to say, for numbers less than 10 and not less than 1, the index is 0; for numbers less than 100 and not less than 10, the index is 1; for numbers less than 1000 and not less than 100, the index is 2; and so on.

The index of the common logarithm of a decimal fraction less than 1 is *negative*, and is one more than the number of noughts between the decimal point and the significant figures; and the negative sign is usually written above instead of before the index; that is to say, for numbers less than 1 and not less than ·1, the index is $\bar{1}$; for numbers less than ·1 and not less than ·01, the index is $\bar{2}$; and so on.

The fractional part of a common logarithm is always positive, and depends solely upon the series of figures of which the number consists, and not upon the place of the decimal point amongst them.

EXAMPLES.

Number.	Logarithm.
377000	5·57634
37700	4·57634
3770	3·57634
377	2·57634
37·7	1·57634
3·77	0·57634
·377	$\bar{1}$·57634
·0377	$\bar{2}$·57634
·00377	$\bar{3}$·57634

and so on.

29. The logarithm of a product is the sum of the logarithms of its factors.

30. The logarithm of a power is equal to the logarithm of the root multiplied by the index of the power.

31. The logarithm of a quotient is found by subtracting the logarithm of the divisor from the logarithm of the dividend.

32. The logarithm of a root is found by dividing the logarithm of one of its powers by the index of that power.

NOTE.—In applying the principles 29 and 31 to logarithms of numbers less than 1, it is to be observed that negative indices are to be subtracted instead of being added, and added instead of being subtracted.

33. To avoid the inconvenience which attends the use of negative indices to logarithms, it is a very common practice to put, instead of a negative index to the logarithm of a fraction, the *complement* (as it is called) of that index to 10; that is to say, 9 instead of $\bar{1}$, 8 instead of $\bar{2}$, 7 instead of $\bar{3}$, and so on. In such cases, it is always to be understood that each such complementary index has — 10 combined with it; and to prevent mistakes, it is useful to prefix — 10 + to it; for example,

Number.	Logarithm with Negative Index.	Logarithm with Complementary Index.
·377	$\bar{1}\cdot57634$	$-10+9\cdot57634$
·0377	$\bar{2}\cdot57634$	$-10+8\cdot57634$
·00377	$\bar{3}\cdot57634$	$-10+7\cdot57634$

34. To find the fractional part of the common logarithm of a number of five places of figures; take from the table the logarithm corresponding to the first three figures, and the difference between that logarithm and the next greater logarithm in the table; multiply that difference by the two remaining figures of the given number, and divide by 100; the quotient will be a correction, to be added to the logarithm already found.

EXAMPLE.—Find the common logarithm of 37725.

Log. 377,...........................	57634
Log. 378,...........................	57749
Difference,...........	115
	× 25 ÷ 100
Correction,...........	29
Add log. 377,......................	57634
Log. 37725,.......................	57663 *Answer.*

35. To find the natural number, or *antilogarithm*, corresponding to a common logarithm of five places of decimals, which is not in the table; find the next less, and the next greater logarithm in

the table, and take their difference. Opposite the next less logarithm will be the first three figures of the antilogarithm. Subtract the next less logarithm from the given logarithm; annex two noughts to the remainder, and divide by the before-mentioned difference; the quotient will give two additional figures of the required antilogarithm. (The first of those figures may be a nought.)

EXAMPLE.—Find the antilogarithm of the common logarithm ·57663.

Next less log. in table,...............	57634
Next greater,.........................	57749
Difference,......................	115
Given logarithm,.....................	57663
Subtract log. 377,....................	57634
Divide by difference,.........	115)2900
Two additional figures,...	25

so that the answer is 37725.

EXPLANATION OF TABLE 1 A AND TABLE 2.

Table 1 A, immediately following Table 1, gives the approximate square roots, cube roots, and reciprocals of the prime numbers from 2 to 97 inclusive; the roots to seven, and the reciprocals to nine places of decimals.

Table 2, following Table 1 A, gives the squares and fifth powers of numbers from 10 to 99 inclusive.

No.	Square.	Cube.	Reciprocal.	C. Log.
101	1 02 01	1 030 301	·009900990	00432
102	1 04 04	1 061 208	·009803922	00860
103	1 06 09	1 092 727	·009708738	01284
104	1 08 16	1 124 864	·009615385	01703
105	1 10 25	1 157 625	·009523810	02119
106	1 12 36	1 191 016	·009433962	02531
107	1 14 49	1 225 043	·009345794	02938
108	1 16 64	1 259 712	·009259259	03342
109	1 18 81	1 295 029	·009174312	03743
110	1 21 00	1 331 000	·009090909	04139
111	1 23 21	1 367 631	·009009009	04532
112	1 25 44	1 404 928	·008928571	04922
113	1 27 69	1 442 897	·008849558	05308
114	1 29 96	1 481 544	·008771930	05690
115	1 32 25	1 520 875	·008695652	06070
116	1 34 56	1 560 896	·008620690	06446
117	1 36 89	1 601 613	·008547009	06819
118	1 39 24	1 643 032	·008474576	07188
119	1 41 61	1 685 159	·008403361	07555
120	1 44 00	1 728 000	·008333333	07918
121	1 46 41	1 771 561	·008264463	08279
122	1 48 84	1 815 848	·008196721	08636
123	1 51 29	1 860 867	·008130081	08991
124	1 53 76	1 906 624	·008064516	09342
125	1 56 25	1 953 125	·008000000	09691
126	1 58 76	2 000 376	·007936508	10037
127	1 61 29	2 048 383	·007874016	10380
128	1 63 84	2 097 152	·007812500	10721
129	1 66 41	2 146 689	·007751938	11059
130	1 69 00	2 197 000	·007692308	11394
131	1 71 61	2 248 091	·007633588	11727
132	1 74 24	2 299 968	·007575758	12057
133	1 76 89	2 352 637	·007518797	12385
134	1 79 56	2 406 104	·007462687	12710
135	1 82 25	2 460 375	·007407407	13033
136	1 84 96	2 515 456	·007352941	13354
137	1 87 69	2 571 353	·007299270	13672
138	1 90 44	2 628 072	·007246377	13988
139	1 93 21	2 685 619	·007194245	14301
140	1 96 00	2 744 000	·007142857	14613
141	1 98 81	2 803 221	·007092199	14922
142	2 01 64	2 863 288	·007042254	15229
143	2 04 49	2 924 207	·006993007	15534
144	2 07 36	2 985 984	·006944444	15836
145	2 10 25	3 048 625	·006896552	16137

No.	Square.	Cube.	Reciprocal.	C. Log.
146	2 13 16	3 112 136	·006849315	16435
147	2 16 09	3 176 523	·006802721	16732
148	2 19 04	3 241 792	·006756757	17026
149	2 22 01	3 307 949	·006711409	17319
150	2 25 00	3 375 000	·006666667	17609
151	2 28 01	3 442 951	·006622517	17898
152	2 31 04	3 511 808	·006578947	18184
153	2 34 09	3 581 577	·006535948	18469
154	2 37 16	3 652 264	·006493506	18752
155	2 40 25	3 723 875	·006451613	19033
156	2 43 36	3 796 416	·006410256	19312
157	2 46 49	3 869 893	·006369427	19590
158	2 49 64	3 944 312	·006329114	19866
159	2 52 81	4 019 679	·006289308	20140
160	2 56 00	4 096 000	·006250000	20412
161	2 59 21	4 173 281	·006211180	20683
162	2 62 44	4 251 528	·006172840	20952
163	2 65 69	4 330 747	·006134969	21219
164	2 68 96	4 410 944	·006097561	21484
165	2 72 25	4 492 125	·006060606	21748
166	2 75 56	4 574 296	·006024096	22011
167	2 78 89	4 657 463	·005988024	22272
168	2 82 24	4 741 632	·005952381	22531
169	2 85 61	4 826 809	·005917160	22789
170	2 89 00	4 913 000	·005882353	23045
171	2 92 41	5 000 211	·005847953	23300
172	2 95 84	5 088 448	·005813953	23553
173	2 99 29	5 177 717	·005780347	23805
174	3 02 76	5 268 024	·005747126	24055
175	3 06 25	5 359 375	·005714286	24304
176	3 09 76	5 451 776	·005681818	24551
177	3 13 29	5 545 233	·005649718	24797
178	3 16 84	5 639 752	·005617978	25042
179	3 20 41	5 735 339	·005586592	25285
180	3 24 00	5 832 000	·005555556	25527
181	3 27 61	5 929 741	·005524862	25768
182	3 31 24	6 028 568	·005494505	26007
183	3 34 89	6 128 487	·005464481	26245
184	3 38 56	6 229 504	·005434783	26482
185	3 42 25	6 331 625	·005405405	26717
186	3 45 96	6 434 856	·005376344	26951
187	3 49 69	6 539 203	·005347594	27184
188	3 53 44	6 644 672	·005319149	27416
189	3 57 21	6 751 269	·005291005	27646
190	3 61 00	6 859 000	·005263158	27875

No.	Square.	Cube.	Reciprocal.	C. Log.
191	3 64 81	6 967 871	·005235602	28103
192	3 68 64	7 077 888	·005208333	28330
193	... 3 72 49	... 7 189 057	......·005181347	28556
194	3 76 36	7 301 384	·005154639	28780
195	3 80 25	7 414 875	·005128205	29003
196	... 3 84 16	... 7 529 536	......·005102041	29226
197	3 88 09	7 645 373	·005076142	29447
198	3 92 04	7 762 392	·005050505	29667
199	... 3 96 01	... 7 880 599	......·005025126	29885
200	4 00 00	8 000 000	·005000000	30103
201	4 04 01	8 120 601	·004975124	30320
202	... 4 08 04	... 8 242 408	......·004950495	30535
203	4 12 09	8 365 427	·004926108	30750
204	4 16 16	8 489 664	·004901961	30963
205	... 4 20 25	... 8 615 125	......·004878049	31175
206	4 24 36	8 741 816	·004854369	31387
207	4 28 49	8 869 743	·004830918	31597
208	... 4 32 64	... 8 998 912	......·004807692	31806
209	4 36 81	9 129 329	·004784689	32015
210	4 41 00	9 261 000	·004761905	32222
211	... 4 45 21	... 9 393 931	......·004739336	32428
212	4 49 44	9 528 128	·004716981	32634
213	4 53 69	9 663 597	·004694836	32838
214	... 4 57 96	... 9 800 344	......·004672897	33041
215	4 62 25	9 938 375	·004651163	33244
216	4 66 56	10 077 696	·004629630	33445
217	... 4 70 89	...10 218 313	......·004608295	33646
218	4 75 24	10 360 232	·004587156	33846
219	4 79 61	10 503 459	·004566210	34044
220	... 4 84 00	...10 648 000	......·004545455	34242
221	4 88 41	10 793 861	·004524887	34439
222	4 92 84	10 941 048	·004504505	34635
223	... 4 97 29	...11 089 567	......·004484305	34830
224	5 01 76	11 239 424	·004464286	35025
225	5 06 25	11 390 625	·004444444	35218
226	... 5 10 76	...11 543 176	......·004424779	35411
227	5 15 29	11 697 083	·004405286	35603
228	5 19 84	11 852 352	·004385965	35793
229	... 5 24 41	...12 008 989	......·004366812	35984
230	5 29 00	12 167 000	·004347826	36173
231	5 33 61	12 326 391	·004329004	36361
232	... 5 38 24	...12 487 168	......·004310345	36549
233	5 42 89	12 649 337	·004291845	36736
234	5 47 56	12 812 904	·004273504	36922
235	5 52 25	12 977 875	·004255319	37107

No.	Square.	Cube.	Reciprocal.	C. Log.
236	5 56 96	13 144 256	·004237288	37291
237	5 61 69	13 312 053	·004219409	37475
238	... 5 66 44	...13 481 272	......·004201681	37658
239	5 71 21	13 651 919	·004184100	37840
240	5 76 00	13 824 000	·004166667	38021
241	... 5 80 81	...13 997 521	......·004149378	38202
242	5 85 64	14 172 488	·004132231	38382
243	5 90 49	14 348 907	·004115226	38561
244	... 5 95 36	...14 526 784	......·004098361	38739
245	6 00 25	14 706 125	·004081633	38917
246	6 05 16	14 886 936	·004065041	39094
247	... 6 10 09	...15 069 223	......·004048583	39270
248	6 15 04	15 252 992	·004032258	39445
249	6 20 01	15 438 249	·004016064	39620
250	... 6 25 00	...15 625 000	......·004000000	39794
251	6 30 01	15 813 251	·003984064	39967
252	6 35 04	16 003 008	·003968254	40140
253	... 6 40 09	...16 194 277	......·003952569	40312
254	6 45 16	16 387 064	·003937008	40483
255	6 50 25	16 581 375	·003921569	40654
256	... 6 55 36	...16 777 216	......·003906250	40824
257	6 60 49	16 974 593	·003891051	40993
258	6 65 64	17 173 512	·003875969	41162
259	... 6 70 81	...17 373 979	......·003861004	41330
260	6 76 00	17 576 000	·003846154	41497
261	6 81 21	17 779 581	·003831418	41664
262	... 6 86 44	...17 984 728	......·003816794	41830
263	6 91 69	18 191 447	·003802281	41996
264	6 96 96	18 399 744	·003787879	42160
265	... 7 02 25	...18 609 625	......·003773585	42325
266	7 07 56	18 821 096	·003759398	42488
267	7 12 89	19 034 163	·003745318	42651
268	... 7 18 24	...19 248 832	......·003731343	42813
269	7 23 61	19 465 109	·003717472	42975
270	7 29 00	19 683 000	·003703704	43136
271	... 7 34 41	...19 902 511	......·003690037	43297
272	7 39 84	20 123 648	·003676471	43457
273	7 45 29	20 346 417	·003663004	43616
274	... 7 50 76	...20 570 824	......·003649635	43775
275	7 56 25	20 796 875	·003636364	43933
276	7 61 76	21 024 576	·003623188	44091
277	... 7 67 29	...21 253 933	......·003610108	44248
278	7 72 84	21 484 952	·003597122	44404
279	7 78 41	21 717 639	·003584229	44560
280	7 84 00	21 952 000	·003571429	44716

No.	Square.	Cube.	Reciprocal.	C. Log.
281	7 89 61	22 188 041	·003558719	44871
282	7 95 24	22 425 768	·003546099	45025
283	... 8 00 89	...22 665 187	......·003533569	45179
284	8 06 56	22 906 304	·003521127	45332
285	8 12 25	23 149 125	·003508772	45484
286	... 8 17 96	...23 393 656	......·003496503	45637
287	8 23 69	23 639 903	·003484321	45788
288	8 29 44	23 887 872	·003472222	45939
289	... 8 35 21	...24 137 569	......·003460208	46090
290	8 41 00	24 389 000	·003448276	46240
291	8 46 81	24 642 171	·003436426	46389
292	... 8 52 64	...24 897 088	......·003424658	46538
293	8 58 49	25 153 757	·003412969	46687
294	8 64 36	25 412 184	·003401361	46835
295	... 8 70 25	...25 672 375	......·003389831	46982
296	8 76 16	25 934 336	·003378378	47129
297	8 82 09	26 198 073	·003367003	47276
298	... 8 88 04	...26 463 592	......·003355705	47422
299	8 94 01	26 730 899	·003344482	47567
300	9 00 00	27 000 000	·003333333	47712
301	... 9 06 01	...27 270 901	......·003322259	47857
302	9 12 04	27 543 608	·003311258	48001
303	9 18 09	27 818 127	·003300330	48144
304	... 9 24 16	...28 094 464	......·003289474	48287
305	9 30 25	28 372 625	·003278689	48430
306	9 36 36	28 652 616	·003267974	48572
307	... 9 42 49	...28 934 443	......·003257329	48714
308	9 48 64	29 218 112	·003246753	48855
309	9 54 81	29 503 629	·003236246	48996
310	... 9 61 00	...29 791 000	......·003225806	49136
311	9 67 21	30 080 231	·003215434	49276
312	9 73 44	30 371 328	·003205128	49415
313	... 9 79 69	...30 664 297	......·003194888	49554
314	9 85 96	30 959 144	·003184713	49693
315	9 92 25	31 255 875	·003174603	49831
316	... 9 98 56	...31 554 496	......·003164557	49969
317	10 04 89	31 855 013	·003154574	50106
318	10 11 24	32 157 432	·003144654	50243
319	... 10 17 61	...32 461 759	......·003134796	50379
320	10 24 00	32 768 000	·003125000	50515
321	10 30 41	33 076 161	·003115265	50651
322	... 10 36 84	...33 386 248	......·003105590	50786
323	10 43 29	33 698 267	·003095975	50920
324	10 49 76	34 012 224	·003086420	51055
325	10 56 25	34 328 125	·003076923	51188

No.	Square.	Cube.	Reciprocal.	C. Log.
326	10 62 76	34 645 976	·003067485	51322
327	10 69 29	34 965 783	·003058104	51455
328	... 10 75 84	...35 287 552	......·003048780	51587
329	10 82 41	35 611 289	·003039514	51720
330	10 89 00	35 937 000	·003030303	51851
331	... 10 95 61	...36 264 691	......·003021148	51983
332	11 02 24	36 594 368	·003012048	52114
333	11 08 89	36 926 037	·003003003	52244
334	... 11 15 56	...37 259 704	......·002994012	52375
335	11 22 25	37 595 375	·002985075	52504
336	11 28 96	37 933 056	·002976190	52634
337	... 11 35 69	...38 272 753	......·002967359	52763
338	11 42 44	38 614 472	·002958580	52892
339	11 49 21	38 958 219	·002949853	53020
340	... 11 56 00	...39 304 000	......·002941176	53148
341	11 62 81	39 651 821	·002932551	53275
342	11 69 64	40 001 688	·002923977	53403
343	... 11 76 49	...40 353 607	......·002915452	53529
344	11 83 36	40 707 584	·002906977	53656
345	11 90 25	41 063 625	·002898551	53782
346	... 11 97 16	...41 421 736	......·002890173	53908
347	12 04 09	41 781 923	·002881844	54033
348	12 11 04	42 144 192	·002873563	54158
349	... 12 18 01	...42 508 549	......·002865330	54283
350	12 25 00	42 875 000	·002857143	54407
351	12 32 01	43 243 551	·002849003	54531
352	... 12 39 04	...43 614 208	......·002840909	54654
353	12 46 09	43 986 977	·002832861	54777
354	12 53 16	44 361 864	·002824859	54900
355	... 12 60 25	...44 738 875	......·002816901	55023
356	12 67 36	45 118 016	·002808989	55145
357	12 74 49	45 499 293	·002801120	55267
358	... 12 81 64	...45 882 712	......·002793296	55388
359	12 88 81	46 268 279	·002785515	55509
360	12 96 00	46 656 000	·002777778	55630
361	... 13 03 21	...47 045 881	......·002770083	55751
362	13 10 44	47 437 928	·002762431	55871
363	13 17 69	47 832 147	·002754821	55991
364	... 13 24 96	...48 228 544	......·002747253	56110
365	13 32 25	48 627 125	·002739726	56229
366	13 39 56	49 027 896	·002732240	56348
367	... 13 46 89	...49 430 863	......·002724796	56467
368	13 54 24	49 836 032	·002717391	56585
369	13 61 61	50 243 409	·002710027	56703
370	13 69 00	50 653 000	·002702703	56820

No.	Square.	Cube.	Reciprocal.	C. Log.
371	13 76 41	51 064 811	·002695418	56937
372	13 83 84	51 478 848	·002688172	57054
373	... 13 91 29	...51 895 117	......·002680965	57171
374	13 98 76	52 313 624	·002673797	57287
375	14 06 25	52 734 375	·002666667	57403
376	... 14 13 76	...53 157 376	......·002659574	57519
377	14 21 29	53 582 633	·002652520	57634
378	14 28 84	54 010 152	·002645503	57749
379	... 14 36 41	...54 439 939	......·002638522	57864
380	14 44 00	54 872 000	·002631579	57978
381	14 51 61	55 306 341	·002624672	58092
382	... 14 59 24	...55 742 968	......·002617801	58206
383	14 66 89	56 181 887	·002610966	58320
384	14 74 56	56 623 104	·002604167	58433
385	... 14 82 25	...57 066 625	......·002597403	58546
386	14 89 96	57 512 456	·002590674	58659
387	14 97 69	57 960 603	·002583979	58771
388	... 15 05 44	...58 411 072	......·002577320	58883
389	15 13 21	58 863 869	·002570694	58995
390	15 21 00	59 319 000	·002564103	59106
391	... 15 28 81	...59 776 471	......·002557545	59218
392	15 36 64	60 236 288	·002551020	59329
393	15 44 49	60 698 457	·002544529	59439
394	... 15 52 36	...61 162 984	......·002538071	59550
395	15 60 25	61 629 875	·002531646	59660
396	15 68 16	62 099 136	·002525253	59770
397	... 15 76 09	...62 570 773	......·002518892	59879
398	15 84 04	63 044 792	·002512563	59988
399	15 92 01	63 521 199	·002506266	60097
400	... 16 00 00	...64 000 000	......·002500000	60206
401	16 08 01	64 481 201	·002493766	60314
402	16 16 04	64 964 808	·002487562	60423
403	... 16 24 09	...65 450 827	......·002481390	60531
404	16 32 16	65 939 264	·002475248	60638
405	16 40 25	66 430 125	·002469136	60746
406	... 16 48 36	...66 923 416	......·002463054	60853
407	16 56 49	67 419 143	·002457002	60959
408	16 64 64	67 917 312	·002450980	61066
409	... 16 72 81	...68 417 929	......·002444988	61172
410	16 81 00	68 921 000	·002439024	61278
411	16 89 21	69 426 531	·002433090	61384
412	... 16 97 44	...69 934 528	......·002427184	61490
413	17 05 69	70 444 997	·002421308	61595
414	17 13 96	70 957 944	·002415459	61700
415	17 22 25	71 473 375	·002409639	61805

No.	Square.	Cube.	Reciprocal.	C. Log.
416	17 30 56	71 991 296	·002403846	61909
417	17 38 89	72 511 713	·002398082	62014
418	17 47 24	73 034 632	·002392344	62118
419	17 55 61	73 560 059	·002386635	62221
420	17 64 00	74 088 000	·002380952	62325
421	17 72 41	74 618 461	·002375297	62428
422	17 80 84	75 151 448	·002369668	62531
423	17 89 29	75 686 967	·002364066	62634
424	17 97 76	76 225 024	·002358491	62737
425	18 06 25	76 765 625	·002352941	62839
426	18 14 76	77 308 776	·002347418	62941
427	18 23 29	77 854 483	·002341920	63043
428	18 31 84	78 402 752	·002336449	63144
429	18 40 41	78 953 589	·002331002	63246
430	18 49 00	79 507 000	·002325581	63347
431	18 57 61	80 062 991	·002320186	63448
432	18 66 24	80 621 568	·002314815	63548
433	18 74 89	81 182 737	·002309469	63649
434	18 83 56	81 746 504	·002304147	63749
435	18 92 25	82 312 875	·002298851	63849
436	19 00 96	82 881 856	·002293578	63949
437	19 09 69	83 453 453	·002288330	64048
438	19 18 44	84 027 672	·002283105	64147
439	19 27 21	84 604 519	·002277904	64246
440	19 36 00	85 184 000	·002272727	64345
441	19 44 81	85 766 121	·002267574	64444
442	19 53 64	86 350 888	·002262443	64542
443	19 62 49	86 938 307	·002257336	64640
444	19 71 36	87 528 384	·002252252	64738
445	19 80 25	88 121 125	·002247191	64836
446	19 89 16	88 716 536	·002242152	64933
447	19 98 09	89 314 623	·002237136	65031
448	20 07 04	89 915 392	·002232143	65128
449	20 16 01	90 518 849	·002227171	65225
450	20 25 00	91 125 000	·002222222	65321
451	20 34 01	91 733 851	·002217295	65418
452	20 43 04	92 345 408	·002212389	65514
453	20 52 09	92 959 677	·002207506	65610
454	20 61 16	93 576 664	·002202643	65706
455	20 70 25	94 196 375	·002197802	65801
456	20 79 36	94 818 816	·002192982	65896
457	20 88 49	95 443 993	·002188184	65992
458	20 97 64	96 071 912	·002183406	66087
459	21 06 81	96 702 579	·002178649	66181
460	21 16 00	97 336 000	·002173913	66276

No.	Square.	Cube.	Reciprocal.	C. Log.
461	21 25 21	97 972 181	·002169197	66370
462	21 34 44	98 611 128	·002164502	66464
463	21 43 69	99 252 847	·002159827	66558
464	21 52 96	99 897 344	·002155172	66652
465	21 62 25	100 544 625	·002150538	66745
466	21 71 56	101 194 696	·002145923	66839
467	21 80 89	101 847 563	·002141328	66932
468	21 90 24	102 503 232	·002136752	67025
469	21 99 61	103 161 709	·002132196	67117
470	22 09 00	103 823 000	·002127660	67210
471	22 18 41	104 487 111	·002123142	67302
472	22 27 84	105 154 048	·002118644	67394
473	22 37 29	105 823 817	·002114165	67486
474	22 46 76	106 496 424	·002109705	67578
475	22 56 25	107 171 875	·002105263	67669
476	22 65 76	107 850 176	·002100840	67761
477	22 75 29	108 531 333	·002096436	67852
478	22 84 84	109 215 352	·002092050	67943
479	22 94 41	109 902 239	·002087683	68034
480	23 04 00	110 592 000	·002083333	68124
481	23 13 61	111 284 641	·002079002	68215
482	23 23 24	111 980 168	·002074689	68305
483	23 32 89	112 678 587	·002070393	68395
484	23 42 56	113 379 904	·002066116	68485
485	23 52 25	114 084 125	·002061856	68574
486	23 61 96	114 791 256	·002057613	68664
487	23 71 69	115 501 303	·002053388	68753
488	23 81 44	116 214 272	·002049180	68842
489	23 91 21	116 930 169	·002044990	68931
490	24 01 00	117 649 000	·002040816	69020
491	24 10 81	118 370 771	·002036660	69108
492	24 20 64	119 095 488	·002032520	69197
493	24 30 49	119 823 157	·002028398	69285
494	24 40 36	120 553 784	·002024291	69373
495	24 50 25	121 287 375	·002020202	69461
496	24 60 16	122 023 936	·002016129	69548
497	24 70 09	122 763 473	·002012072	69636
498	24 80 04	123 505 992	·002008032	69723
499	24 90 01	124 251 499	·002004008	69810
500	25 00 00	125 000 000	·002000000	69897
501	25 10 01	125 751 501	·001996008	69984
502	25 20 04	126 506 008	·001992032	70070
503	25 30 09	127 263 527	·001988072	70157
504	25 40 16	128 024 064	·001984127	70243
505	25 50 25	128 787 625	·001980198	70329

No.	Square.	Cube.	Reciprocal.	C. Log.
506	25 60 36	129 554 216	·001976285	70415
507	25 70 49	130 323 843	·001972387	70501
508	25 80 64	131 096 512	·001968504	70586
509	25 90 81	131 872 229	·001964637	70672
510	26 01 00	132 651 000	·001960784	70757
511	26 11 21	133 432 831	·001956947	70842
512	26 21 44	134 217 728	·001953125	70927
513	26 31 69	135 005 697	·001949318	71012
514	26 41 96	135 796 744	·001945525	71096
515	26 52 25	136 590 875	·001941748	71181
516	26 62 56	137 388 096	·001937984	71265
517	26 72 89	138 188 413	·001934236	71349
518	26 83 24	138 991 832	·001930502	71433
519	26 93 61	139 798 359	·001926782	71517
520	27 04 00	140 608 000	·001923077	71600
521	27 14 41	141 420 761	·001919386	71684
522	27 24 84	142 236 648	·001915709	71767
523	27 35 29	143 055 667	·001912046	71850
524	27 45 76	143 877 824	·001908397	71933
525	27 56 25	144 703 125	·001904762	72016
526	27 66 76	145 531 576	·001901141	72099
527	27 77 29	146 363 183	·001897533	72181
528	27 87 84	147 197 952	·001893939	72263
529	27 98 41	148 035 889	·001890359	72346
530	28 09 00	148 877 000	·001886792	72428
531	28 19 61	149 721 291	·001883239	72509
532	28 30 24	150 568 768	·001879699	72591
533	28 40 89	151 419 437	·001876173	72673
534	28 51 56	152 273 304	·001872659	72754
535	28 62 25	153 130 375	·001869159	72835
536	28 72 96	153 990 656	·001865672	72916
537	28 83 69	154 854 153	·001862197	72997
538	28 94 44	155 720 872	·001858736	73078
539	29 05 21	156 590 819	·001855288	73159
540	29 16 00	157 464 000	·001851852	73239
541	29 26 81	158 340 421	·001848429	73320
542	29 37 64	159 220 088	·001845018	73400
543	29 48 49	160 103 007	·001841621	73480
544	29 59 36	160 989 184	·001838235	73560
545	29 70 25	161 878 625	·001834862	73640
546	29 81 16	162 771 336	·001831502	73719
547	29 92 09	163 667 323	·001828154	73799
548	30 03 04	164 566 592	·001824818	73878
549	30 14 01	165 469 149	·001821494	73957
550	30 25 00	166 375 000	·001818182	74036

No.	Square.	Cube.	Reciprocal.	C. Log.
551	30 36 01	167 284 151	·001814882	74115
552	30 47 04	168 196 608	·001811594	74194
553	30 58 09	169 112 377	·001808318	74273
554	30 69 16	170 031 464	·001805054	74351
555	30 80 25	170 953 875	·001801802	74429
556	30 91 36	171 879 616	·001798561	74507
557	31 02 49	172 808 693	·001795332	74586
558	31 13 64	173 741 112	·001792115	74663
559	31 24 81	174 676 879	·001788909	74741
560	31 36 00	175 616 000	·001785714	74819
561	31 47 21	176 558 481	·001782531	74896
562	31 58 44	177 504 328	·001779359	74974
563	31 69 69	178 453 547	·001776199	75051
564	31 80 96	179 406 144	·001773050	75128
565	31 92 25	180 362 125	·001769912	75205
566	32 03 56	181 321 496	·001766784	75282
567	32 14 89	182 284 263	·001763668	75358
568	32 26 24	183 250 432	·001760563	75435
569	32 37 61	184 220 009	·001757469	75511
570	32 49 00	185 193 000	·001754386	75587
571	32 60 41	186 169 411	·001751313	75664
572	32 71 84	187 149 248	·001748252	75740
573	32 83 29	188 132 517	·001745201	75815
574	32 94 76	189 119 224	·001742160	75891
575	33 06 25	190 109 375	·001739130	75967
576	33 17 76	191 102 976	·001736111	76042
577	33 29 29	192 100 033	·001733102	76118
578	33 40 84	193 100 552	·001730104	76193
579	33 52 41	194 104 539	·001727116	76268
580	33 64 00	195 112 000	·001724138	76343
581	33 75 61	196 122 941	·001721170	76418
582	33 87 24	197 137 368	·001718213	76492
583	33 98 89	198 155 287	·001715266	76567
584	34 10 56	199 176 704	·001712329	76641
585	34 22 25	200 201 625	·001709402	76716
586	34 33 96	201 230 056	·001706485	76790
587	34 45 69	202 262 003	·001703578	76864
588	34 57 44	203 297 472	·001700680	76938
589	34 69 21	204 336 469	·001697793	77012
590	34 81 00	205 379 000	·001694915	77085
591	34 92 81	206 425 071	·001692047	77159
592	35 04 64	207 474 688	·001689189	77232
593	35 16 49	208 527 857	·001686341	77305
594	35 28 36	209 584 584	·001683502	77379
595	35 40 25	210 644 875	·001680672	77452

No.	Square.	Cube.	Reciprocal.	C. Log.
596	35 52 16	211 708 736	·001677852	77525
597	35 64 09	212 776 173	·001675042	77597
598	35 76 04	213 847 192	·001672241	77670
599	35 88 01	214 921 799	·001669449	77743
600	36 00 00	216 000 000	·001666667	77815
601	36 12 01	217 081 801	·001663894	77887
602	36 24 04	218 167 208	·001661130	77960
603	36 36 09	219 256 227	·001658375	78032
604	36 48 16	220 348 864	·001655629	78104
605	36 60 25	221 445 125	·001652893	78176
606	36 72 36	222 545 016	·001650165	78247
607	36 84 49	223 648 543	·001647446	78319
608	36 96 64	224 755 712	·001644737	78390
609	37 08 81	225 866 529	·001642036	78462
610	37 21 00	226 981 000	·001639344	78533
611	37 33 21	228 099 131	·001636661	78604
612	37 45 44	229 220 928	·001633987	78675
613	37 57 69	230 346 397	·001631321	78746
614	37 69 96	231 475 544	·001628664	78817
615	37 82 25	232 608 375	·001626016	78888
616	37 94 56	233 744 896	·001623377	78958
617	38 06 89	234 885 113	·001620746	79029
618	38 19 24	236 029 032	·001618123	79099
619	38 31 61	237 176 659	·001615509	79169
620	38 44 00	238 328 000	·001612903	79239
621	38 56 41	239 483 061	·001610306	79309
622	38 68 84	240 641 848	·001607717	79379
623	38 81 29	241 804 367	·001605136	79449
624	38 93 76	242 970 624	·001602564	79518
625	39 06 25	244 140 625	·001600000	79588
626	39 18 76	245 314 376	·001597444	79657
627	39 31 29	246 491 883	·001594896	79727
628	39 43 84	247 673 152	·001592357	79796
629	39 56 41	248 858 189	·001589825	79865
630	39 69 00	250 047 000	·001587302	79934
631	39 81 61	251 239 591	·001584786	80003
632	39 94 24	252 435 968	·001582278	80072
633	40 06 89	253 636 137	·001579779	80140
634	40 19 56	254 840 104	·001577287	80209
635	40 32 25	256 047 875	·001574803	80277
636	40 44 96	257 259 456	·001572327	80346
637	40 57 69	258 474 853	·001569859	80414
638	40 70 44	259 694 072	·001567398	80482
639	40 83 21	260 917 119	·001564945	80550
640	40 96 00	262 144 000	·001562500	80618

No.	Square.	Cube.	Reciprocal.	C. Log.
641	41 08 81	263 374 721	·001560062	80686
642	41 21 64	264 609 288	·001557632	80754
643	... 41 34 49	...265 847 707	......·001555210	80821
644	41 47 36	267 089 984	·001552795	80889
645	41 60 25	268 336 125	·001550388	80956
646	... 41 73 16	...269 586 136	......·001547988	81023
647	41 86 09	270 840 023	·001545595	81090
648	41 99 04	272 097 792	·001543210	81158
649	... 42 12 01	...273 359 449	......·001540832	81224
650	42 25 00	274 625 000	·001538462	81291
651	42 38 01	275 894 451	·001536098	81358
652	... 42 51 04	...277 167 808	......·001533742	81425
653	42 64 09	278 445 077	·001531394	81491
654	42 77 16	279 726 264	·001529052	81558
655	... 42 90 25	...281 011 375	......·001526718	81624
656	43 03 36	282 300 416	·001524390	81690
657	43 16 49	283 593 393	·001522070	81757
658	... 43 29 64	...284 890 312	......·001519757	81823
659	43 42 81	286 191 179	·001517451	81889
660	43 56 00	287 496 000	·001515152	81954
661	... 43 69 21	...288 804 781	......·001512859	82020
662	43 82 44	290 117 528	·001510574	82086
663	43 95 69	291 434 247	·001508296	82151
664	... 44 08 96	...292 754 944	......·001506024	82217
665	44 22 25	294 079 625	·001503759	82282
666	44 35 56	295 408 296	·001501502	82347
667	... 44 48 89	...296 740 963	......·001499250	82413
668	44 62 24	298 077 632	·001497006	82478
669	44 75 61	299 418 309	·001494768	82543
670	... 44 89 00	...300 763 000	......·001492537	82607
671	45 02 41	302 111 711	·001490313	82672
672	45 15 84	303 464 448	·001488095	82737
673	... 45 29 29	...304 821 217	......·001485884	82802
674	45 42 76	306 182 024	·001483680	82866
675	45 56 25	307 546 875	·001481481	82930
676	... 45 69 76	...308 915 776	......·001479290	82995
677	45 83 29	310 288 733	·001477105	83059
678	45 96 84	311 665 752	·001474926	83123
679	... 46 10 41	...313 046 839	......·001472754	83187
680	46 24 00	314 432 000	·001470588	83251
681	46 37 61	315 821 241	·001468429	83315
682	... 46 51 24	...317 214 568	......·001466276	83378
683	46 64 89	318 611 987	·001464129	83442
684	46 78 56	320 013 504	·001461988	83506
685	46 92 25	321 419 125	·001459854	83569

No.	Square.	Cube.	Reciprocal.	C. Log.
686	47 05 96	322 828 856	·001457726	83632
687	47 19 69	324 242 703	·001455604	83696
688	47 33 44	325 660 672	·001453488	83759
689	47 47 21	327 082 769	·001451379	83822
690	47 61 00	328 509 000	·001449275	83885
691	47 74 81	329 939 371	·001447178	83948
692	47 88 64	331 373 888	·001445087	84011
693	48 02 49	332 812 557	·001443001	84073
694	48 16 36	334 255 384	·001440922	84136
695	48 30 25	335 702 375	·001438849	84198
696	48 44 16	337 153 536	·001436782	84261
697	48 58 09	338 608 873	·001434720	84323
698	48 72 04	340 068 392	·001432665	84386
699	48 86 01	341 532 099	·001430615	84448
700	49 00 00	343 000 000	·001428571	84510
701	49 14 01	344 472 101	·001426534	84572
702	49 28 04	345 948 408	·001424501	84634
703	49 42 09	347 428 927	·001422475	84696
704	49 56 16	348 913 664	·001420455	84757
705	49 70 25	350 402 625	·001418440	84819
706	49 84 36	351 895 816	·001416431	84880
707	49 98 49	353 393 243	·001414427	84942
708	50 12 64	354 894 912	·001412429	85003
709	50 26 81	356 400 829	·001410437	85065
710	50 41 00	357 911 000	·001408451	85126
711	50 55 21	359 425 431	·001406470	85187
712	50 69 44	360 944 128	·001404494	85248
713	50 83 69	362 467 097	·001402525	85309
714	50 97 96	363 994 344	·001400560	85370
715	51 12 25	365 525 875	·001398601	85431
716	51 26 56	367 061 696	·001396648	85491
717	51 40 89	368 601 813	·001394700	85552
718	51 55 24	370 146 232	·001392758	85612
719	51 69 61	371 694 959	·001390821	85673
720	51 84 00	373 248 000	·001388889	85733
721	51 98 41	374 805 361	·001386963	85794
722	52 12 84	376 367 048	·001385042	85854
723	52 27 29	377 933 067	·001383126	85914
724	52 41 76	379 503 424	·001381215	85974
725	52 56 25	381 078 125	·001379310	86034
726	52 70 76	382 657 176	·001377410	86094
727	52 85 29	384 240 583	·001375516	86153
728	52 99 84	385 828 352	·001373626	86213
729	53 14 41	387 420 489	·001371742	86273
730	53 29 00	389 017 000	·001369863	86332

No.	Square.	Cube.	Reciprocal.	C. Log.
731	53 43 61	390 617 891	·001367989	86392
732	53 58 24	392 223 168	·001366120	86451
733	53 72 89	393 832 837	·001364256	86510
734	53 87 56	395 446 904	·001362398	86570
735	54 02 25	397 065 375	·001360544	86629
736	54 16 96	398 688 256	·001358696	86688
737	54 31 69	400 315 553	·001356852	86747
738	54 46 44	401 947 272	·001355014	86806
739	54 61 21	403 583 419	·001353180	86864
740	54 76 00	405 224 000	·001351351	86923
741	54 90 81	406 869 021	·001349528	86982
742	55 05 64	408 518 488	·001347709	87040
743	55 20 49	410 172 407	·001345895	87099
744	55 35 36	411 830 784	·001344086	87157
745	55 50 25	413 493 625	·001342282	87216
746	55 65 16	415 160 936	·001340483	87274
747	55 80 09	416 832 723	·001338688	87332
748	55 95 04	418 508 992	·001336898	87390
749	56 10 01	420 189 749	·001335113	87448
750	56 25 00	421 875 000	·001333333	87506
751	56 40 01	423 564 751	·001331558	87564
752	56 55 04	425 259 008	·001329787	87622
753	56 70 09	426 957 777	·001328021	87679
754	56 85 16	428 661 064	·001326260	87737
755	57 00 25	430 368 875	·001324503	87795
756	57 15 36	432 081 216	·001322751	87852
757	57 30 49	433 798 093	·001321004	87910
758	57 45 64	435 519 512	·001319261	87967
759	57 60 81	437 245 479	·001317523	88024
760	57 76 00	438 976 000	·001315789	88081
761	57 91 21	440 711 081	·001314060	88138
762	58 06 44	442 450 728	·001312336	88195
763	58 21 69	444 194 947	·001310616	88252
764	58 36 96	445 943 744	·001308901	88309
765	58 52 25	447 697 125	·001307190	88366
766	58 67 56	449 455 096	·001305483	88423
767	58 82 89	451 217 663	·001303781	88480
768	58 98 24	452 984 832	·001302083	88536
769	59 13 61	454 756 609	·001300390	88593
770	59 29 00	456 533 000	·001298701	88649
771	59 44 41	458 314 011	·001297017	88705
772	59 59 84	460 099 648	·001295337	88762
773	59 75 29	461 889 917	·001293661	88818
774	59 90 76	463 684 824	·001291990	88874
775	60 06 25	465 484 375	·001290323	88930

No.	Square.	Cube.	Reciprocal.	C. Log.
776	60 21 76	467 288 576	·001288660	88986
777	60 37 29	469 097 433	·001287001	89042
778	60 52 84	470 910 952	·001285347	89098
779	60 68 41	472 729 139	·001283697	89154
780	60 84 00	474 552 000	·001282051	89209
781	60 99 61	476 379 541	·001280410	89265
782	61 15 24	478 211 768	·001278772	89321
783	61 30 89	480 048 687	·001277139	89376
784	61 46 56	481 890 304	·001275510	89432
785	61 62 25	483 736 625	·001273885	89487
786	61 77 96	485 587 656	·001272265	89542
787	61 93 69	487 443 403	·001270648	89597
788	62 09 44	489 303 872	·001269036	89653
789	62 25 21	491 169 069	·001267427	89708
790	62 41 00	493 039 000	·001265823	89763
791	62 56 81	494 913 671	·001264223	89818
792	62 72 64	496 793 088	·001262626	89873
793	62 88 49	498 677 257	·001261034	89927
794	63 04 36	500 566 184	·001259446	89982
795	63 20 25	502 459 875	·001257862	90037
796	63 36 16	504 358 336	·001256281	90091
797	63 52 09	506 261 573	·001254705	90146
798	63 68 04	508 169 592	·001253133	90200
799	63 84 01	510 082 399	·001251564	90255
800	64 00 00	512 000 000	·001250000	90309
801	64 16 01	513 922 401	·001248439	90363
802	64 32 04	515 849 608	·001246883	90417
803	64 48 09	517 781 627	·001245330	90472
804	64 64 16	519 718 464	·001243781	90526
805	64 80 25	521 660 125	·001242236	90580
806	64 96 36	523 606 616	·001240695	90634
807	65 12 49	525 557 943	·001239157	90687
808	65 28 64	527 514 112	·001237624	90741
809	65 44 81	529 475 129	·001236094	90795
810	65 61 00	531 441 000	·001234568	90849
811	65 77 21	533 411 731	·001233046	90902
812	65 93 44	535 387 328	·001231527	90956
813	66 09 69	537 367 797	·001230012	91009
814	66 25 96	539 353 144	·001228501	91062
815	66 42 25	541 343 375	·001226994	91116
816	66 58 56	543 338 496	·001225490	91169
817	66 74 89	545 338 513	·001223990	91222
818	66 91 24	547 343 432	·001222494	91275
819	67 07 61	549 353 259	·001221001	91328
820	67 24 00	551 368 000	·001219512	91381

No.	Square.	Cube.	Reciprocal.	C. Log.
821	67 40 41	553 387 661	·001218027	91434
822	67 56 84	555 412 248	·001216545	91487
823	67 73 29	557 441 767	·001215067	91540
824	67 89 76	559 476 224	·001213592	91593
825	68 06 25	561 515 625	·001212121	91645
826	68 22 76	563 559 976	·001210654	91698
827	68 39 29	565 609 283	·001209190	91751
828	68 55 84	567 663 552	·001207729	91803
829	68 72 41	569 722 789	·001206273	91855
830	68 89 00	571 787 000	·001204819	91908
831	69 05 61	573 856 191	·001203369	91960
832	69 22 24	575 930 368	·001201923	92012
833	69 38 89	578 009 537	·001200480	92065
834	69 55 56	580 093 704	·001199041	92117
835	69 72 25	582 182 875	·001197605	92169
836	69 88 96	584 277 056	·001196172	92221
837	70 05 69	586 376 253	·001194743	92273
838	70 22 44	588 480 472	·001193317	92324
839	70 39 21	590 589 719	·001191895	92376
840	70 56 00	592 704 000	·001190476	92428
841	70 72 81	594 823 321	·001189061	92480
842	70 89 64	596 947 688	·001187648	92531
843	71 06 49	599 077 107	·001186240	92583
844	71 23 36	601 211 584	·001184834	92634
845	71 40 25	603 351 125	·001183432	92686
846	71 57 16	605 495 736	·001182033	92737
847	71 74 09	607 645 423	·001180638	92788
848	71 91 04	609 800 192	·001179245	92840
849	72 08 01	611 960 049	·001177856	92891
850	72 25 00	614 125 000	·001176471	92942
851	72 42 01	616 295 051	·001175088	92993
852	72 59 04	618 470 208	·001173709	93044
853	72 76 09	620 650 477	·001172333	93095
854	72 93 16	622 835 864	·001170960	93146
855	73 10 25	625 026 375	·001169591	93197
856	73 27 36	627 222 016	·001168224	93247
857	73 44 49	629 422 793	·001166861	93298
858	73 61 64	631 628 712	·001165501	93349
859	73 78 81	633 839 779	·001164144	93399
860	73 96 00	636 056 000	·001162791	93450
861	74 13 21	638 277 381	·001161440	93500
862	74 30 44	640 503 928	·001160093	93551
863	74 47 69	642 735 647	·001158749	93601
864	74 64 96	644 972 544	·001157407	93651
865	74 82 25	647 214 625	·001156069	93702

No.	Square.	Cube.	Reciprocal.	C. Log.
866	74 99 56	649 461 896	·001154734	93752
867	75 16 89	651 714 363	·001153403	93802
868	75 34 24	653 972 032	·001152074	93852
869	75 51 61	656 234 909	·001150748	93902
870	75 69 00	658 503 000	·001149425	93952
871	75 86 41	660 776 311	·001148106	94002
872	76 03 84	663 054 848	·001146789	94052
873	76 21 29	665 338 617	·001145475	94101
874	76 38 76	667 627 624	·001144165	94151
875	76 56 25	669 921 875	·001142857	94201
876	76 73 76	672 221 376	·001141553	94250
877	76 91 29	674 526 133	·001140251	94300
878	77 08 84	676 836 152	·001138952	94349
879	77 26 41	679 151 439	·001137656	94399
880	77 44 00	681 472 000	·001136364	94448
881	77 61 61	683 797 841	·001135074	94498
882	77 79 24	686 128 968	·001133787	94547
883	77 96 89	688 465 387	·001132503	94596
884	78 14 56	690 807 104	·001131222	94645
885	78 32 25	693 154 125	·001129944	94694
886	78 49 96	695 506 456	·001128668	94743
887	78 67 69	697 864 103	·001127396	94792
888	78 85 44	700 227 072	·001126126	94841
889	79 03 21	702 595 369	·001124859	94890
890	79 21 00	704 969 000	·001123596	94939
891	79 38 81	707 347 971	·001122334	94988
892	79 56 64	709 732 288	·001121076	95036
893	79 74 49	712 121 957	·001119821	95085
894	79 92 36	714 516 984	·001118568	95134
895	80 10 25	716 917 375	·001117318	95182
896	80 28 16	719 323 136	·001116071	95231
897	80 46 09	721 734 273	·001114827	95279
898	80 64 04	724 150 792	·001113586	95328
899	80 82 01	726 572 699	·001112347	95376
900	81 00 00	729 000 000	·001111111	95424
901	81 18 01	731 432 701	·001109878	95472
902	81 36 04	733 870 808	·001108647	95521
903	81 54 09	736 314 327	·001107420	95569
904	81 72 16	738 763 264	·001106195	95617
905	81 90 25	741 217 625	·001104972	95665
906	82 08 36	743 677 416	·001103753	95713
907	82 26 49	746 142 643	·001102536	95761
908	82 44 64	748 613 312	·001101322	95809
909	82 62 81	751 089 429	·001100110	95856
910	82 81 00	753 571 000	·001098901	95904

No.	Square.	Cube.	Reciprocal.	C. Log.
911	82 99 21	756 058 031	·001097695	95952
912	83 17 44	758 550 528	·001096491	95999
913	83 35 69	761 048 497	·001095290	96047
914	83 53 96	763 551 944	·001094092	96095
915	83 72 25	766 060 875	·001092896	96142
916	83 90 56	768 575 296	·001091703	96190
917	84 08 89	771 095 213	·001090513	96237
918	84 27 24	773 620 632	·001089325	96284
919	84 45 61	776 151 559	·001088139	96332
920	84 64 00	778 688 000	·001086957	96379
921	84 82 41	781 229 961	·001085776	96426
922	85 00 84	783 777 448	·001084599	96473
923	85 19 29	786 330 467	·001083424	96520
924	85 37 76	788 889 024	·001082251	96567
925	85 56 25	791 453 125	·001081081	96614
926	85 74 76	794 022 776	·001079914	96661
927	85 93 29	796 597 983	·001078749	96708
928	86 11 84	799 178 752	·001077586	96755
929	86 30 41	801 765 089	·001076426	96802
930	86 49 00	804 357 000	·001075269	96848
931	86 67 61	806 954 491	·001074114	96895
932	86 86 24	809 557 568	·001072961	96942
933	87 04 89	812 166 237	·001071811	96988
934	87 23 56	814 780 504	·001070664	97035
935	87 42 25	817 400 375	·001069519	97081
936	87 60 96	820 025 856	·001068376	97128
937	87 79 69	822 656 953	·001067236	97174
938	87 98 44	825 293 672	·001066098	97220
939	88 17 21	827 936 019	·001064963	97267
940	88 36 00	830 584 000	·001063830	97313
941	88 54 81	833 237 621	·001062699	97359
942	88 73 64	835 896 888	·001061571	97405
943	88 92 49	838 561 807	·001060445	97451
944	89 11 36	841 232 384	·001059322	97497
945	89 30 25	843 908 625	·001058201	97543
946	89 49 16	846 590 536	·001057082	97589
947	89 68 09	849 278 123	·001055966	97635
948	89 87 04	851 971 392	·001054852	97681
949	90 06 01	854 670 349	·001053741	97727
950	90 25 00	857 375 000	·001052632	97772
951	90 44 01	860 085 351	·001051525	97818
952	90 63 04	862 801 408	·001050420	97864
953	90 82 09	865 523 177	·001049318	97909
954	91 01 16	868 250 664	·001048218	97955
955	91 20 25	870 983 875	·001047120	98000

No.	Square.	Cube.	Reciprocal.	C. Log.
956	91 39 36	873 722 816	·001046025	98046
957	91 58 49	876 467 493	·001044932	98091
958	91 77 64	879 217 912	·001043841	98137
959	91 96 81	881 974 079	·001042753	98182
960	92 16 00	884 736 000	·001041667	98227
961	92 35 21	887 503 681	·001040583	98272
962	92 54 44	890 277 128	·001039501	98318
963	92 73 69	893 056 347	·001038422	98363
964	92 92 96	895 841 344	·001037344	98408
965	93 12 25	898 632 125	·001036269	98453
966	93 31 56	901 428 696	·001035197	98498
967	93 90 89	904 231 063	·001034126	98543
968	93 70 24	907 039 232	·001033058	98588
969	93 89 61	909 853 209	·001031992	98632
970	94 09 00	912 673 000	·001030928	98677
971	94 28 41	915 498 611	·001029866	98722
972	94 47 84	918 330 048	·001028807	98767
973	94 67 29	921 167 317	·001027749	98811
974	94 86 76	924 010 424	·001026694	98856
975	95 06 25	926 859 375	·001025641	98900
976	95 25 76	929 714 176	·001024590	98945
977	95 45 29	932 574 833	·001023541	98989
978	95 64 84	935 441 352	·001022495	99034
979	95 84 41	938 313 739	·001021450	99078
980	96 04 00	941 192 000	·001020408	99123
981	96 23 61	944 076 141	·001019368	99167
982	96 43 24	946 966 168	·001018330	99211
983	96 62 89	949 862 087	·001017294	99255
984	96 82 56	952 763 904	·001016260	99300
985	97 02 25	955 671 625	·001015228	99344
986	97 21 96	958 585 256	·001014199	99388
987	97 41 69	961 504 803	·001013171	99432
988	97 61 44	964 430 272	·001012146	99476
989	97 81 21	967 361 669	·001011122	99520
990	98 01 00	970 299 000	·001010101	99564
991	98 20 81	973 242 271	·001009082	99607
992	98 40 64	976 191 488	·001008065	99651
993	98 60 49	979 146 657	·001007049	99695
994	98 80 36	982 107 784	·001006036	99739
995	99 00 25	985 074 875	·001005025	99782
996	99 20 16	988 047 936	·001004016	99826
997	99 40 09	991 026 973	·001003009	99870
998	99 60 04	994 011 992	·001002004	99913
999	99 80 01	997 002 999	·001001001	99957
1000	100 00 00	1000 000 000	·001000000	00000

TABLE 1 A.—APPROXIMATE SQUARE AND CUBE ROOTS AND RECIPROCALS OF PRIME NUMBERS FROM 2 TO 97.

No.	Square Root.	Cube Root.	Reciprocal.
2	1·4142136	1·2599210	0·500000000
3	1·7320508	1·4422496	·333333333
5	2·2360680	1·7099759	·200000000
7	2·6457513	1·9129312	·142857143
11	3·3166248	2·2239801	·090909091
13	3·6055513	2·3513347	·076923077
17	4·1231056	2·5712816	·058823529
19	4·3588989	2·6684016	·052631579
23	4·7958315	2·8438670	·043478261
29	5·3851648	3·0723168	·034482759
31	5·5677644	3·1413806	·032258065
37	6·0827625	3·3322218	·027027027
41	6·4031242	3·4482172	·024390244
43	6·5574385	3·5033981	·023255814
47	6·8556546	3·6088261	·021276600
53	7·2801099	3·7562858	·018867925
59	7·6811457	3·8929965	·016949153
61	7·8102497	3·9364972	·016393443
67	8·1853528	4·0615480	·014925373
71	8·4261498	4·1408178	·014084507
73	8·5440037	4·1793392	·013698630
79	8·8881944	4·2908404	·012658228
83	9·1104336	4·3620707	·012048193
89	9·4339811	4·4647451	·011235955
93	9·6436508	4·5306549	·010752688
97	9·8488578	4·5947009	·010309278

	Square.	Fifth Power.		Square.	Fifth Power.
10	1 00	1 00000	55	30 25	5032 84375
11	1 21	1 61051	56	31 36	5507 31776
12	1 44	2 48832	57	32 49	6016 92057
13	1 69	3 71293	58	33 64	6563 56768
14	1 96	5 37824	59	34 81	7149 24299
15	2 25	7 59375	60	36 00	7776 00000
16	2 56	10 48576	61	37 21	8445 96301
17	2 89	14 19857	62	38 44	9161 32832
18	3 24	18 89568	63	39 69	9924 36543
19	3 61	24 76099	64	40 96	10737 41824
20	4 00	32 00000	65	42 25	11602 90625
21	4 41	40 84101	66	43 56	12523 32576
22	4 84	51 53632	67	44 89	13501 25107
23	5 29	64 36343	68	46 24	14539 33568
24	5 76	79 62624	69	47 61	15640 31349
25	6 25	97 65625	70	49 00	16807 00000
26	6 76	118 81376	71	50 41	18042 29351
27	7 29	143 48907	72	51 84	19349 17632
28	7 84	172 10368	73	53 29	20730 71593
29	8 41	205 11149	74	54 76	22190 06624
30	9 00	243 00000	75	56 25	23730 46875
31	9 61	286 29151	76	57 76	25355 25376
32	10 24	335 54432	77	59 29	27067 84157
33	10 89	391 35393	78	60 84	28871 74368
34	11 56	454 35424	79	62 41	30770 56399
35	12 25	525 21875	80	64 00	32768 00000
36	12 96	604 66176	81	65 61	34867 84401
37	13 69	693 43957	82	67 24	37073 98439
38	14 44	792 35168	83	68 89	39390 40643
39	15 21	902 24199	84	70 56	41821 19424
40	16 00	1024 00000	85	72 25	44370 53125
41	16 81	1158 56201	86	73 96	47042 70176
42	17 64	1306 91232	87	75 69	49842 09207
43	18 49	1470 08443	88	77 44	52773 19168
44	19 36	1649 16224	89	79 21	55840 59449
45	20 25	1845 28125	90	81 00	59049 00000
46	21 16	2059 62976	91	82 81	62403 21451
47	22 09	2293 45007	92	84 64	65908 15232
48	23 04	2548 03968	93	86 49	69568 83693
49	24 01	2824 75249	94	88 36	73390 40224
50	25 00	3125 00000	95	90 25	77378 09375
51	26 01	3450 25251	96	92 16	81537 26976
52	27 04	3802 04032	97	94 09	85873 40257
53	28 09	4181 95493	98	96 04	90392 07968
54	29 16	4591 65024	99	98 01	95099 00499

Table 2 a.—Prime Factors of Numbers up to 256.

(Numbers without Factors are themselves Prime.)

2		42	$= 2\cdot3\cdot7$	82	$= 2\cdot41$
3		43		83	
4	$= 2^2$	44	$2^2\cdot11$	84	$2^2\cdot3\cdot7$
5		45	$3^2\cdot5$	85	$5\cdot17$
6	$2\cdot3$	46	$2\cdot23$	86	$2\cdot43$
7		47		87	$3\cdot29$
8	2^3	48	$2^4\cdot3$	88	$2^3\cdot11$
9	3^2	49	7^2	89	
10	$2\cdot5$	50	$2\cdot5^2$	90	$2\cdot3^2\cdot5$
11		51	$3\cdot17$	91	$7\cdot13$
12	$2^2\cdot3$	52	$2^2\cdot13$	92	$2^2\cdot23$
13		53		93	$3\cdot31$
14	$2\cdot7$	54	$2\cdot3^3$	94	$2\cdot47$
15	$3\cdot5$	55	$5\cdot11$	95	$5\cdot19$
16	2^4	56	$2^3\cdot7$	96	$2^5\cdot3$
17		57	$3\cdot19$	97	
18	$2\cdot3^2$	58	$2\cdot29$	98	$2\cdot7^2$
19		59		99	$3^2\cdot11$
20	$2^2\cdot5$	60	$2^2\cdot3\cdot5$	100	$2^2\cdot5^2$
21	$3\cdot7$	61		101	
22	$2\cdot11$	62	$2\cdot31$	102	$2\cdot3\cdot17$
23		63	$3^2\cdot7$	103	
24	$2^3\cdot3$	64	2^6	104	$2^3\cdot13$
25	5^2	65	$5\cdot13$	105	$3\cdot5\cdot7$
26	$2\cdot13$	66	$2\cdot3\cdot11$	106	$2\cdot53$
27	3^3	67		107	
28	$2^2\cdot7$	68	$2^2\cdot17$	108	$2^2\cdot3^3$
29		69	$3\cdot23$	109	
30	$2\cdot3\cdot5$	70	$2\cdot5\cdot7$	110	$2\cdot5\cdot11$
31		71		111	$3\cdot37$
32	2^5	72	$2^3\cdot3^2$	112	$2^4\cdot7$
33	$3\cdot11$	73		113	
34	$2\cdot17$	74	$2\cdot37$	114	$2\cdot3\cdot19$
35	$5\cdot7$	75	$3\cdot5^2$	115	$5\cdot23$
36	$2^2\cdot3^2$	76	$2^2\cdot19$	116	$2^2\cdot29$
37		77	$7\cdot11$	117	$3^2\cdot13$
38	$2\cdot19$	78	$2\cdot3\cdot13$	118	$2\cdot59$
39	$3\cdot13$	79		119	$7\cdot17$
40	$2^3\cdot5$	80	$2^4\cdot5$	120	$2^3\cdot3\cdot5$
41		81	3^4	121	11^2

122 =	$2 \cdot 61$	167		212 =	$2^2 \cdot 53$
123	$3 \cdot 41$	168 =	$2^3 \cdot 3 \cdot 7$	213	$3 \cdot 71$
124	$2^2 \cdot 31$	169	13^2	214	$2 \cdot 107$
125	5^3	170	$2 \cdot 5 \cdot 17$	215	$5 \cdot 43$
126	$2 \cdot 3^2 \cdot 7$	171	$3^2 \cdot 19$	216	$2^3 \cdot 3^3$
127		172	$2^2 \cdot 43$	217	$7 \cdot 31$
128	2^7	173		218	$2 \cdot 109$
129	$3 \cdot 43$	174	$2 \cdot 3 \cdot 29$	219	$3 \cdot 73$
130	$2 \cdot 5 \cdot 13$	175	$5^2 \cdot 7$	220	$2^2 \cdot 5 \cdot 11$
131		176	$2^4 \cdot 11$	221	$13 \cdot 17$
132	$2^2 \cdot 3 \cdot 11$	177	$3 \cdot 59$	222	$2 \cdot 3 \cdot 37$
133	$7 \cdot 19$	178	$2 \cdot 89$	223	
134	$2 \cdot 67$	179		224	$2^5 \cdot 7$
135	$3^3 \cdot 5$	180	$2^2 \cdot 3^2 \cdot 5$	225	$3^2 \cdot 5^2$
136	$2^3 . 17$	181		226	$2 \cdot 113$
137		182	$2 \cdot 7 \cdot 13$	227	
138	$2 \cdot 3 \cdot 23$	183	$3 \cdot 61$	228	$2^2 \cdot 3 \cdot 19$
139		184	$2^3 \cdot 23$	229	
140	$2^2 \cdot 5 \cdot 7$	185	$5 \cdot 37$	230	$2 \cdot 5 \cdot 23$
141	$3 \cdot 47$	186	$2 \cdot 3 \cdot 31$	231	$3 \cdot 7 \cdot 11$
142	$2 \cdot 71$	187	$11 \cdot 17$	232	$2^3 \cdot 29$
143	$11 \cdot 13$	188	$2^2 \cdot 47$	233	
144	$2^4 \cdot 3^2$	189	$3^3 \cdot 7$	234	$2 \cdot 3^2 \cdot 13$
145	$5 \cdot 29$	190	$2 \cdot 5 \cdot 19$	235	$5 \cdot 47$
146	$2 \cdot 73$	191		236	$2^2 \cdot 59$
147	$3 \cdot 7^2$	192	$2^6 \cdot 3$	237	$3 \cdot 79$
148	$2^2 \cdot 37$	193		238	$2 \cdot 7 \cdot 17$
149		194	$2 \cdot 97$	239	
150	$2 \cdot 3 \cdot 5^2$	195	$5 \cdot 3 \cdot 13$	240	$2^4 \cdot 3 \cdot 5$
151		196	$2^2 \cdot 7^2$	241	
152	$2^3 \cdot 19$	197		242	$2 \cdot 11^2$
153	$3^2 \cdot 17$	198	$2 \cdot 3^2 \cdot 11$	243	3^5
154	$2 \cdot 7 \cdot 11$	199		244	$2^2 \cdot 61$
155	$5 \cdot 31$	200	$2^3 \cdot 5^2$	245	$5 \cdot 7^2$
156	$2^2 \cdot 3 \cdot 13$	201	$3 \cdot 67$	246	$2 \cdot 3 \cdot 41$
157		202	$2 \cdot 101$	247	$13 \cdot 19$
158	$2 \cdot 79$	203	$7 \cdot 29$	248	$2^3 \cdot 391$
159	$3 \cdot 53$	204	$2^2 \cdot 3 \cdot 17$	249	$3 \cdot 83$
160	$2^5 \cdot 5$	205	$5 \cdot 41$	250	$2 \cdot 5^3$
161	$7 \cdot 23$	206	$2 \cdot 103$	251	
162	$2 \cdot 3^4$	207	$3^2 \cdot 23$	252	$2^2 \cdot 3^2 \cdot 7$
163		208	$2^4 \cdot 13$	253	$11 \cdot 23$
164	$2^2 \cdot 41$	209	$11 \cdot 19$	254	$2 \cdot 127$
165	$3 \cdot 5 \cdot 11$	210	$2 \cdot 3 \cdot 5 \cdot 7$	255	$3 \cdot 5 \cdot 17$
166	$2 \cdot 83$	211		256	$2^8 \cdot$

Tables 3 and 3 a—Hyperbolic, Naperian, or Natural Logarithms.

1. Table 3 gives the hyperbolic logarithms of *integer* numbers from 1 to 100. To find the hyperbolic logarithm of an integer number consisting of not more than two significant figures followed by noughts; take the hyperbolic logarithm corresponding to the significant figures, and add to it the product of the hyperbolic logarithm of 10 by the number of noughts (this may be found by the aid of the second column of Table 3 a). For example, to find the hyperbolic logarithm of 3700;

Hyp. log. 37, ...	3·61092
2 × Hyp. log. 10,......................	4·60517
Hyp. log. 3700, ...	8·21609

Note.—Multiples of the hyperbolic logarithm of 10 may be taken from the second column of Table 3 a.

2. The hyperbolic logarithm of the product of two numbers is the sum of their hyperbolic logarithms. For example,

Hyp. log. 74, ...	4·30407
Hyp. log. 50, ...	3·91202
Hyp. log. 3700, ...	8·21609

3. To find the hyperbolic logarithm of a decimal fraction containing not more than two significant figures; take from the table the hyperbolic logarithm corresponding to those figures, and take the difference between it and as many times the hyperbolic logarithm of 10 as there are places of decimals. That difference will be the required logarithm, and will be positive or negative according as the fraction is greater or less than 1. For example,

Hyp. log. 37, ...		3·61092
Hyp. log. 10, ...		2·30259
Hyp. log. 3·7,.................	+	1·30833

Hyp. log. 37, ...		3·61092
3 × Hyp. log. 10, ...		6·90776
Hyp. log. 0·037,..............	–	3·29684

In such examples as the last, the fractional as well as the integral part of the hyperbolic logarithm is negative.

4. Examples of the use of Table 3 a.

I. To find the hyperbolic logarithm of 377 from its common logarithm;

2·57634, common logarithm.

2·	4·605170
5	1·151293
7	161181
6	13816
3	691
4	92
Sum,	5·932243

The required hyperbolic logarithm is thus found to be 5·93224, correct to five places of decimals; the sixth being rejected as liable to error.

II. To find the common logarithm corresponding to the hyperbolic logarithm 5·93224;

5·	2·171472
9	390865
3	13029
2	869
2	87
4	17
	2·576339

from which, rejecting the last place of figures as liable to error, the required common logarithm is found to be 2·57634.

5. To calculate the hyperbolic logarithm of the ratio of two numbers without logarithmic tables; divide the difference of the numbers by their sum; then add together twice the quotient, two-thirds of its cube, two-fifths of its fifth power, two-sevenths of its seventh power, and so on, until the required degree of accuracy has been attained; the result of the summation will be the required hyperbolic logarithm.

EXAMPLE.—Required the hyperbolic logarithm of $\frac{377}{370}$.

$\frac{\text{Difference, } 7}{\text{Sum, } 747} = \cdot0093708$ quotient, correct to the seventh place of decimals.

$$\text{Quotient, } \cdot0093708 \times 2 = \cdot0187416$$

$$\text{Cube, } \cdot0000009 \times \frac{2}{3} = \cdot0000006$$

Hyp. log. of $\frac{377}{370}$, correct to the seventh place of decimals, ·0187422

NOTE.—This process may be used in finding hyperbolic logarithms of numbers not in the table. For example, to find the hyperbolic logarithm of 377, we have

From the tables,	hyp. log. 37,	3·61092
	hyp. log. 10,	2·30258
	hyp. log. 370,	5·91350

$$\text{Hyp. log. } \frac{377}{370}, \text{ already calculated, } 0{\cdot}01874$$

Hyp. log. 377, ...5·93224

6. To find the *antilogarithm* (or natural number) corresponding to a given positive hyperbolic logarithm by calculation, without using logarithmic tables; take the sum of the following series, to as many terms as may be necessary in order to give the required degree of accuracy;

First term = ...1.
Second term = The given hyp. log.

$$\text{Third term} = \text{second term} \times \frac{\text{given hyp. log.}}{2};$$

$$\text{Fourth term} = \text{third term} \times \frac{\text{given hyp. log.}}{3};$$

$$\text{Fifth term} = \text{fourth term} \times \frac{\text{given hyp. log.}}{4};$$

and so on.

The accuracy of this process is the greater the smaller the given hyperbolic logarithm.

Example.—To calculate the hyperbolic antilogarithm of 1 (in other words, the number whose hyperbolic logarithm is 1) to seven places of decimals;

1st term,	....	1·0000000
2d „	....	1·0000000
3d „	= 2d × $\frac{1}{2}$	0·5000000
4th „	= 3d × $\frac{1}{3}$	0·1666667
5th „	= 4th × $\frac{1}{4}$	0·0416667
6th „	= 5th × $\frac{1}{5}$	0·0083333
7th „	= 6th × $\frac{1}{6}$	0·0013889
8th „	= 7th × $\frac{1}{7}$	0·0001984
9th „	= 8th × $\frac{1}{8}$	0·0000248
10th „	= 9th × $\frac{1}{9}$	0·0000027
11th „	= 10th × $\frac{1}{10}$	0·0000003
Hyperbolic antilogarithm of 1	=	2·7182818

This number is called the *base of the Naperian Logarithms*, and denoted in algebra by the symbol *ε* or *e*.

TABLE 3.—HYPERBOLIC LOGARITHMS.

No.	Hyp. Log.	No.	Hyp. Log.	No.	Hyp. Log.	No.	Hyp. Log.
1	0·00000	26	3·25810	51	3·93183	76	4·33073
2	0·69315	27	3·29584	52	3·95124	77	4·34381
3	1·09861	28	3·33220	53	3·97029	78	4·35671
4	1·38629	29	3·36730	54	3·98898	79	4·36945
5	1·60944	30	3·40120	55	4·00733	80	4·38203
6	1·79176	31	3·43399	56	4·02535	81	4·39445
7	1·94591	32	3·46574	57	4·04305	82	4·40672
8	2·07944	33	3·49651	58	4·06044	83	4·41884
9	2·19722	34	3·52636	59	4·07754	84	4·43082
10	2·30259	35	3·55535	60	4·09434	85	4·44265
11	2·39790	36	3·58352	61	4·11087	86	4·45435
12	2·48491	37	3·61092	62	4·12713	87	4·46591
13	2·56495	38	3·63759	63	4·14313	88	4·47734
14	2·63906	39	3·66356	64	4·15888	89	4·48864
15	2·70805	40	3·68888	65	4·17439	90	4·49981
16	2·77259	41	3·71357	66	4·18965	91	4·51086
17	2·83321	42	3·73767	67	4·20469	92	4·52179
18	2·89037	43	3·76120	68	4·21951	93	4·53260
19	2·94444	44	3·78419	69	4·23411	94	4·54329
20	2·99573	45	3·80666	70	4·24850	95	4·55388
21	3·04452	46	3·82864	71	4·26268	96	4·56435
22	3·09104	47	3·85015	72	4·27667	97	4·57471
23	3·13549	48	3·87120	73	4·29046	98	4·58497
24	3·17805	49	3·89182	74	4·30407	99	4·59512
25	3·21888	50	3·91202	75	4·31749	100	4·60517

Hyp. log. 10, correct to eight places of decimals, = 2·30258509.

TABLE 3 A.—MULTIPLIERS FOR CONVERTING LOGARITHMS.

	Common into Hyperbolic.	Hyperbolic into Common.	
1	2·302585	0·434294	1
2	4·605170	0·868589	2
3	6·907755	1·302883	3
4	9·210340	1·737178	4
5	11·512925	2·171472	5
6	13·815510	2·605767	6
7	16·118096	3·040061	7
8	18·420681	3·474356	8
9	20·723266	3·908650	9
10	23·025851	4·342945	10

TABLE 4.—MULTIPLIERS FOR THE CONVERSION OF CIRCULAR LENGTHS AND AREAS.

	A.—Diameters into Circumferences.	B.—Circumferences into Diameters	C.—Radius-Lengths into Circumferences.	D.—Circumferences into Radius-Lengths.	
1	3·1416	0·31831	6·2832	0·15916	1
2	6·2832	0·63662	12·5664	0·31831	2
3	9·4248	0·95493	18·8496	0·47747	3
4	12·5664	1·27324	25·1327	0·63662	4
5	15·7080	1·59155	31·4159	0·79578	5
6	18·8496	1·90986	37·6991	0·95493	6
7	21·9911	2·22817	43·9823	1·11409	7
8	25·1327	2·54648	50·2655	1·27324	8
9	28·2743	2·86479	56·5487	1·43240	9
10	31·4159	3·18310	62·8319	1·59155	10

	E.—Circular Areas into Square Areas.	F.—Square Areas into Circular Areas.	G.—Degrees into Radius-Lengths.	H.—Radius-Lengths into Degrees.	
1	0·7854	1·2732	0·0174533	57·2958	1
2	1·5708	2·5465	0·0349066	114·5916	2
3	2·3562	3·8197	0·0523599	171·8873	3
4	3·1416	5·0930	0·0698132	229·1831	4
5	3·9270	6·3662	0·0872665	286·4789	5
6	4·7124	7·6394	0·1047197	343·7747	6
7	5·4978	8·9127	0·1221730	401·0705	7
8	6·2832	10·1859	0·1396263	458·3662	8
9	7·0686	11·4592	0·1570796	515·6620	9
10	7·8540	12·7324	0·1745329	572·9578	10

	I.—Minutes into Radius-Lengths.	K.—Radius-Lengths into Minutes.	L.—Seconds into Radius-Lengths.	M.—Radius-Lengths into Seconds.	
1	0·000291	3437·75	0·000005	206265	1
2	0·000582	6875·50	0·000010	412530	2
3	0·000873	10313·24	0·000015	618794	3
4	0·001164	13750·99	0·000019	825059	4
5	0·001454	17188·74	0·000024	1031324	5
6	0·001745	20626·48	0·000029	1237589	6
7	0·002036	24064·23	0·000034	1443854	7
8	0·002327	27501·97	0·000039	1650118	8
9	0·002618	30939·72	0·000044	1856383	9
10	0·002909	34377·47	0·000048	2062648	10
20	0·005818	......	0·000097	......	20
30	0·008727	......	0·000145	......	30
40	0·011636	......	0·000194	......	40
50	0·014544	......	0·000242	......	50

EXAMPLES OF THE USE OF TABLE 4.

I. What is the circumference of a circle whose diameter is 113 inches? From division A of the table, we have the following:—

100		314·16
10		31·416
3		9·4248
113	*Sum,*	355·0008

The answer is 355 inches; the fourth and third places of decimals being rejected as beyond the limits of exactness of the table.

II. What is the radius of a circle whose circumference is 710 inches? From division D of the table, we have the following:—

700		111·409
10		1·5916
710	*Sum,*	113·0006

The answer is 113 inches; the fourth place of decimals being rejected as beyond the limits of the exactness of the table.

III. What is the area in square inches of a circle of 8 inches diameter? Square of 8 = 64 = area in *circular inches.* Then, by division E of the table,

60	47·124
4	3·1416
Area in square inches (to five figures only),	50·266

IV. What is the diameter of a circle whose area is 5027 square inches? From division F of the table we have

5000	6366·2
20	25·465
7	8·9127
Area in circular inches (to five figures only),	6400·6

the square root of which (by Table 1, the fractions being found by calculation) is 80·004, being the diameter required in inches, correct to five places of figures.

V. How many radius-lengths are there in an arc of 57° 17′ 45″?

			Radius-Lengths.
From division G,		50°	0·872665
—	—	7°	0·122173
—	— I,	10′	0·002909
—	—	7′	0·002036
—	— L,	40″	0·000194
—	—	5″	0·000024
Total,		57° 17′ 45″	1·000001

or almost exactly *one* radius-length.

VI. How many minutes are there in the arc which is one-eightieth (or 0·0125) of a radius-length? By division K we have

·01	34·3775
·002	6·8755
·0005	1·7189
	42·9719 *Answer;*

or 42′ 58″ nearly.

Explanation of Table 5.

This table gives the circumferences and areas of circles, of diameters from 101 to 1000; the circumferences computed to two places of decimals, the areas to the nearest unit. Circumferences and areas for diameters not in the table may be computed by the aid of the following principles:—

1. The circumferences of circles are proportional to their diameters.
2. The areas of circles are proportional to the squares of their diameters.

Table 5.—Circumferences and Areas of Circles.

Diam.	Circum.	Area.	Diam.	Circum.	Area.
101	317·30	8012	146	458·67	16742
102	320·44	8171	147	461·81	16972
103	323·58	8332	148	464·96	17203
104	326·73	8495	149	468·10	17437
105	329·87	8659	150	471·24	17671
106	333·01	8825	151	474·38	17908
107	336·15	8992	152	477·52	18146
108	339·29	9161	153	480·66	18385
109	342·43	9331	154	483·81	18627
110	345·58	9503	155	486·95	18869
111	348·72	9677	156	490·09	19113
112	351·86	9852	157	493·23	19359
113	355·00	10029	158	496·37	19607
114	358·14	10207	159	499·51	19856
115	361·28	10387	160	502·65	20106
116	364·42	10568	161	505·80	20358
117	367·57	10751	162	508·94	20612
118	370·71	10936	163	512·08	20867
119	373·85	11122	164	515·22	21124
120	376·99	11310	165	518·36	21382
121	380·13	11499	166	521·50	21642
122	383·27	11690	167	524·65	21904
123	386·42	11882	168	527·79	22167
124	389·56	12076	169	530·93	22432
125	392·70	12272	170	534·07	22698
126	395·84	12469	171	537·21	22966
127	398·98	12668	172	540·35	23235
128	402·12	12868	173	543·50	23506
129	405·27	13070	174	546·64	23779
130	408·41	13273	175	549·78	24053
131	411·55	13478	176	552·92	24329
132	414·69	13685	177	556·06	24606
133	417·83	13893	178	559·20	24885
134	420·97	14103	179	562·35	25165
135	424·12	14314	180	565·49	25447
136	427·26	14527	181	568·63	25730
137	430·40	14741	182	571·77	26016
138	433·54	14957	183	574·91	26302
139	436·68	15175	184	578·05	26590
140	439·82	15394	185	581·19	26880
141	442·96	15615	186	584·34	27172
142	446·11	15837	187	587·48	27465
143	449·25	16061	188	590·62	27759
144	452·39	16286	189	593·76	28055
145	455·53	16513	190	596·90	28353

Diam.	Circum.	Area.	Diam.	Circum.	Area.
191	600·04	28652	236	741·42	43744
192	603·19	28953	237	744·56	44115
193	606·33	29255	238	747·70	44488
194	609·47	29559	239	750·84	44863
195	612·61	29865	240	753·98	45239
196	615·75	30172	241	757·12	45617
197	618·89	30481	242	760·27	45996
198	622·04	30791	243	763·41	46377
199	625·18	31103	244	766·55	46759
200	628·32	31416	245	769·69	47144
201	631·46	31731	246	772·83	47529
202	634·60	32047	247	775·97	47916
203	637·74	32365	248	779·12	48305
204	640·89	32685	249	782·26	48695
205	644·03	33006	250	785·40	49087
206	647·17	33329	251	788·54	49481
207	650·31	33654	252	791·68	49876
208	653·45	33979	253	794·82	50273
209	656·59	34307	254	797·96	50671
210	659·73	34636	255	801·11	51071
211	662·88	34967	256	804·25	51472
212	666·02	35299	257	807·39	51875
213	669·16	35633	258	810·53	52279
214	672·30	35968	259	813·67	52685
215	675·44	36305	260	816·81	53093
216	678·58	36644	261	819·96	53502
217	681·73	36984	262	823·10	53913
218	684·87	37325	263	826·24	54325
219	688·01	37668	264	829·38	54739
220	691·15	38013	265	832·52	55155
221	694·29	38360	266	835·66	55572
222	697·43	38708	267	838·81	55990
223	700·58	39057	268	841·95	56410
224	703·72	39408	269	845·09	56832
225	706·86	39761	270	848·23	57256
226	710·00	40115	271	851·37	57680
227	713·14	40471	272	854·51	58107
228	716·28	40828	273	857·66	58535
229	719·42	41187	274	860·80	58965
230	722·57	41548	275	863·94	59396
231	725·71	41910	276	867·08	59828
232	728·85	42273	277	870·22	60263
233	731·99	42638	278	873·36	60699
234	735·13	43005	279	876·50	61136
235	738·27	43374	280	879·65	61575

Diam.	Circum.	Area.	Diam.	Circum.	Area.
281	882·79	62016	326	1024·16	83469
282	885·93	62458	327	1027·30	83982
283	889·07	62902	328	1030·44	84496
284	892·21	63347	329	1033·58	85012
285	895·35	63794	330	1036·73	85530
286	898·50	64242	331	1039·87	86049
287	901·64	64692	332	1043·01	86570
288	904·78	65144	333	1046·15	87092
289	907·92	65597	334	1049·29	87616
290	911·06	66052	335	1052·43	88141
291	914·20	66508	336	1055·58	88668
292	917·35	66966	337	1058·72	89197
293	920·49	67426	338	1061·86	89727
294	923·63	67887	339	1065·00	90259
295	926·77	68349	340	1068·14	90792
296	929·91	68813	341	1071·28	91327
297	933·05	69279	342	1074·42	91863
298	936·19	69747	343	1077·57	92401
299	939·34	70215	344	1080·71	92941
300	942·48	70686	345	1083·85	93482
301	945·62	71158	346	1086·99	94025
302	948·76	71631	347	1090·13	94569
303	951·90	72107	348	1093·27	95115
304	955·04	72583	349	1096·42	95662
305	958·19	73062	350	1099·56	96211
306	961·33	73542	351	1102·70	96762
307	964·47	74023	352	1105·84	97314
308	967·61	74506	353	1108·98	97868
309	970·75	74991	354	1112·12	98423
310	973·89	75477	355	1115·27	98980
311	977·04	75964	356	1118·41	99538
312	980·18	76454	357	1121·55	100098
313	983·32	76945	358	1124·69	100660
314	986·46	77437	359	1127·83	101223
315	989·60	77931	360	1130·97	101788
316	992·74	78427	361	1134·12	102354
317	995·88	78924	362	1137·26	102922
318	999·03	79423	363	1140·40	103491
319	1002·17	79923	364	1143·54	104062
320	1005·31	80425	365	1146·68	104635
321	1008·45	80928	366	1149·82	105209
322	1011·59	81433	367	1152·97	105785
323	1014·73	81940	368	1156·11	106362
324	1017·88	82448	369	1159·25	106941
325	1021·02	82958	370	1162·39	107521

Diam.	Circum.	Area.	Diam.	Circum.	Area.
371	1165·53	108103	416	1306·91	135918
372	1168·67	108687	417	1310·05	136572
373	1171·81	109272	418	1313·19	137228
374	1174·96	109858	419	1316·33	137885
375	1178·10	110447	420	1319·47	138544
376	1181·24	111036	421	1322·61	139205
377	1184·38	111628	422	1325·75	139867
378	1187·52	112221	423	1328·89	140531
379	1190·66	112815	424	1332·04	141196
380	1193·81	113411	425	1335·18	141863
381	1196·95	114009	426	1338·32	142531
382	1200·09	114608	427	1341·46	143201
383	1203·23	115209	428	1344·60	143872
384	1206·37	115812	429	1347·74	144545
385	1209·51	116416	430	1350·89	145220
386	1212·66	117021	431	1354·03	145896
387	1215·80	117628	432	1357·17	146574
388	1218·94	118237	433	1360·31	147254
389	1222·08	118847	434	1363·45	147934
390	1225·22	119459	435	1366·59	148617
391	1228·36	120072	436	1369·73	149301
392	1231·50	120687	437	1372·88	149987
393	1234·65	121304	438	1376·02	150674
394	1237·79	121922	439	1379·16	151363
395	1240·93	122542	440	1382·30	152053
396	1244·07	123163	441	1385·44	152745
397	1247·21	123786	442	1388·58	153439
398	1250·35	124410	443	1391·73	154134
399	1253·50	125036	444	1394·87	154830
400	1256·64	125664	445	1398·01	155528
401	1259·78	126293	446	1401·15	156228
402	1262·92	126923	447	1404·29	156930
403	1266·06	127556	448	1407·43	157633
404	1269·20	128190	449	1410·58	158337
405	1272·35	128825	450	1413·72	159043
406	1275·49	129462	451	1416·86	159751
407	1278·63	130100	452	1420·00	160460
408	1281·77	130741	453	1423·14	161171
409	1284·91	131382	454	1426·28	161883
410	1288·05	132025	455	1429·42	162597
411	1291·19	132670	456	1432·57	163313
412	1294·34	133317	457	1435·71	164030
413	1297·48	133965	458	1438·85	164748
414	1300·62	134614	459	1441·99	165468
415	1303·76	135265	460	1445·13	166190

Diam.	Circum.	Area.	Diam.	Circum.	Area.
461	1448·27	166914	506	1589·65	201090
462	1451·42	167639	507	1592·79	201886
463	1454·56	168365	508	1595·93	202683
464	1457·70	169093	509	1599·07	203482
465	1460·84	169823	510	1602·21	204282
466	1463·98	170554	511	1605·35	205084
467	1467·12	171287	512	1608·50	205887
468	1470·27	172021	513	1611·64	206692
469	1473·41	172757	514	1614·78	207499
470	1476·55	173494	515	1617·92	208307
471	1479·69	174234	516	1621·06	209117
472	1482·83	174974	517	1624·20	209928
473	1485·97	175716	518	1627·35	210741
474	1489·12	176460	519	1630·49	211556
475	1492·26	177205	520	1633·63	212372
476	1495·40	177952	521	1636·77	213189
477	1498·54	178701	522	1639·91	214008
478	1501·68	179451	523	1643·05	214829
479	1504·82	180203	524	1646·20	215651
480	1507·96	180956	525	1649·34	216475
481	1511·11	181711	526	1652·48	217301
482	1514·25	182467	527	1655·62	218128
483	1517·39	183225	528	1658·76	218956
484	1520·53	183984	529	1661·90	219787
485	1523·67	184745	530	1665·04	220618
486	1526·81	185508	531	1668·19	221452
487	1529·96	186272	532	1671·33	222287
488	1533·10	187038	533	1674·47	223123
489	1536·24	187805	534	1677·61	223961
490	1539·38	188574	535	1680·75	224801
491	1542·52	189345	536	1683·89	225642
492	1545·66	190117	537	1687·04	226484
493	1548·81	190890	538	1690·18	227329
494	1551·95	191665	539	1693·32	228175
495	1555·09	192442	540	1696·46	229022
496	1558·23	193221	541	1699·60	229871
497	1561·37	194000	542	1702·74	230722
498	1564·51	194782	543	1705·88	231574
499	1567·65	195565	544	1709·03	232428
500	1570·80	196350	545	1712·17	233283
501	1573·94	197136	546	1715·31	234140
502	1577·08	197923	547	1718·45	234998
503	1580·22	198713	548	1721·59	235858
504	1583·36	199504	549	1724·73	236720
505	1586·50	200296	550	1727·88	237583

Diam.	Circum.	Area.	Diam.	Circum.	Area.
551	1731·02	238448	596	1872·39	278986
552	1734·16	239314	597	1875·53	279923
553	1737·30	240182	598	1878·67	280862
554	1740·44	241051	599	1881·81	281802
555	1743·58	241922	600	1884·96	282743
556	1746·73	242795	601	1888·10	283687
557	1749·87	243669	602	1891·24	284631
558	1753·01	244545	603	1894·38	285578
559	1756·15	245422	604	1897·52	286526
560	1759·29	246301	605	1900·66	287475
561	1762·43	247181	606	1903·81	288426
562	1765·58	248063	607	1906·95	289379
563	1768·72	248947	608	1910·09	290334
564	1771·86	249832	609	1913·23	291289
565	1775·00	250719	610	1916·37	292247
566	1778·14	251607	611	1919·51	293206
567	1781·28	252497	612	1922·65	294166
568	1784·42	253388	613	1925·80	295128
569	1787·57	254281	614	1928·94	296092
570	1790·71	255176	615	1932·08	297057
571	1793·85	256072	616	1935·22	298024
572	1796·99	256970	617	1938·36	298992
573	1800·13	257869	618	1941·50	299962
574	1803·27	258770	619	1944·65	300934
575	1806·42	259672	620	1947·79	301907
576	1809·56	260576	621	1950·93	302882
577	1812·70	261482	622	1954·c7	303858
578	1815·84	262389	623	1957·21	304836
579	1818·98	263298	624	1960·35	305815
580	1822·12	264208	625	1963·50	306796
581	1825·27	265120	626	1966·64	307779
582	1828·41	266033	627	1969·78	308763
583	1831·55	266948	628	1972·92	309748
584	1834·69	267865	629	1976·06	310736
585	1837·83	268783	630	1979·20	311725
586	1840·97	269702	631	1982·35	312715
587	1844·11	270624	632	1985·49	313707
588	1847·26	271547	633	1988·63	314700
589	1850·40	272471	634	1991·77	315696
590	1853·54	273397	635	1994·91	316692
591	1856·68	274325	636	1998·05	317690
592	1859·82	275254	637	2001·19	318690
593	1862·96	276184	638	2004·34	319692
594	1866·11	277117	639	2007·48	320695
595	1869·25	278052	640	2010·62	321699

Diam.	Circum.	Area.	Diam.	Circum.	Area.
641	2013·76	322705	686	2155·13	369605
642	2016·90	323713	687	2158·27	370684
643	2020·04	324722	688	2161·42	371764
644	2023·19	325733	689	2164·56	372845
645	2026·33	326745	690	2167·70	373928
646	2029·47	327759	691	2170·84	375013
647	2032·61	328775	692	2173·98	376099
648	2035·75	329792	693	2177·12	377187
649	2038·89	330810	694	2180·27	378276
650	2042·04	331831	695	2183·41	379367
651	2045·18	332853	696	2186·55	380459
652	2048·32	333876	697	2189·69	381554
653	2051·46	334901	698	2192·83	382649
654	2054·60	335927	699	2195·97	383746
655	2057·74	336955	700	2199·11	384845
656	2060·88	337985	701	2202·26	385945
657	2064·03	339016	702	2205·40	387047
658	2067·17	340049	703	2208·54	388151
659	2070·31	341084	704	2211·68	389256
660	2073·45	342119	705	2214·82	390363
661	2076·59	343157	706	2217·96	391471
662	2079·73	344196	707	2221·11	392580
663	2082·88	345237	708	2224·25	393692
664	2086·02	346279	709	2227·39	394805
665	2089·16	347323	710	2230·53	395919
666	2092·30	348368	711	2233·67	397035
667	2095·44	349415	712	2236·81	398153
668	2098·58	350464	713	2239·96	399272
669	2101·73	351514	714	2243·10	400393
670	2104·87	352565	715	2246·24	401515
671	2108·01	353618	716	2249·38	402639
672	2111·15	354673	717	2252·52	403765
673	2114·29	355730	718	2255·66	404892
674	2117·43	356788	719	2258·81	406020
675	2120·58	357847	720	2261·95	407150
676	2123·72	358908	721	2265·09	408282
677	2126·86	359971	722	2268·23	409416
678	2130·00	361035	723	2271·37	410550
679	2133·14	362101	724	2274·51	411687
680	2136·28	363168	725	2277·65	412825
681	2139·42	364237	726	2280·80	413965
682	2142·57	365308	727	2283·94	415106
683	2145·71	366380	728	2287·08	416248
684	2148·85	367453	729	2290·22	417393
685	2151·99	368528	730	2293·36	418539

Diam.	Circum.	Area.	Diam.	Circum.	Area.
731	2296·50	419686	776	2437·88	472948
732	2299·65	420835	777	2441·02	474168
733	2302·79	421986	778	2444·16	475389
734	2305·93	423139	779	2447·30	476612
735	2309·07	424293	780	2450·44	477836
736	2312·21	425448	781	2453·58	479062
737	2315·35	426604	782	2456·73	480290
738	2318·50	427762	783	2459·87	481519
739	2321·64	428922	784	2463·01	482750
740	2324·78	430084	785	2466·15	483982
741	2327·92	431247	786	2469·29	485216
742	2331·06	432412	787	2472·43	486451
743	2334·20	433578	788	2475·58	487688
744	2337·34	434746	789	2478·72	488927
745	2340·49	435916	790	2481·86	490167
746	2343·63	437087	791	2485·00	491409
747	2346·77	438259	792	2488·14	492652
748	2349·91	439433	793	2491·28	493897
749	2353·05	440609	794	2494·42	495143
750	2356·19	441786	795	2497·57	496391
751	2359·34	442965	796	2500·71	497641
752	2362·48	444146	797	2503·85	498892
753	2365·62	445328	798	2506·99	500145
754	2368·76	446511	799	2510·13	501399
755	2371·90	447697	800	2513·27	502655
756	2375·04	448883	801	2516·42	503912
757	2378·19	450072	802	2519·56	505171
758	2381·33	451262	803	2522·70	506432
759	2384·47	452453	804	2525·84	507694
760	2387·61	453646	805	2528·98	508958
761	2390·75	454841	806	2532·12	510223
762	2393·89	456037	807	2535·27	511490
763	2397·04	457234	808	2538·41	512758
764	2400·18	458434	809	2541·55	514028
765	2403·32	459635	810	2544·69	515300
766	2406·46	460837	811	2547·83	516573
767	2409·60	462041	812	2550·97	517848
768	2412·74	463247	813	2554·11	519124
769	2415·88	464454	814	2557·26	520402
770	2419·03	465663	815	2560·40	521681
771	2422·17	466873	816	2563·54	522962
772	2425·31	468085	817	2566·68	524245
773	2428·45	469298	818	2569·82	525529
774	2431·59	470513	819	2572·96	526814
775	2434·73	471730	820	2576·11	528102

Diam.	Circum.	Area.	Diam.	Circum.	Area.
821	2579·25	529391	866	2720·62	589014
822	2582·39	530681	867	2723·76	590375
823	2585·53	531973	868	2726·90	591738
824	2588·67	533267	869	2730·04	593102
825	2591·81	534562	870	2733·19	594468
826	2594·96	535858	871	2736·33	595835
827	2598·10	537157	872	2739·47	597204
828	2601·24	538456	873	2742·61	598575
829	2604·38	539758	874	2745·75	599947
830	2607·52	541061	875	2748·89	601320
831	2610·66	542365	876	2752·04	602696
832	2613·81	543671	877	2755·18	604073
833	2616·95	544979	878	2758·32	605451
834	2620·09	546288	879	2761·46	606831
835	2623·23	547599	880	2764·60	608212
836	2626·37	548912	881	2767·74	609595
837	2629·51	550226	882	2770·88	610980
838	2632·65	551541	883	2774·03	612366
839	2635·80	552858	884	2777·17	613754
840	2638·94	554177	885	2780·31	615143
841	2642·08	555497	886	2783·45	616534
842	2645·22	556819	887	2786·59	617927
843	2648·36	558142	888	2789·73	619321
844	2651·51	559467	889	2792·88	620717
845	2654·65	560794	890	2796·02	622114
846	2657·79	562122	891	2799·16	623513
847	2660·93	563452	892	2802·30	624913
848	2664·07	564783	893	2805·44	626315
849	2667·21	566116	894	2808·58	627718
850	2670·35	567450	895	2811·73	629124
851	2673·50	568786	896	2814·87	630530
852	2676·64	570124	897	2818·01	631938
853	2679·78	571463	898	2821·15	633348
854	2682·92	572803	899	2824·29	634760
855	2686·06	574146	900	2827·43	636173
856	2689·20	575490	901	2830·58	637587
857	2692·34	576835	902	2833·72	639003
858	2695·49	578182	903	2836·86	640421
859	2698·63	579530	904	2840·00	641840
860	2701·77	580880	905	2843·14	643261
861	2704·91	582232	906	2846·28	644683
862	2708·05	583585	907	2849·42	646107
863	2711·19	584940	908	2852·57	647533
864	2714·34	586297	909	2855·71	648960
865	2717·48	587655	910	2858·85	650388

Diam.	Circum.	Area.	Diam.	Circum.	Area.
911	2861·99	651818	956	3003·36	717804
912	2865·13	653250	957	3006·50	719306
913	2868·27	654684	958	3009·65	720810
914	2871·42	656119	959	3012·79	722316
915	2874·56	657555	960	3015·93	723823
916	2877·70	658993	961	3019·07	725332
917	2880·84	660433	962	3022·21	726842
918	2883·98	661874	963	3025·35	728354
919	2887·12	663317	964	3028·50	729867
920	2890·27	664761	965	3031·64	731382
921	2893·41	666207	966	3034·78	732899
922	2896·55	667654	967	3037·92	734417
923	2899·69	669103	968	3041·06	735937
924	2902·83	670554	969	3044·20	737458
925	2905·97	672006	970	3047·34	738981
926	2909·11	673460	971	3050·49	740506
927	2912·26	674915	972	3053·63	742032
928	2915·40	676372	973	3056·77	743559
929	2918·54	677831	774	3059·91	745088
930	2921·68	679291	975	3063·05	746619
931	2924·82	680753	976	3066·19	748151
932	2927·96	682216	977	3069·34	749685
933	2931·11	683680	978	3072·48	751221
934	2934·25	685147	979	3075·62	752758
935	2937·39	686615	980	3078·76	754296
936	2940·53	688084	981	3081·90	755837
937	2943·67	689555	982	3085·04	757378
938	2946·81	691028	983	3088·19	758922
939	2949·96	692502	984	3091·33	760466
940	2953·10	693978	985	3094·47	762013
941	2956·24	695455	986	3097·61	763561
942	2959·38	696934	987	3100·75	765111
943	2962·52	698415	988	3103·89	766662
944	2965·66	699897	989	3107·04	768215
945	2968·81	701380	990	3110·18	769769
946	2971·95	702865	991	3113·32	771325
947	2975·09	704352	992	3116·46	772882
948	2978·23	705840	993	3119·60	774441
949	2981·37	707330	994	3122·74	776002
950	2984·51	708822	995	3125·88	777564
951	2987·65	710315	996	3129·03	779128
952	2990·80	711810	997	3132·17	780693
953	2993·94	713306	998	3135·31	782260
954	2997·08	714803	999	3138·45	783828
955	3000·22	716303	1000	3141·59	785398

TRIGONOMETRICAL RULES.

(The following is a summary of the principles and chief rules of trigonometry. In applying those rules to ordinary mechanical questions, a very brief table, such as Table 6, is sufficient; but for purposes of surveying, astronomy, and navigation, it is necessary to use tables too voluminous to be included in such a work as this.)

I. *Trigonometrical Functions Defined.*—Suppose that A, B, C stand for the three angles of a right-angled triangle, C being the right angle, and that a, b, c stand for the sides respectively opposite to those angles, c being the hypothenuse; then the various names of trigonometrical functions of the angle A have the following meanings:—

$$\sin A = \frac{a}{c}; \ \cos A = \frac{b}{c};$$

$$\text{versin } A = \frac{c-b}{c}; \ \text{coversin } A = \frac{c-a}{c};$$

$$\tan A = \frac{a}{b}; \ \text{cotan } A = \frac{b}{a};$$

$$\sec A = \frac{c}{b}; \ \text{cosec } A = \frac{c}{a}.$$

The *complement* of A means the angle B, such that A + B = a right angle; and the sine of each of those angles is the cosine of the other, and so of the other functions by pairs.

II. *Relations amongst the Trigonometrical Functions of One Angle,* A, *and of its Supplement,* 180° — A:—

$$\sin A = \sqrt{1 - \cos^2 A} = \frac{\tan A}{\sec A} = \frac{1}{\text{cosec } A};$$

$$\cos A = \sqrt{1 - \sin^2 A} = \frac{\text{cotan } A}{\text{cosec } A} = \frac{1}{\sec A};$$

$$\text{versin } A = 1 - \cos A;$$

$$\text{coversin } A = 1 - \sin A;$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{1}{\text{cotan } A} = \sin A \cdot \sec A = \sqrt{\sec^2 A - 1};$$

$$\text{cotan } A = \frac{\cos A}{\sin A} = \frac{1}{\tan A} = \cos A \cdot \text{cosec } A = \sqrt{\text{cosec}^2 A - 1};$$

$$\sec A = \frac{1}{\cos A} = \sqrt{1 + \tan^2 A};$$

$$\text{cosec } A = \frac{1}{\sin A} = \sqrt{1 + \text{cotan}^2 A}.$$

$\sin(180° - A) = \sin A;$
$\cos(180° - A) = -\cos A;$
$\text{versin}(180° - A) = 1 + \cos A = 2 - \text{versin} A;$
$\text{coversin}(180° - A) = \text{coversin} A;$
$\tan(180° - A) = -\tan A;$
$\text{cotan}(180° - A) = -\text{cotan} A;$
$\sec(180° - A) = -\sec A;$
$\text{cosec}(180° - A) = \text{cosec} A.$

To compute sines, &c., approximately by series; reduce the angle to circular measure—that is, to radius-lengths and fractions of a radius-length (see Table 5); let it be denoted by A. Then

$$\sin A = A - \frac{A^3}{2.3} + \frac{A^5}{2.3.4.5} - \frac{A^7}{2.3.4.5.6.7} + \&c.$$

$$\cos A = 1 - \frac{A^2}{2} + \frac{A^4}{2.3.4} - \frac{A^6}{2.3.4.5.6} + \&c.$$

III. *Trigonometrical Functions of Two Angles:—*

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B;$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B;$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

IV. *Formulæ for the Solution of Plane Triangles.*—Let A, B, C be the angles, and a, b, c the sides respectively opposite them.

1. **Relations amongst the Angles**—

$$A + B + C = 180°;$$

or if A and B are given,

$$C = 180° - A - B.$$

2. **When the Angles and One Side are given,** let a be the given side; then the other two sides are

$$b = a \cdot \frac{\sin B}{\sin A}; \quad c = a \cdot \frac{\sin C}{\sin A}.$$

3. **When Two Sides and the Included Angle are given,** let a, b be the given sides, C the given included angle; then

To find the third side. First Method:

$$c = \sqrt{(a^2 + b^2 - 2\,a\,b\cos C)};$$

Second Method: Make $\sin D = \frac{2\sqrt{a\,b}}{a+b} \cdot \cos\frac{C}{2}$; then

$$c = (a + b)\cos D.$$

Third Method: Make $\tan E = \frac{2\sqrt{ab}}{a-b} \cdot \sin \frac{C}{2}$; then

$$c = (a - b) \sec E.$$

To find the remaining angles, A and B.

If the third side has been computed,

$$\sin A = \frac{a}{c} \cdot \sin C; \ \sin B = \frac{b}{c} \cdot \sin C.$$

If the third side has not been computed,

$$\tan \cdot \frac{A+B}{2} = \text{cotan} \frac{C}{2}; \ \tan \frac{A-B}{2} = \frac{a-b}{a+b} \text{cotan} \frac{C}{2};$$

$$A = \frac{A+B}{2} + \frac{A-B}{2}; \ B = \frac{A+B}{2} - \frac{A-B}{2}.$$

4. **When the Three Sides are given,** *to find any one of the angles,* such as C—

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab};$$

otherwise, let

$$s = \frac{a+b+c}{2}; \text{ then}$$

$$\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}; \ \sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}};$$

$$\text{cotan} \frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}; \ \tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}};$$

$$\sin C = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{ab}.$$

NOTE.—In all trigonometrical problems, it is to be borne in mind, that small acute angles, and large obtuse angles, are most accurately determined by means of their *sines, tangents,* and *cosecants,* and angles approaching a right angle by their *cosines, cotangents,* and *secants.*

5. **To Solve a Right-angled Triangle.**—Let C denote the right angle; c the hypothenuse; A and B the two oblique angles; a and b the sides respectively opposite them.

Given, the right angle, another angle B, the hypothenuse c. Then

$$A = 90° - B; \ a = c \cdot \cos B; \ b = c \cdot \sin B.$$

Given, the right angle, another angle B, a side a,

$$A = 90^\circ - B;\ b = a \cdot \tan B;\ c = a \cdot \sec B.$$

Given, the right angle, and the sides a, b,

$$\tan A = \frac{a}{b};\ \tan B = \frac{b}{a};\ c = \sqrt{a^2 + b^2}.$$

Given, the right angle, the hypothenuse c; a side a,

$$\sin A = \cos B = \frac{a}{c};\ b = \sqrt{c^2 - a^2}.$$

Given, the three sides a, b, c, which fulfilling the equation $c^2 = a^2 + b^2$, the triangle is known to be right-angled at C.

$$\sin A = \frac{a}{c};\ \sin B = \frac{b}{c}.$$

6. **To Express the Area of a Plane Triangle in Terms of its Sides and Angles.**

Given, one side, c, and the angles.

$$\text{Area} = \frac{c^2}{2} \cdot \frac{\sin A \sin B}{\sin C}.$$

Given, two sides, b, c, and the included angle A.

$$\text{Area} = \frac{b\, c \cdot \sin A}{2}.$$

Given, the three sides a, b, c. Let $\frac{a + b + c}{2} = s$; then

$$\text{Area} = \sqrt{\left\{ s\,(s - a)\,(s - b)\,(s - c) \right\}}.$$

V. *Rules for the Solution of Spherical Triangles.*—Let A, B, C denote the three angles of a spherical triangle, and α, β, γ, the angles subtended by its sides at the centre of the sphere, called for brevity's sake, the sides.

The *spherical excess* means, the excess of the sum of the angles A + B + C above two right angles.

1. $$\frac{\text{Spherical excess}}{4 \text{ right angles}} = \frac{\text{area of triangle}}{\text{surface of hemisphere}}.$$

2. To compute the *approximate spherical excess, in seconds*, of a triangle on the earth's surface whose area is given; divide that area by one or other of the following divisors, according as it is given in square feet, in square nautical miles, or in square mètres:—

Area given in	Divisor.	Com. Log.
Square feet, ...	2,115,500,000	9·3254101
Square nautical miles,.....	57·29578	1·7581226
Square mètres, ...	196,530,000	8·2934243

3. Given, two angles of a spherical triangle, and the side between them; to find the remaining sides and angle—

Let A, B be the given angles, and γ the given side. Then to find the remaining sides, α and β—

$$\tan\frac{\alpha+\beta}{2} = \tan\frac{\gamma}{2}\cdot\frac{\cos\frac{A-B}{2}}{\cos\frac{A+B}{2}};$$

$$\tan\frac{\alpha-\beta}{2} = \tan\frac{\gamma}{2}\cdot\frac{\sin\frac{A-B}{2}}{\sin\frac{A+B}{2}};$$

$$\alpha = \frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2};\ \beta = \frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}.$$

To find the remaining angle, C, we have the proportion—

$$\sin\alpha : \sin\beta : \sin\gamma :: \sin A : \sin B : \sin C.$$

4. Given, two sides of a spherical triangle and the angle between them; to find the remaining side and angle—

Let α, β be the given sides; C, the given angle.

First Method.—To find the remaining side, γ;

$$\cos\gamma = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta\cdot\cos C;$$

but this formula being unsuited to calculation by logarithms, the following has been deduced from it:—

$$\text{Make } \sin D = \cos\frac{C}{2}\cdot\sqrt{\sin\alpha\cdot\sin\beta};\ \text{then}$$

$$\sin\frac{\gamma}{2} = \sqrt{\left\{\sin\left(\frac{\alpha+\beta}{2}+D\right)\cdot\sin\left(\frac{\alpha+\beta}{2}-D\right)\right\}};$$

and to find the remaining angles, we have the proportion,

$$\sin\gamma : \sin\alpha : \sin\beta :: \sin C : \sin A : \sin B.$$

Second Method.—To find the remaining angles, A, B.

$$\tan\frac{A+B}{2} = \frac{\cos\frac{\alpha-\beta}{2}\cdot\text{cotan}\,\frac{C}{2}}{\cos\frac{\alpha+\beta}{2}};$$

$$\tan\frac{A-B}{2} = \frac{\sin\frac{\alpha-\beta}{2}\cdot\text{cotan}\,\frac{C}{2}}{\sin\frac{\alpha+\beta}{2}};$$

$$A = \frac{A+B}{2} + \frac{A-B}{2}; \ B = \frac{A+B}{2} - \frac{A-B}{2}.$$

The remaining side, γ, is found by the proportion stated above.

5. The three sides of a spherical triangle being given, to find the angles—

Let C be the angle sought in the first instance. Then

$$\cos C = \frac{\cos \gamma - \cos \alpha \cdot \cos \beta}{\sin \alpha \cdot \sin \beta};$$

or otherwise—

$$\text{Let } \sigma = \frac{\alpha + \beta + \gamma}{2} \text{ denote the half sum of the sides;}$$

$$\cos \frac{C}{2} = \sqrt{\frac{\sin \sigma \cdot \sin (\sigma - \gamma)}{\sin \alpha \cdot \sin \beta}}; \ \sin \frac{C}{2} = \sqrt{\frac{\sin (\sigma - \alpha)(\sin \sigma - \beta)}{\sin \alpha \cdot \sin \beta}}.$$

$\cos \frac{C}{2}$ is best when $\frac{C}{2}$ approaches a right angle; $\sin \frac{C}{2}$ when $\frac{C}{2}$ is small.

These formulæ will serve alike to compute any angle. If it is desired to express the angle sought by A or by B, the following substitutions are to be made in the formulæ:—

For the following symbols in the formulæ for C,...	α	β	γ
Substitute respectively in the formulæ for A,...	β	γ	α
— — — — for B,...	γ	α	β

6. In a right-angled spherical triangle, the right angle and any two other parts being given, to find the remaining parts—

Let C be the right angle, and γ the side opposite to it.

Case I. Two sides being given, the third is found by the equation—

$$\cos \alpha \cdot \cos \beta = \cos \gamma;$$

and the oblique angles by the equations—

$$\cos A = \text{cotan } \gamma \cdot \tan \beta; \ \cos B = \text{cotan } \gamma \cdot \tan \alpha;$$

or by the equations—

$$\text{cotan } A = \text{cotan } \alpha \cdot \sin \beta; \ \text{cotan } B = \text{cotan } \beta \cdot \sin \alpha.$$

Case II. Given, a side (α) and the opposite angle (A). Find the side β by the formula—

$$\sin \beta = \tan \alpha \cdot \text{cotan } A;$$

then find γ and B as in Case I.

CASE III. Given, a side (α) and the adjacent angle (B). Find the side γ by the formula—

$$\text{cotan}\ \gamma = \cos A \cdot \text{cotan}\ \beta;$$

then find α and B as in CASE I.

CASE IV. Given, two angles, A, B—

$$\cos \alpha = \frac{\cos A}{\sin B};\ \cos \beta = \frac{\cos B}{\sin A};\ \cos \gamma = \text{cotan}\ A \cdot \text{cotan}\ B.$$

VI. *Approximate Solutions of Spherical Triangles, used in Trigonometrical Surveying.*

1. Given, in a triangle on the earth's surface the length of one side, c, and the adjacent angles, A, B; to find approximately the third angle, C.

Calculate the *approximate area* of the triangle, as if it were plane. From that area calculate the "spherical excess," X. Then

$$C = 180° + X - A - B.$$

2. To find approximately the remaining sides, a, b, of the same triangle. Let α, β, γ be the angles subtended by the sides.

From each of the angles subtract one-third of the spherical excess, and then treat the triangle as if it were plane. That is to say—

$$a = c \cdot \frac{\sin\left(A - \frac{X}{3}\right)}{\sin\left(C - \frac{X}{3}\right)};\ b = c \cdot \frac{\sin\left(B - \frac{X}{3}\right)}{\sin\left(C - \frac{X}{3}\right)}.$$

PROBLEM THIRD.—Given, in a triangle on the earth's surface, two sides, a, b, and the included angle, C, to find the remaining side, c, and angles, A, B.

Compute the *approximate area* as if the triangle were plane; thence compute the spherical excess, X, and deduct one-third of it from the given angle. Then consider the triangle as a plane triangle, in which are given the two sides a, b, and the included angle $C' = C - \frac{X}{3}$, and find the third side, c, and the remaining angles, A′, B′. Then for the remaining angles of the real spherical triangle, take

$$A = A' + \frac{X}{3};\ B = B' + \frac{X}{3}.$$

TABLE 6.—ARCS, SINES, AND TANGENTS, FOR EVERY DEGREE FROM 1° TO 89°.

EXPLANATION.

1. The table gives arcs and their complements in circular measure, sines and cosines, tangents and cotangents, for every whole degree, correct to five places of decimals.

2. Arcs containing fractions of a degree may be found either by the aid of Table 4, Divisions I and L, or by multiplying the fractional part by 0·01745, and adding the product to the arc corresponding to the whole number of degrees.

3. For finding the sines, &c., of angles containing fractions of a degree, the following process is correct to the following numbers of places of decimals:—

For sines and tangents of angles between 0° and 6°, For cosines and cotangents of angles between 84° and 90°, ...	To five places;
For sines of angles between 6° and 90°, ... For cosines of angles between 0° and 84°, ... For tangents of angles between 6° and 30°, ... For cotangents of angles between 60° and 84°, ...	To four places;
For tangents of angles between 30° and 45°, ... For cotangents of angles between 45° and 60°, ...	To three places.

Multiply the fraction of a degree by the difference between the values of the quantity to be found for the next lower and next higher whole numbers of degrees, and add the product to the value for the next lower whole number of degrees.

EXAMPLE.—Required the sine of 30° 20′ = 30° $\frac{1}{3}$.

Sine of 31°, ...	·51504
Sine of 30°, ...	·50000
Difference, ...	·01504
	× $\frac{1}{3}$
	·00501
Add sine of 30°, ...	·50000
Sin 30°$\frac{1}{3}$, correct to four places of decimals,	·50501

4. The *sine* or *cosine* of an angle containing a fraction of a degree may be found correct to five places of decimals, when required, as follows:—Find a first approximation to the sine or cosine by the preceding rule. Then multiply together the given fraction of a degree, the difference between that fraction and unity, the fraction ·00015, and the approximate sine or cosine already found; the

product will be a correction, to be added to the approximate sine or cosine for a more exact value.

EXAMPLE.—Required the sine of $30°\frac{1}{3}$, correct to five places of decimals.

First approximation, as already found,............... ·50501
Correction to be added, $\frac{1}{3} \times \frac{2}{3} \times \cdot00015 \times \cdot50501 = \cdot000017$
Sum ·505027

so that the sine required, to five places of decimals, is ·50503.

CORRECTION-FACTORS, TO MULTIPLY APPROXIMATE SINES AND COSINES.

Minutes.	Factors.	Minutes.
5	1·000011	55
10	1·000021	50
15	1·000028	45
20	1·000033	40
25	1·000036	35
30	1·000037	30

Angle.	Arc.	Sine.	Tangent.	Co-tangent.	Co-sine.	Co-arc.	Co-angle.
1°	01745	01745	01746	57·28996	99985	1·55335	89°
2	03491	03490	03492	28·63625	99939	1·53589	88
3	05236	05234	05241	19·08114	99863	1·51844	87
4	06981	06976	06993	14·30067	99756	1·50099	86
5	08727	08716	08749	11·43005	99619	1·48353	85
6	10472	10453	10510	9·51436	99452	1·46608	84
7	12217	12187	12278	8·14435	99255	1·44863	83
8	13963	13917	14054	7·11540	99027	1·43117	82
9	15708	15643	15838	6·31375	98769	1·41372	81
10	17453	17365	17633	5·67128	98481	1·39627	80
11	19199	19081	19438	5·14455	98163	1·37881	79
12	20944	20791	21256	4·70463	97815	1·36136	78
13	22689	22495	23087	4·33148	97437	1·34391	77
14	24435	24192	24933	4·01078	97030	1·32645	76
15	26180	25882	26795	3·73205	96593	1·30900	75
16	27925	27564	28675	3·48741	96126	1·29155	74
17	29671	29237	30573	3·27085	95630	1·27409	73
18	31416	30902	32492	3·07768	95106	1·25664	72
19	33161	32557	34433	2·90421	94552	1·23919	71
20	34907	34202	36397	2·74748	93969	1·22173	70
21	36652	35837	38386	2·60509	93358	1·20428	69
22	38397	37461	40403	2·47509	92718	1·18683	68
23	40143	39073	42447	2·35585	92050	1·16937	67
24	41888	40674	44523	2·24604	91355	1·15192	66
25	43633	42262	46631	2·14451	90631	1·13447	65
26	45379	43837	48773	2·05030	89879	1·11701	64
27	47124	45399	50953	1·96261	89101	1·09956	63
28	48869	46947	53171	1·88073	88295	1·08211	62
29	50615	48481	55431	1·80405	87462	1·06465	61
30	52360	50000	57735	1·73205	86603	1·04720	60
31	54105	51504	60086	1·66428	85717	1·02975	59
32	55850	52992	62487	1·60033	84805	1·01230	58
33	57596	54464	64941	1·53986	83867	99484	57
34	59341	55919	67451	1·48256	82904	97739	56
35	61087	57358	70021	1·42815	81915	95993	55
36	62832	58779	72654	1·37638	80902	94248	54
37	64577	60182	75355	1·32704	79864	92503	53
38	66322	61566	78129	1·27994	78801	90758	52
39	68068	62932	80978	1·23490	77715	89012	51
40	69813	64279	83910	1·19175	76604	87267	50
41	71558	65606	86929	1·15037	75471	85522	49
42	73304	66913	90040	1·11061	74314	83776	48
43	75049	68200	93252	1·07237	73135	82031	47
44	76794	69466	96569	1·03553	71934	80286	46
45	78540	70711	1·00000	1·00000	70711	78540	45
Co-angle.	Co-arc.	Co-sine.	Co-tangent.	Tangent.	Sine.	Arc.	Angle.

RULES FOR THE MENSURATION OF FIGURES.

SECTION I.—PLANE AREAS.

1. **Parallelogram.** RULE A.—Multiply the length of one of the sides by the perpendicular distance between that side and the opposite side.

RULE B.—Multiply together the lengths of two adjacent sides and the sine of the angle which they make with each other. (When the parallelogram is right-angled, that sine is = 1.)

2. **Trapezoid** (or four-sided figure bounded by a pair of parallel straight lines, and a pair of straight lines not parallel). Multiply the half sum of the two parallel sides by the perpendicular distance between them.

3. **Triangle.** RULE A.—Multiply the length of any one of the sides by one-half of its perpendicular distance from the opposite angle.

RULE B.—Multiply one-half of the product of any two of the sides by the sine of the angle between them.

RULE C.—Multiply together the following four quantities: the half sum of the three sides, and the three remainders left after subtracting each of the three sides from that half sum; extract the square root of the quotient; that root will be the area required.

NOTE.—*Any polygon* may be measured by dividing it into triangles, measuring those triangles, and adding their areas together.

4. **Parabolic Figures of the Third Degree.**—The parabolic figures to which the following rules apply are of the following kind (see figs. 1 and 2.) One boundary is a straight line, A X, called the *base* or *axis;* two other boundaries are either points in that line, or straight lines at right angles to it, such as A B and X C, called *ordinates;* and the fourth boundary is a curve, B C, of the *parabolic class*, and of the *third degree;* that is, a curve whose *ordinate*

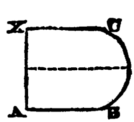


Fig 1.

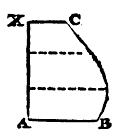


Fig. 2.

(or perpendicular distance from the base A X) at any point is expressed by what is called an *algebraical function of the third degree* of the *abscissa* (or distance of that ordinate from a fixed point in the base). An algebraical function of the third degree of a quantity consists of terms not exceeding four in number, of which

one may be constant, and the rest must be proportional to powers of that quantity not higher than the cube.

Rule A.—Divide the base, as in fig. 1, into two equal parts or intervals; measure the endmost ordinates, A B and X C, and the middle ordinate (which is dotted in the figure) at the point of division; add together the endmost ordinates and *four times* the middle ordinate, and divide the sum by *six;* the quotient will be the *mean breadth* of the figure, which, being multiplied by the length of the base, A X, will give the area.

Rule B.—Divide the base, as in fig. 2, into three equal intervals; measure the endmost ordinates, A B and X C, and the two intermediate ordinates (which are dotted) at the points of division; add together the endmost ordinates and three times each of the intermediate ordinates; divide the sum by *eight;* the quotient will be the *mean breadth* of the figure, which, being multiplied by the length of the base, A X, will give the area.

In applying either of those rules to figures whose curved boundaries meet the base at one or both ends, the ordinate at each such point of meeting is to be made = 0.

5. **Any Plane Area.**—Draw an axis or base-line, A X, in a convenient position. The most convenient position is usually parallel to the greatest length of the area to be measured. Divide the length of the figure into a convenient number of equal intervals, and measure breadths in a direction perpendicular to the axis at the two ends of that length, and at the points of division, which breadths will, of course, be one more in number than the intervals. (For example, in fig. 3, the length of the figure is divided into ten equal intervals, and eleven breadths are measured at b_0, b_1, &c.) Then the following rules are exact, if the sides of the figure are bounded by straight lines, and by parabolic curves not exceeding the third degree, and are approximate for boundaries of any other figures.

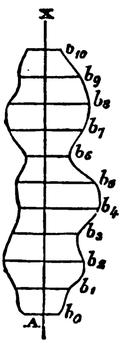


Fig. 3.

Rule A. (*"Simpson's First Rule,"* to be used when the number of intervals is even.)—Add together the two endmost breadths, *twice* every second intermediate breadth, and *four times* each of the remaining intermediate breadths; multiply the sum by the common interval between the breadths, and divide by 3; the result will be the area required.

For two intervals the multipliers for the breadths are 1, 4, 1 (as in Rule A of the preceding Article); for four intervals, 1, 4, 2, 4, 1; for six intervals, 1, 4, 2, 4, 2, 4, 1; and so on. These are called "Simpson's Multipliers."

Example.—Length, 120 feet, divided into six intervals of 20 feet each.

Breadths in Feet and Decimals.	Simpson's Multipliers.	Products.
17·28	1	17·28
16·40	4	65·60
14·08	2	28·16
10·80	4	43·20
7·04	2	14·08
3·28	4	13·12
0	1	0·00
	Sum,	181·44
	× Common interval,	20 feet.
	÷ 3)	3628·8
	Area required,	1209·6 square feet.

Rule B. ("*Simpson's Second Rule,*" to be used when the number of intervals is a multiple of 3.)—Add together the two endmost breadths, *twice* every third intermediate breadth, and *thrice* each of the remaining intermediate breadths; multiply the sum by the common interval between the breadths, and by 3; divide the product by 8; the result will be the area required.

"Simpson's multipliers" in this case are, for three intervals, 1, 3, 3, 1; for six intervals, 1, 3, 3, 2, 3, 3, 1; for nine intervals, 1, 3, 3, 2, 3, 3, 2, 3, 3, 1; and so on.

Example.—Length, 120 feet, divided into six intervals of 20 feet each.

Breadths in Feet and Decimals.	Simpson's Multipliers.	Products.
17·28	1	17·28
16·40	3	49·20
14·08	3	42·24
10·80	2	21·60
7·04	3	21·12
3·28	3	9·84
0	1	0·00
	Sum,	161·28
	× Common interval,	20 feet.
		3225·6
		× 3
	÷ 8)	9676·8
	Area required,	1209·6 square feet.

Remarks.—The preceding examples are taken from a parabolic figure of the third degree, for which both Simpson's Rules are exact; and the results of using them agree together precisely. For other figures, for which the rules are approximate only, the first

rule is in general somewhat more accurate than the second, and is therefore to be used unless there is some special reason for preferring the second.

The probable extent of error in applying Simpson's First Rule to a given figure is, in most cases, nearly proportional to the fourth power of the length of an interval.

The errors are greatest where the boundaries of the figure are most curved, and where they are nearly perpendicular to the axis. In such positions of a figure the errors may be diminished by subdividing the axis into smaller intervals.

RULE C. ("*Merrifield's Trapezoidal Rule,*" for calculating separately the areas of the parts into which a figure is subdivided by its equidistant ordinates or breadths.)—Write down the breadths in their order. Then take the *differences* of the successive breadths, distinguishing them into positive and negative according as the breadths are increasing or diminishing, and write them opposite the intervals between the breadths. Then take the differences of those differences, or *second differences,* and write them opposite the intervals between the first differences, distinguishing them into positive and negative according to the following principles:—

First Differences.	Second Difference.
Positive increasing, or Negative diminishing,	.........Positive.
Negative increasing, or Positive diminishing,	.........Negative.

In the column of second differences there will now be two blanks opposite the two endmost breadths; those blanks are to be filled up with numbers each forming an arithmetical progression with the two adjoining second differences, if these are unequal, or equal to them, if they are equal.

Divide each second difference by 12; this gives a *correction,* which is to be *subtracted* from the breadth opposite it if the second difference is *positive,* and *added* to that breadth if the second difference is negative.

Then to find the area of the division of the figure contained between a given pair of ordinates or breadths; *multiply the half sum of the corrected breadths by the interval between them.*

The area of the whole figure may be formed either by adding together the areas of all its divisions, or by adding together the halves of the endmost corrected breadths, and the whole of the intermediate breadths, and multiplying the sum by the common interval.

EXAMPLE.—Length, 120 feet, divided into six intervals of 20 feet each.

Breadths in Feet and Decimals.	First Differences.	Second Differences.	Corrections.	Corrected Breadths. Feet.	Areas of Divisions. Sq. Feet.
17·28		(— 1·92)	+ 0·16	17·44	
	— 0·88				339·6
16·40		— 1·44	+ 0·12	16·52	
	— 2·32				306·8
14·08		— 0·96	+ 0·08	14·16	
	— 3·28				250·0
10·80		— 0·48	+ 0·04	10·84	
	— 3·76				178·8
7·04		0	0	7·04	
	— 3·76				102·8
3·28		+ 0·48	— 0·04	3·24	
	— 3·28				31·6
0		(+ 0·96)	— 0·08	— 0·08	
			Total area, square feet,		1209·6

The second differences enclosed in parentheses at the top and bottom of the column are those filled in by making them form an arithmetical progression with the second differences adjoining them. The last corrected breadth in the present example is negative, and is therefore subtracted instead of added in the ensuing computation.

Rule D.—("*Common Trapezoidal Rule,*" to be used when a rough approximation is sufficient.) Add together the halves of the endmost breadths, and the whole of the intermediate breadths, and multiply the sum by the common interval.

Example.—The same as before.

	Feet	
Half breadth at one end, 17·28 ÷ 2 =	8·64	
Intermediate breadths,	16·40	
	14·08	
	10·80	
	7·04	
	3·28	
Half breadth at the other end, . .	0	
	60·24	
× Common interval, .	20	
Approximate area, ...	1204·8	square feet.
True area as before computed, . .	1209·6	
Error,	—4·8	square feet.

6. **Circle.**—The area of a circle is equal to its circumference multiplied by one-fourth of its diameter, and therefore to the square of the diameter multiplied by one-fourth of the ratio of the circum-

ference to the diameter. The ratio of the area of a circle to the square of its diameter (which ratio is denoted by the symbol $\frac{\pi}{4}$) is *incommensurable;* that is, not expressible exactly in figures; but it can be found approximately, to any required degree of precision. Its value has been computed to 250 places of decimals; but the following approximations are close enough for most purposes, scientific or practical:—

Approximate Values of $\frac{\pi}{4}$.	Errors in Fractions of the Circle, about
·7853981634 −	+ one-300,000,000,000th.
·785398 +	− one-5,000,000th.
·7854 −	+ one-400,000th.
$\frac{355}{4 \times 113}$ −	+ one-13,000,000th.
$\frac{11}{14}$ −	+ one-2,500th.

Tables 4 and 5 contain examples of the results of such calculations.

The *diameter of a circle equal in area to a given square* is very nearly 1·12838 × the side of the square. The following table gives examples of this:—

TABLE 4 N.—MULTIPLIERS FOR CONVERTING

	Sides of Squares into Diameters of Equal Circles.	Diameters of Circles into Sides of Equal Squares.	
1	1·12838	0·88623	1
2	2·25676	1·77245	2
3	3·38514	2·65868	3
4	4·51352	3·54491	4
5	5·64190	4·43113	5
6	6·77028	5·31736	6
7	7·89866	6·20359	7
8	9·02704	7·08981	8
9	10·15542	7·97604	9
10	11·28380	8·86227	10

7. The area of a **Circular Sector** (O A C B, fig. 4) is the same fraction of the whole circle that the angle A O B of the sector is of a whole revolution. In other words, multiply *half the square of the radius*, or *one-eighth of the square of the diameter*, by the circular measure (to radius unity) of the angle A O B; the product will be the area of the sector. (For circular measures of angles, see Tables 4 and 6.)

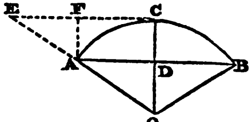


Fig. 4.

8. A **Circular Segment** (A D B C, fig. 4) is equal to the sector O A C B less the triangle O A B. Hence, from the circular measure of the angle A O B subtract its sine; multiply the remainder by half the square of the radius; the product will be the area of the segment.

9. **Circular Spandrils.** CASE I.—Spandril A C E, bounded by the arc A C, the tangent C E, and the external secant A E. From the tangent of the angle A O C subtract the circular measure of that angle; multiply the remainder by half the square of the radius; the product will be the area.

CASE II.—Spandril A C F, bounded by the arc A C, the tangent C F, and the straight line A F perpendicular to C F. From twice the sine of the angle A O C subtract the circular measure of that angle, and half the sine of double the angle; multiply the remainder by half the square of the radius; the product will be the area.

10. **Ellipse.** CASE I.—Given (in fig. 5), the two axes, A O a, B O b. Multiply the lengths of those axes together, and their product by $\frac{\pi}{4}$. (See Article 6 of this section.)

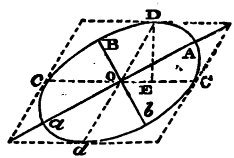


Fig. 5.

CASE II.—Given, a pair of conjugate diameters, C O c, D O d (that is, a pair of diameters each of which is parallel to the tangents at the ends of the other). From one end of one of those diameters (as D) let fall D E perpendicular to the other diameter, C c; multiply C c by twice D E, and the product by $\frac{\pi}{4}$; or otherwise—multiply together the given conjugate diameters, and their product by the sine of the angle between them, and by $\frac{\pi}{4}$.

11. **Elliptic Sectors and Segments.**—In fig. 6, let O A, O B, be the greater and lesser semi-axes of an ellipse, A C D B a quadrant of that ellipse, C O D an elliptic sector, and C D an elliptic segment. About O with the radius O A describe the circular quadrant A c d b; through C and D draw C c and D d parallel to O B, cutting the circle in c and d. Join O c, O d, c d. Then

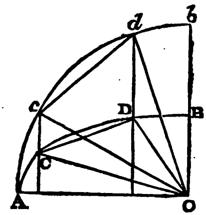


Fig. 6.

as O A
: is to O B
: : so is the circular { sector O c d / or segment c d }
: to the elliptic { sector O C D / or segment C D.

12. **Hyperbolic Sector.**—In fig 7, let the straight lines O X, O Y, be the *asymptotes* of a hyperbola; A and B two points in that hyperbola, and O A B a *hyperbolic sector*, whose area is required. A characteristic property of the hyperbola is the following: that if from any point in it, such as A or B, there be drawn straight lines parallel to the asymptotes, so as to enclose a parallelogram, such as O C A E or O D B F, the areas of all such parallelograms shall be equal for a given hyperbola. Let the common area of them all for the given hyperbola be called the *modulus;* then the area of the sector A O B is equal to the modulus multiplied by the hyperbolic logarithm of the ratio $\frac{\text{A C}}{\text{B D}} = \frac{\text{B F}}{\text{A E}}$. (For hyperbolic logarithms, see Tables 3 and 3 A.) The areas A C D B and A E F B are each of them equal to the sector A O B.

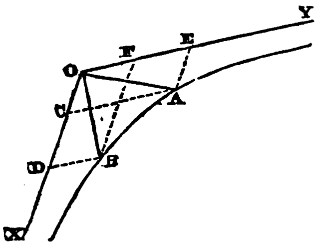


Fig. 7.

13. **Harmonic Curve** (see fig. 8). Case I. *Single Harmonic Curve.*—Let A B be the base and O C the height of a harmonic curve, O being the middle of the base. The ordinate X Y, at any point, X, in the base, is equal to O C multiplied by the cosine of an angle bearing the same proportion to two right-angles that O X bears to A B. Then the area A C B is equal to A B × O C × $\frac{2}{\pi}$. The approximate value of $\frac{2}{\pi}$, correct to about one-2,000,000th, is ·63662.

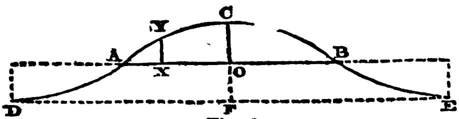


Fig. 8.

Case II. *Double Harmonic Curve, or Curve of Versed Sines.*—Let the harmonic curve be continued to D and E as far below A B as C is above that line; the arcs A D and B E being similar to A C and B C, but inverted; so that the new base D E is twice the length of A B, and is a tangent to the curve at D and E; and the new height F C is twice O C. Then the area D C E = D E × F C × $\frac{1}{2}$.

14. **Trochoid, or Rolling Wave-line** (see fig. 9).—Let a circular disc, H, roll along a straight line, E F; then a tracing point fixed in the rolling disc traces a trochoïd, of which A C B is one wave,

extending from one of the lowest positions of the tracing-point to the next. Let the base of the figure to be measured be the straight line, A B, touching the trochoïd at A and B; then the length of that base is equal to the circumference of the rolling circle, H; and the extreme breadth of the figure, C D, is twice the *tracing radius*, or distance of the tracing-point from the centre of the rolling circle.

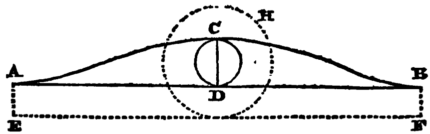


Fig. 9.

To find the area, A C B; multiply the base, A B, by the tracing radius, $\frac{1}{2}$ C D, and to the product add the area of the circle described on C D as a diameter.

15. **Catenary**, or Chain-curve.—See Section IV., further on.

Section II.—Cylindrical, Conical, and Spherical Areas.

1. **Cylinder.**—The curved surface of a cylinder is measured by multiplying its circumference by its length.

2. **Cone.**—The curved surface of a right cone is greater than the area of its circular base, in the same proportion in which the slanting side of the cone is longer than the radius of its base.

3. **Sphere.**—The surface of a sphere is equal to the curved surface of the circumscribed cylinder—that is, to the diameter of the sphere multiplied by its circumference, or to four times the area of a great circle of the sphere.

4. **Spherical Zones and Segments.**—The area of a zone or belt, or of a segment of a sphere, is equal to that of a zone of equal height on the curved surface of the circumscribed cylinder. In other words, multiply the height of the zone or segment by the circumference of a great circle of the sphere.

Thus, in fig. 10, B A C is a hemisphere; B D E C, a circumscribed cylinder; O A, the axis of that cylinder; F K, a plane perpendicular to that axis, cutting it in H, and cutting the sphere in the small circle I J. Then I A J is a segment of the sphere; and its area is equal to that of the cylindrical belt F D E K, or to the circumference of the sphere × A H; and B I J C is a zone or belt of the sphere, whose area is equal to that of the cylindrical belt B F K C, or to the circumference of the sphere × H O.

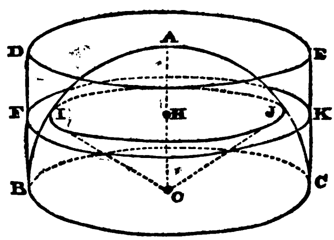


Fig. 10.

5. **Spherical Triangle.**—As a complete revolution (or four right-angles)

: is to the spherical excess (see Trigonometrical Rules, Division V.),

: : so is the surface of the hemisphere

: to the area of the triangle.

Section III.—Volumes.

1. **Any Prism or Cylinder with Plane Parallel Ends.** Rule A.—Measure the sectional area of the prism or cylinder upon a plane perpendicular to its axis; multiply that area by the length; the product will be the volume.

Rule B.—Multiply the area of either end by the perpendicular distance between the planes of the ends.

2. **Rectangular Prism, with Plane Ends not Parallel.**—Measure the sectional area on a plane perpendicular to the axis; multiply it by the half-sum of the lengths measured along a pair of opposite edges.

3. **Triangular Prism, with Plane Ends not Parallel.**—Measure the sectional area on a plane at right angles to the axis; multiply by the third part of the sum of the lengths of the three edges.

4. **Rectangular Prism with Curved Ends** (*"Woolley's Rule"*).—Add together the lengths along the middles of the four faces of the prism, and twice the length along the axis, and divide the sum by *six*, for the mean length; multiply the mean length by the sectional area measured on a plane perpendicular to the axis.

This rule is exact when the ends of the prism are curved surfaces, of a degree not exceeding the third, and approximate for other curved surfaces.

5. **Any Solid.** Method I. *By Layers.*—Choose a straight axis in any convenient position. (The most convenient is usually parallel to the greatest length of the solid.) Divide the whole length of the solid, as marked on the axis, into a convenient number of equal intervals, and measure the sectional area of the solid upon a series of planes crossing the axis at right angles at the two ends and at the points of division. Then treat those areas as if they were the breadths of a plane figure, applying to them Rule A, B, or C, of Section I., Article 5; and the result of the calculation will be the volume required. If Rule C is used, the volume will be obtained in separate layers.

This method is exact when the sectional area is an algebraical function of the distance along the axis of a degree not higher than the third. Some of the figures which fulfil that condition are specified further on. For other figures the method is approximate only.

METHOD II. *By Prisms or Columns* ("*Woolley's Rule*").—Assume a plane in a convenient position as a base, divide it into a network of equal rectangular divisions, and conceive the solid to be built of a set of rectangular prismatic columns, having these rectangular divisions for their sectional areas. Measure the thickness of the solid at the *centre* and at the *middle of each of the sides* of each of those rectangular columns; calculate the volume of each column by the rule of Section III., Article 4, and take the sum of those volumes.

Or otherwise, to calculate the volume of the solid at one operation—add together the doubles of all the thicknesses before-mentioned, which are in the interior of the solid, and the simple thicknesses which are at its boundaries; divide the sum by *six*, and multiply by the area of one rectangular division of the base.

6. **Cone or Pyramid.**—Multiply the area of the base by one-third of the height, measured perpendicularly to the plane of the base.

7. **Sphere and Ellipsoid.** RULE A.—Multiply the area of a diametral section (found by Section I., Article 6, for a circle, or by Section I., Article 10, for an ellipse) by two-thirds of the height measured perpendicularly to the plane of that section.

RULE B.—Multiply together the three axes of an ellipsoid (or take the cube of the diameter of a sphere); then multiply by the factor $\frac{\pi}{6}$.

Approximate Values of $\frac{\pi}{6}$.	Errors, about
$0{\cdot}5235987756 -$	+ one-300,000,000,000th.
$0{\cdot}523599 -$	+ one-2,300,000th.
$0{\cdot}5236 -$	+ one-400,000th.
$\frac{355}{6 \times 113} -$	+ one-13,000,000th.
$\frac{377}{720} -$	+ one-40,000th.
$\frac{11}{21} -$	+ one-2,500th.

8. **Frustum—Prismoid—Spherical and Ellipsoidal Segments and Zones.**—The following rule is applicable to

A frustum, or part cut off from a cone or pyramid by a plane parallel to the base (fig. 11);

A prismoid, or solid bounded by two parallel quadrangular ends (E F L K, C D I H, fig. 12) and four plane faces, parallel or not (C F L H, H L K I, I K E D, D E F C);

A segment cut off by one plane, or a zone cut out by a pair of parallel planes, from a sphere or an ellipsoid (fig. 13);

And generally, to any solid bounded endwise by a pair of parallel planes, and sideways by a conical, spherical, or ellipsoidal surface, or by any number of planes.

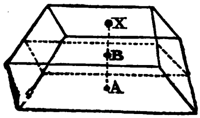


Fig. 11.

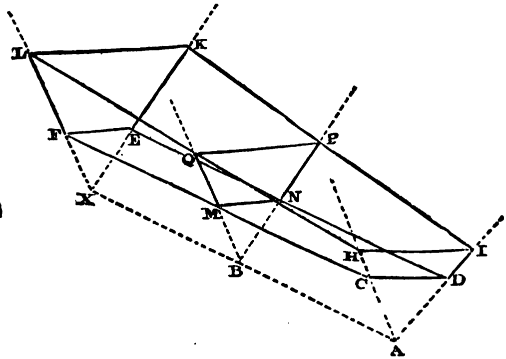


Fig. 12.

To the areas of the ends add four times the area of a cross section made by a plane midway between and parallel to the ends; divide the sum by *six* for the *mean section,* which multiply by the length A X measured perpendicular to the planes of the ends.

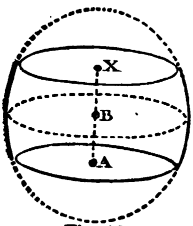


Fig. 13.

9. **Spherical Cone** (O I A J, fig. 10).—Find by Section II., Article 4, the area of the segment I A J, which is the base of the cone; multiply that area by one-third of the radius of the sphere.

SECTION IV.—LENGTHS OF CURVES.

The measurement of the lengths of curves is called *rectification.*

1. **Any Curve.** RULE A. *By Chords.*—Let A B (fig. 14) be the curved line whose length is to be measured. Divide it into any EVEN number of intervals, equal or unequal, by points (such as 1, 2, 3), measure the series of straight chords (such as $\overline{A\,1}$, $\overline{1\,2}$, $\overline{2\,3}$, $\overline{3\,B}$), which span those intervals, and take the sum of their lengths; measure also the straight chords (such as $\overline{A\,2}$, $\overline{2\,B}$) which span the intervals by pairs, and take the sum of their lengths; to the first sum add *one-third*

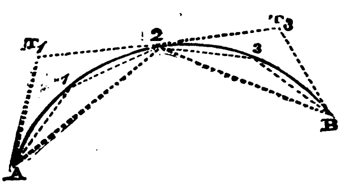


Fig. 14.

of the difference between it and the second sum; the result will be the approximate length of the curve.

RULE B. *By Chords and Tangents.*—Divide the curve into any number of intervals, equal or unequal, by points (such as 2 in fig. 14). At the ends and points of division draw straight tangents (such as $A\,T_1$, $T_1\,T_3$, $T_3\,B$), stopping at their first intersections with each other. Measure the total length of those tangents, and also the total length of the straight chords (such as $\overline{A\,2}$, $\overline{2\,B}$). To the total length of the tangents add *twice* the total length of the chords, and divide the sum by 3; the quotient will be the approximate length required.

RULE C. *By Tangents.*—Let A B (fig. 15) be the curved line to be measured. Through its two ends, A and B, draw a pair of parallel lines in any convenient direction (but the more nearly that direction is perpendicular to a straight line from A to B the more accurate will the result be). Divide the distance between those parallel lines into an *even* number of *equal* intervals, by means of intermediate parallel lines, cutting the curve in intermediate points, such as 1, 2, 3. At each of these intermediate points, and also at the ends of the curve, draw straight tangents extending the whole way from one of the outer parallel lines to the other (as $A\,T_0$, $t_1\,T_1$, $t_2\,T_2$, $t_3\,T_3$, $t_4\,B$). Multiply the lengths of those tangents in their order by "Simpson's Multipliers"

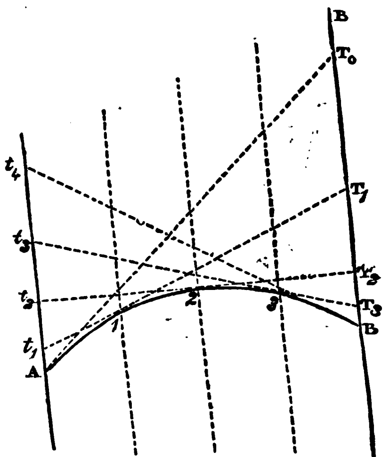


Fig. 15.

(as in Section I., Article 5, Rule A); add together the products, and divide their sum by the sum of the multipliers; the quotient will be the approximate length required.

REMARK.—The errors of the three preceding rules vary nearly as the fourth power of the *angular interval*, or angle made by the tangents at the two ends of an interval; hence the *lengths* of the intervals should be made least where the curvature is most rapid, so that the angular intervals may be nearly equal. The following are the proportionate errors in applying the rules to circular arcs with angular intervals of 30°; + meaning too great, and − too small:—

Rule A,..................	Error about	− one-6,500th.
„ B,..................	„	+ one-4,000th.
„ C,..................	„	+ one-250th.

With half the angular interval, the *errors* are reduced in each case to one-sixteenth.

RULE D. *For Arcs of Small Curvature.*—In fig. 16, let A B be the arc to be measured. Draw the straight chord B A; produce it to C, making A C $= \frac{1}{2}$ B A; about C, with the radius C B $= \frac{3}{2}$ B A, draw a circle; then draw the straight line A D, touching the arc A B in A, and meeting the last mentioned circle in D; A D will be nearly equal to the arc A B.

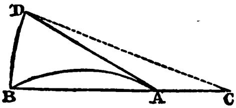


Fig. 16.

For a circular arc of 30° the error of this rule is about + one-14,400th; and it varies nearly as the fourth power of the angular interval.

RULE E. *From a given Point, to set off a given Length along a Curved Line.*—In fig. 16A, let A D be part of the given curve; A the given point, and A B a straight line of the given length, drawn so as to touch the curve at A. In A B take A C $= \frac{1}{4}$ A B; and about C, with the radius C B $= \frac{3}{4}$ A B, draw a circular arc B D, meeting the given curve in D. The arc A D will be very nearly equal to A B.

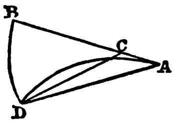


Fig. 16A.

RULE F. *To reduce a "Rolled Curve" to an equal Circular Arc.*—Let D E be a base line of any figure, upon which a disc of any figure rolls; a point, B, in that disc traces a "rolled curve," F B H. The *rolling radius* at any instant is the distance, B A, from the tracing-point, B, to the point of contact, A, of the disc and base line, and is everywhere perpendicular to the rolled curve.

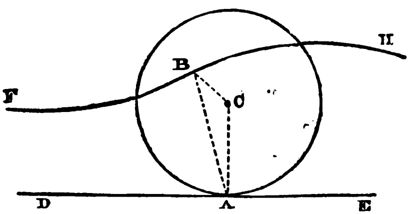


Fig. 17.

Divide the whole angle through which the disc turns in describing the given curve by rolling, into an even number of angular intervals, corresponding to an odd number of intermediate positions of the disc; measure the rolling radii corresponding to those intermediate positions, and to the endmost positions. Multiply the series of rolling radii by the multipliers in Simpson's first rule

(Section I., Article 5, Rule A); add together the products; divide their sum by the sum of the multipliers; the quotient will be the *mean rolling radius*. Then with the mean rolling radius describe a circular arc subtending an angle equal to the total angle through which the disc turns in rolling; that arc will be nearly equal in length to the given rolled curve.

Instances of the application of this to particular cases will be given in Article 3 of this Section, Rule C, and in Articles 4 and 5.

2. **Circle.**—The incommensurable ratio of the circumference of a circle to its diameter is denoted by π. The following are approximations to its value, of various degrees of accuracy:—

Approximate Value of π.	Error, about
$3{\cdot}1415926536-$	+ one-300,000,000,000th.
$3{\cdot}141593-$	+ one-9,000,000th.
$3{\cdot}1416-$	+ one-400,000th.
$\frac{355}{113}-$	+ one-13,000,000th.
$\frac{377}{120}-$	+ one-40,000th.
$\frac{360}{114{\cdot}6}+$	− one-13,000th.
$\frac{22}{7}-$	+ one-2,500th.

For the approximate value of π to 250 places of decimals, see Bierens de Haan on *Definite Integrals*.

For particular results, see Table 5.

3. **A Circular Arc** may be measured by any of the preceding general rules, especially Rule D, page 76; also by the following special rules:—

Rule A. *By Calculation.*—Multiply 2π by the ratio which the arc bears to a whole circle; the product will be the ratio which the arc bears to its radius.

Rule B. *By Construction.*—In fig. 18, let C be the centre of the circle, and A B the arc to be measured. Bisect the arc A B in D, and the arc A D in E. Draw the straight tangent A F, and the straight secant C E F, cutting each other in F. Draw the straight line F B. Then A F + F B will be approximately equal in length to the arc A B.

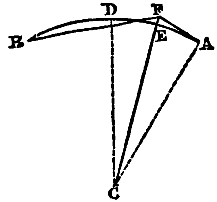


Fig. 18.

The error of this rule for a circular arc equal in length to its radius is about + one-4,000th part of the length of the arc;

and it varies nearly as the fourth power of the angle subtended by the arc.

RULE C. *From a given Point on a given Circle to lay off an Arc approximately equal in length to a given Straight Line.*—In fig. 19,

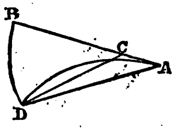


Fig. 19.

let A be the given point, and A D part of the given circle. At A draw the straight tangent A B of the given length. In A B take A C $= \frac{1}{4}$ A B; and about C, with the radius C B $= \frac{3}{4}$ A B, draw the circular arc B D, cutting the given circle in D. Then the arc A D will be nearly equal in length to A B.

The error of this rule for an arc equal in length to its radius is about + one-1,000th part of the length of the arc; and it varies nearly as the fourth power of the angle subtended by the arc.

RULE D. *To Construct a Circular Arc nearly equal in length to a given Straight Line, and subtending a given Angle.*—In fig. 19, let A B be the given straight line. In A B, take A C $= \frac{1}{4}$ A B; and about C, with the radius C B $= \frac{3}{4}$ A B, draw a circle B D. From A draw the straight line A D, making the angle B A D = one-half of the given angle, and cutting the circle B D in D. A and D will be the two ends of the required arc. Then, by the usual method, draw the circular arc A D so as to touch A B in A, and pass through the point D; this will be the arc required. The error of this rule is the same with that of the preceding rule.

4. **Elliptic Arc.**—To construct a circular arc approximately equal

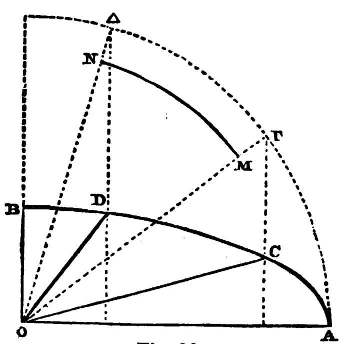


Fig. 20.

to a given arc, C D, fig. 20, not exceeding a quadrant of an ellipse whose semi-axes O A and O B are given.

In fig. 21 draw a straight line, in which take E F = O B and F G = O A. Bisect it in H; and about that point, with the radius H F = H K = $\frac{\text{O A} - \text{O B}}{2}$, describe a circle. Mark the points c and d in that circle, by laying off E c = O C and E d = O D.

Then divide the arc $c\,d$ into an even number of equal intervals, as the case may be, and measure the distances from the ends of the arc and the points of division to G; these will be rolling radii of the ellipse, as generated by rolling a circle of the radius E H inside a circle of the radius E G, the tracing-point being at the distance H F from the centre of the rolling circle; multiply those rolling radii in their order by

Simpson's multipliers (Section I., Article 5, Rule A); divide the sum of the products by the sum of the multipliers; the quotient will be *the radius of the required circular arc.*

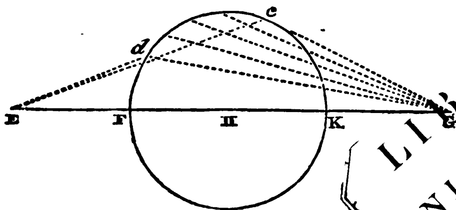


Fig. 21.

Then in fig. 20 describe a circle about O with the radius O A; through C and D draw straight lines parallel to O B, cutting that circle in Γ and Δ; join O Γ, O Δ; and about the centre, O, with the mean rolling radius already found, describe the circular arc M N, bounded by the straight lines O Γ, O Δ; this will be *the required circular arc approximately equal to the elliptic arc* C D.

The circular arc may then be measured by the rules of Article 3 of this Section.

The following are examples of the errors of this rule, when applied to an entire elliptic quadrant divided into two intervals only. For greater numbers of intervals, the errors vary inversely as the fourth power of the number of intervals, or nearly so:—

Major Semi-axis O A.	Minor Semi-axis O B.	Eccentricity.	True Length from Legendre's Tables.	Approximate Length by Rule.	Errors, +
1	·7071	·7071	1·3506	1·3538	·0032
1	·8000	·6000	1·4184	1·4195	·0011
1	·8660	·5000	1·4675	1·4681	·0006

5. **Common Parabola.**—In fig. 16 (page 76) let A be the vertex of a common parabola, and A B an arc to be measured, commencing at the vertex.

For a rough approximation, use Rule D of Article 1 of this Section. For purposes of precision, proceed as follows:—

Draw the tangent at the vertex A C, on which let fall the perpendicular B C, and measure the lengths of those lines. Call A C the *base*, and B C the *height*.

To the square of the height add one-fourth of the square of the base, and extract the square root of the sum. Call this the *sloping tangent*.

Divide the square of the base by four times the height. Call this the *focal distance*.

To the sloping tangent add the height; divide the sum by half the base; take the hyperbolic logarithm of the quotient. Multiply that logarithm by the focal distance.

To the product add the sloping tangent; the sum will be the required arc.*

6. **Catenary.**—In fig. 22 the horizontal straight line through O is the *directrix* of the catenary; the vertical line O A is its *parameter*, on which all its dimensions depend; A is the *vertex*, or lowest point of the curve; B another point; X B a vertical *ordinate* from the directrix to the point B; O X the corresponding *abcissa*, or horizontal distance from O.

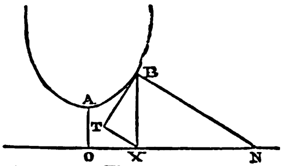


Fig. 22.

Rule A.—Given, O A and X B; to find the arc A B.

By construction:—On X B as a hypothenuse construct the right-angled triangle X T B, making X T = O A; then will T B = the arc A B. (T B is a tangent to the curve at B.)

By calculation:—A B = $\sqrt{}$ (X B² − O A²).

Rule B.—The *area* O A B X = O A × arc A B = 2 × triangle X T B.

Rule C.—Given, O A and O X, to find X B and A B.

Divide O X by O A; find the hyperbolic antilogarithm of the quotient (see Table 3), and the reciprocal of that antilogarithm.

For the ordinate X B, multiply O A by the half-sum of the antilogarithm and its reciprocal.

For the arc A B, multiply O A by the half-difference of the same quantities. †

Addendum to Section I.

A **Platometer** or **Planimeter** is an instrument for measuring plane areas on paper. A point is made to travel round the

* In symbols, let A C = x, C B = y, and the arc A B = s. Then

$$s = \sqrt{\left(y^2 + \frac{x^2}{4}\right)} + \frac{x^2}{4y} \cdot \text{hyp. log.} \frac{y + \sqrt{\left(y^2 + \frac{x^2}{4}\right)}}{\frac{x}{2}}.$$

† In symbols, let O A = m; O X = x; X B = y; arc A B = s; then

$$y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right); \; s = \sqrt{y^2 - m^2} = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right);$$

$$\text{area} \int y\,dx = m\,s = \frac{m^2}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right);$$

$$x = m \cdot \text{hyp. log.} \frac{y + s}{m}.$$

boundary of the figure to be measured; and when that point has returned to the spot from which it started, the area enclosed by the boundary is indicated on one or more graduated circles. The simplest instrument of this kind is Amstler's.

ADDENDUM TO SECTION IV.

Rectification of Curves by an Instrument.—An instrument for rectifying curves on paper consists of a small wheel, milled, and sometimes spiked on the rim, and turning upon a fixed spindle which has a fine screw thread cut upon it. At one end of the spindle is a shoulder, to limit the motion of the wheel in that direction.

The wheel being made to bear against the shoulder, is placed with its rim resting on the commencement of the curve to be rectified. It is then made to run along the curve in such a direction that, in revolving, it screws itself away from the shoulder. Having arrived at the farther end of the curve, it is lifted, and set down at a point marked on a straight line; it is then run along the straight line so as to revolve the contrary way, and screw itself back towards the shoulder. When it has returned to the shoulder from which it started, its point of contact with the straight line is marked; and thus is obtained a straight line equal in length to the given curve.

SECTION V.—CENTRES OF MAGNITUDE.

By the *magnitude* of a figure is to be understood its length, area, or volume, according as it is a line, a surface, or a solid.

The *Centre of Magnitude* of a figure is a point such that, if the figure be divided in any way into equal parts, the distance of the centre of magnitude of the whole figure from any given plane is the mean of the distances of the centres of magnitude of the several equal parts from that plane.*

1. **Symmetrical Figure.**—If a plane divides a figure into two symmetrical halves, the centre of magnitude of the figure is in that plane; if the figure is symmetrically divided in the like manner by two planes, the centre of magnitude is in the line where these planes cut each other; if the figure is symmetrically divided by three planes, the centre of magnitude is their point of intersection; and if a figure has a *centre of figure* (for example, a circle, a sphere,

* The centre of magnitude of an uniformly heavy body is the same with its *centre of gravity;* of which point mention will again be made further on.

The *geometrical moment* of any figure relatively to a given plane is the product of its magnitude into the perpendicular distance of its centre from that plane.

an ellipse, an ellipsoid, a parallelogram, &c.), that point is its centre of magnitude.

2. **Compound Figure.**—To find the perpendicular distance from a given plane of the centre of a compound figure made up of parts whose centres are known. Multiply the magnitude of each part by the perpendicular distance of its centre from the given plane; distinguish the products (or *moments*) into positive or negative according as the centres of the parts lie to one side or to the other of the plane; add together, separately, the positive moments and the negative moments: take the difference of the two sums, and call it positive or negative according as the positive or negative sum is the greater; this is the *resultant moment* of the compound figure relatively to the given plane; and its being positive or negative shows at which side of the plane the required centre lies. Divide the resultant moment by the magnitude of the compound figure; the quotient will be the distance required.

The centre of a figure in three dimensions is determined by finding its distances from three planes that are not parallel to each other. The best position for those planes is perpendicular to each other; for example, one horizontal, and the other two cutting each other at right angles in a vertical line. To determine the centre of a plane figure, its distances from two planes perpendicular to the plane of the figure are sufficient.

3. **Compound Figure of Two Parts.**—Let a compound figure, as in fig. 23, consist of two parts, and let their separate centres, A and B, be known. Draw and measure the straight line A B; multiply its length by the magnitude of either of the parts, and divide by the whole magnitude; the quotient will be the distance of the centre, C, of the whole figure from the centre of the other part; and C will lie in the straight line A B.

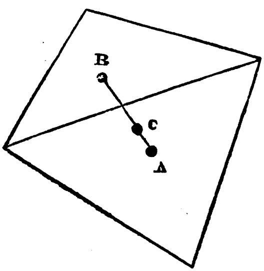


Fig. 23.

[In symbols, let A and B denote the magnitudes of the parts, and A + B that of the whole figure; then

$$A\,C = \frac{B \cdot A\,B}{A + B};\ B\,C = \frac{A \cdot A\,B}{A + B}.\Big]$$

4. **Compound Figure formed by Subtraction.**—From the larger figure in fig. 24, whose known centre is A, let a part whose known centre is B be taken away. Draw and measure the straight line B A. The centre, C, of the remaining figure will lie in B A, produced beyond A. To find the distance A C, multiply B A by the

magnitude of the part taken away, and divide by the magnitude of the remaining figure.

[In symbols, let A be the magnitude of the original figure, B that of the part taken away, and A − B that of the remaining figure. Then

$$C\,A = \frac{B \cdot B\,A}{A - B}.\Big]$$

5. **Figure Changed by Shifting a Part.**—In fig. 25 let C be the original position of the centre of a figure; let the figure be changed

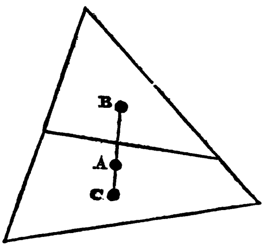


Fig. 24.

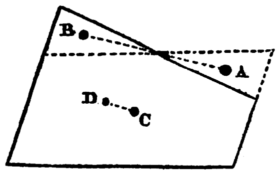


Fig. 25.

in shape, but not in magnitude (from the dotted outline to the plain outline), by shifting part of it; and let A be the original position, and B the new position of the centre of the part shifted. Draw and measure the straight line A B. Through C draw C D parallel to and pointing in the same direction with A B; and make

$$C\,D = \frac{A\,B \times \text{magnitude of part shifted}}{\text{magnitude of whole figure}};$$

D will be the new position of the centre of the figure.

6. **Any Plane Area.**—To find, approximately, the centre of any plane area.

RULE A.—Let the plane area be that represented in fig. 3 (of Section I., Article 5, preceding). Draw an axis, A X, in a convenient position, divide it into equal intervals, measure breadths at the ends and at the points of division, and calculate the area, as in Section I., Article 5.

Then multiply each breadth by its distance from one end of the axis (as A); consider the products as if they were the breadths of a new figure, and proceed by the rules of Section I., Article 5, to calculate the area of that new figure. The result of the operation will be the *moment* of the original figure relatively to a plane perpendicular to A X at the point A.

Divide the *moment* by the *area* of the original figure; the quotient will be the distance of the centre required from the plane perpendicular to A X at A.

Draw a second axis intersecting A X (the most convenient position being in general perpendicular to A X), and by a similar process find the distance of the centre from a plane perpendicular to the second axis at one of its ends; the centre will then be completely determined.

RULE B.—If convenient, the distance of the required centre from a plane cutting an axis at one of the intermediate points of division, instead of at one of its ends, may be computed as follows:—Take separately the moments of the two parts into which that plane divides the figure; the required centre will lie in the part which has the greater moment. Subtract the less moment from the greater; the remainder will be the *resultant moment* of the whole figure, which being divided by the whole area, the quotient will be the distance of the required centre from the plane of division.

REMARK.—When the resultant moment is = 0, the centre is in the plane of division.

RULE C.—To find the perpendicular distance of the centre from the axis A X. Multiply each breadth by the distance of the middle point of that breadth from the axis, and by the proper "Simpson's Multiplier" (Section I., Article 5); distinguish the products into right-handed and left-handed, according as the middle points of the breadths lie to the right or left of the axis; take separately the sum of the right-handed products and the sum of the left-handed products; the required centre will lie to that side of the axis for which the sum is the greater; subtract the less sum from the greater, and multiply the remainder by $\frac{1}{3}$ of the common interval if Simpson's first rule is used, or by $\frac{3}{8}$ of the common interval if Simpson's second rule is used; the product will be the *resultant moment* relatively to the axis A X, which, being divided by the area, the quotient will be the required distance of the centre from that axis.*

7. **Any Solid.**—To find the perpendicular distance of the centre of magnitude of any solid from a plane perpendicular to a given axis at a given point, proceed as in Rule A of the preceding Article to find the moment relatively to the plane, substituting

* The rules of this Article are expressed in symbols as follows:—Let x and y be the perpendicular distances of any point in the plane area from two planes perpendicular to the area and to each other, and x_0 and y_0 the perpendicular distances of the centre of magnitude of the area from the same planes; then

$$\text{(A) } x_0 = \frac{\iint x\,dx\,dy}{\iint dx\,dy}; \quad \text{(B) } y_0 = \frac{\iint y\,dx\,dy}{\iint dx\,dy}.$$

sectional areas for *breadths;* then divide the moment by the volume (as found by Section III., Article 5); the quotient will be the required distance.

To determine the centre completely, find its distances from three planes, no two of which are parallel. In general it is best that those planes should be perpendicular to each other.

8. **Any Curved Line.** RULE A. *To find approximately the Centre of Magnitude of a very Flat Curved Line.*—In fig. 26 let A D B be the arc. Draw the chord A B, which bisect in C; draw C D (the *deflection*) perpendicular to A B; make D E = $\frac{1}{3}$ C D; E will be the centre, nearly.

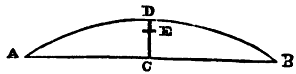


Fig. 26.

For an arc of a cycloid, with the chord A B parallel to the base-line, this rule is exact. For a flat circular arc subtending a degrees, D E is too small by the fraction $\frac{a^2}{400000}$ of its length, nearly.

RULE B.—*When the Curved Line is not very flat,* divide it into very flat arcs; find their several centres of magnitude by Rule A, and measure their lengths by one of the rules of Section IV., Article 1; then treat the whole curve as a compound figure, agreeably to the rules of Article 2 of this Section.

9. **Special Figures.** I. TRIANGLE (fig. 27).—From any two of the angles draw straight lines to the middle points of the opposite sides; these lines will cut each other in the centre required;—or otherwise,—from any one of the angles draw a straight line to the middle of the opposite side, and cut off one-third part from that line, commencing at the side.

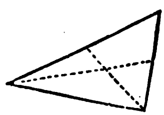


Fig. 27.

II. QUADRILATERAL (fig. 28).—Draw the two diagonals A C and B D, cutting each other in E. If the quadrilateral is a parallelogram, E will divide each diagonal into two equal parts, and will itself be the centre. If not, one or both of the diagonals will be divided into unequal parts by the point E. Let B D be a diagonal that is unequally divided. From D lay off D F in that diagonal = B E. Then the centre of the triangle F A C, found as in the preceding rule, will be the centre required.

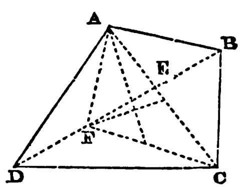


Fig. 28.

III. PLANE POLYGON.—Divide it into triangles; find their centres, and measure their areas; then treat the polygon as a compound figure made up of the triangles, by the rules of Article 2 of this Section.

IV. Prism, or Cylinder with Plane Parallel Ends.—Find the centres of the ends; a straight line joining them will be the axis of the prism or cylinder, and the middle point of that line will be the centre required.

V. Tetrahedron, or Triangular Pyramid (fig. 29).—Bisect any two opposite edges, as A D and B C, in E and F; join E F, and bisect it in G; this point will be the centre required.

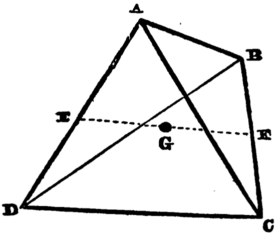


Fig. 29.

VI. Any Pyramid or Cone with a Plane Base.—Find the centre of the base, from which draw a straight line to the summit; this will be the axis of the pyramid or cone. From the axis cut off one-fourth of its length, beginning at the base; this will give the centre required.

VII. Any Polyhedron or Plane-faced Solid.—Divide it into pyramids; find their centres and measure their volumes; then treat the whole solid as a compound figure, by the rules of Article 2 of this Section.

VIII. Truncated Pyramid or Cone.—Find the respective volumes and centres of magnitude of the entire pyramid or cone, and of the part cut off; then find the centre of the remaining part by the rule of Article 4 of this Section.

IX. Circular Arc.—In fig. 30 let A B be the arc, and C the centre of the circle of which it is part. Bisect the arc in D, and join C D and A B. Multiply the radius C D by the chord A B, and divide by the length of the arc A D B; lay off the quotient C E upon C D; E will be the centre of magnitude of the arc.

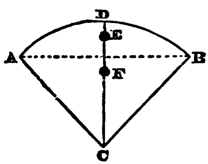


Fig. 30.

X. Circular Sector, C A D B, fig. 30.—Find C E as in the preceding rule, and make C F = $\frac{2}{3}$ C E; F will be the centre required.

XI. Sector of a Plane Circular Ring.—In fig. 31, let C B be the outer, and C A the inner radius of the ring. Divide twice the difference of the cubes of the outer and inner radii by three times the difference of their squares; the quotient will be an intermediate radius, C D, with which describe an arc, D E, subtending the same angle with the sector. The centre of magnitude, F, of the arc D E, found by Rule IX. of this Article, will be the centre required.

XII. Circular Segment, A D B, fig. 30.—Find the respective centres of magnitude of the sector C A D B, and the triangle

C A B, which, being taken from the sector, leaves the segment; then, by the rule of Article 2 of this Section, find the centre of magnitude of the segment.

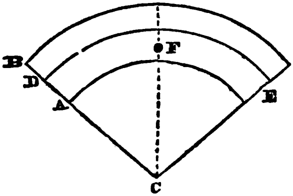


Fig. 31.

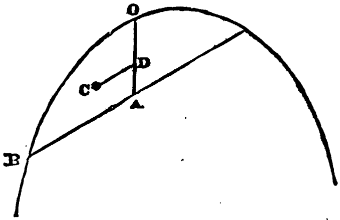


Fig. 32.

XIII. Parabolic Half-Segment.—In fig 32 O A B represents a half-segment of a parabola; O A being part of a diameter parallel to the axis, and A B an ordinate conjugate to that diameter—that is, parallel to a tangent at O. Make O D = $\frac{3}{5}$ O A, and draw D C parallel to A B and = $\frac{3}{8}$ A B; C will be the centre of magnitude of the half-segment.

10. **Centres Found by Parallel Projection.**—By a *parallel projection* of a plane figure, or of a solid, is meant a figure resembling the original figure, but transformed by having its dimensions in one or more directions altered in given proportions, or by distortion; subject to the limitation—*that to every set of parallel straight lines, bearing given proportions to each other in the original figure, there shall correspond a set of parallel straight lines in the new figure, bearing the same proportions to each other.* For example,—all triangles are parallel projections of each other; so are all triangular pyramids; so are all circles and ellipses; so are all spheres, spheroids, and ellipsoids; so are all circular and elliptic cylinders; so are all cones.

Rule.—*The centre of magnitude of a plane or solid figure, which is derived by parallel projection from another figure, is the parallel projection of the centre of magnitude of the original figure.*

Remark.—It is to be observed that this rule applies neither to curved lines nor to curved surfaces, but only to plane areas and to solids.

Example.—*Elliptic Sector*, O C′ D′, fig. 33. Let O be the centre of the whole ellipse; A O A its greater, and B′ O B′ its lesser axis. About O, with the radius O A, describe a circle, A B A B. This will be a parallel projection of the ellipse.* Through C′ and D′ draw E C′ C and F D′ D parallel to O B, cutting the circle in

* Because every ordinate of the ellipse, such as X Y′, parallel to O B′, bears a constant proportion to the corresponding ordinate X Y of the circle—viz., that of O B′ : O B.

C and D; join O C, O D; the circular sector O C D will be a parallel projection of the given elliptic sector. Find, by Rule X. of Article 9, the centre of magnitude, G, of the circular sector; and through it draw G H parallel to B O. Then

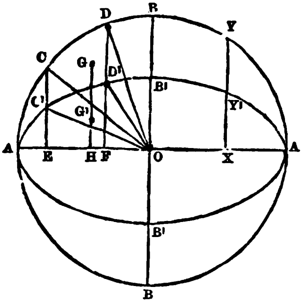


Fig. 33.

$$OB : OB' :: HG : HG';$$

and G′ will be the centre of magnitude of the elliptic sector.

11. **Volume Swept by a Moving Plane.**—Let the centre of magnitude of a plane figure move along any path, straight or curved, and let the plane figure at every instant be perpendicular to the direction of that path; the volume of the space swept through by the plane figure is the product of the area of that figure into the length of the path of its centre.

If any part of the plane figure moves *backwards*, the volume swept by that part is to be subtracted from the volume swept by the part that moves forwards, in estimating the volume swept by the whole figure.

ADDENDUM.

TABLE 7.—REGULAR POLYGONS.

No. of Sides.	Name.	Side = 1.		Semi-diameter = 1.	
		Semi-diameter.	Area.	Side.	Area.
3	Triangle, or Trigon,	0·5774	0·4330	1·73205	1·2990
4	Square, or Tetragon,	0·7071	1·0000	1·41421	2·0000
5	Pentagon,	0·8507	1·7205	1·17557	2·3776
6	Hexagon,	1·0000	2·5981	1·00000	2·5981
7	Heptagon,	1·1524	3·6339	0·86777	2·7364
8	Octagon,	1·3066	4·8284	0·76537	2·8284
9	Enneagon,	1·4619	6·1818	0·68404	2·8925
10	Decagon,	1·6180	7·6942	0·61803	2·9389
11	Hendecagon,	1·7747	9·3656	0·56347	2·9735
12	Dodecagon,	1·9319	11·1962	0·51764	3·0000
13	Decatrigon,	2·0893	13·1858	0·47863	3·0207
14	Decatetragon,	2·2470	15·3345	0·44504	3·0371
15	Decapentagon,	2·4049	17·6424	0·41582	3·0505
16	Decaëxagon,	2·5629	20·1094	0·39018	3·0615
20	Icosagon,	3·1962	31·5688	0·31287	3·0902
24	Icositetragon,	3·8306	45·5745	0·26105	3·1058

The semi-diameter is measured from the centre of the polygon to an angle.

To find the Side of a Regular Decagon by Construction.—In fig. 34 let A B be the semi-diameter of the decagon. Draw B C perpendicular to A B, and = $\frac{1}{2}$ A B; join A C, from which cut off C D = C B; A D will be the side required.

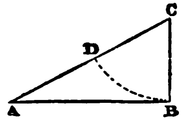


Fig. 34.

To find, very nearly, the Side of a Regular Heptagon by Construction.—In fig. 35 let A B be the semi-diameter of the heptagon. Draw the equilateral triangle A C B. Divide A B into 200 equal parts, and take the point D at 89 of those parts from one end, and 111 from the other end of A B. Join C D; this will be very nearly the side required, the error being practically inappreciable.

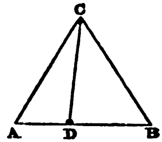


Fig. 35.

Table of Rhumbs (see next page).

Points.	Angles East of North.				Points.
32. N.,	360°	00′	0°	00′,	0. N.
31. N.*b*.W.,	348	45	11	15,	1. N.*b*.E.
30. N.N.W.,	337	30	22	30,	2. N.N.E.
29. N.W.*b*.N.,	326	15	33	45,	3. N.E.*b*.N.
28. N.W.,	315	00	45	00,	4. N.E.
27. N.W.*b*.W.,	303	45	56	15,	5. N.E.*b*.E.
26. W.N.W.,	292	30	67	30,	6. E.N.E.
25. W.*b*.N.,	281	15	78	45,	7. E.*b*.N.
24. W.,	270	00	90	00,	8. E.
23. W.*b*.S.,	258	45	101	15,	9. E.*b*.S.
22. W.S.W.,	247	30	112	30,	10. E.S.E.
21. S.W.*b*.W.,	236	15	123	45,	11. S.E.*b*.E.
20. S.W.,	225	00	135	00,	12. S.E.
19. S.W.*b*.S.,	213	45	146	15,	13. S.E.*b*.S.
18. S.S.W.,	202	30	157	30,	14. S.S.E.
17. S.*b*.W.,	191	15	168	45,	15. S.*b*.E.
16. S.,	180	00	180	00,	16. S.

Quarter-point,......... =	2°	48′	45″
Half-point,............ =	5	37	30
Three quarter-points, =	8	26	15

PART II.

MEASURES.

Section I.—Measures of Angles.

1. The **Sexagesimal System** of angular measurement is as follows:—1 revolution = 4 right angles = 360 degrees; 1 degree = 60 minutes; 1 minute = 60 seconds. Seconds are usually subdivided into decimal fractions. As to *circular measure*, see Table 4 in the preceding part of this work.

2. The **Nautical** or **Binary** system used in the Mariner's compass is as follows:—1 revolution = 32 points, each divided into halves and quarters; 1 point = $11\frac{1}{4}$ degrees (see preceding page).

3. The **Centesimal System** of angular measurement is as follows:—1 revolution = 4 right angles = 400 grades; 1 grade = 100 minutes; 1 minute = 100 seconds. This system is found in some French works published towards the beginning of the nineteenth century, but is now little used.

Section II.—Measures of Time.

1. **Sidereal Day.**—The standard unit of time is the Sidereal Day, being the period in which the earth turns once round on its axis. It is divided into sidereal hours, minutes, and seconds; but these measures of time are used by astronomers only.

2. **Mean Solar Time.**—A second is the time of one swing of a pendulum adjusted so as to make 86,164·09 swings in a sidereal day. Seconds are usually subdivided decimally.

One minute = 60 seconds.
One hour = 60 minutes = 3,600 seconds.
One mean solar day = 24 hours = 1,440 minutes = 86,400 seconds = 1·00273791 sidereal day.

3. **Years.**—One tropical year = 365 days 5 hours 48 minutes 49·7 seconds mean solar time, = 365·24224 mean solar days, nearly.

One common year = 365 days.
One leap year = 366 days.

Years of the Gregorian Calendar.	Days.
Number of year in the Christian Era—	
Not divisible by 4 without remainder,	365
Divisible by 4, but not by 100,	366
Divisible by 100, but not by 400,	365
Divisible by 400 [but not by 4,000],*	366
[Divisible by 4,000,	365] *

4. **Dates, Civil and Astronomical.**—The civil day is held (in Western Europe and in America) to commence at midnight. The astronomical day commences at noon of the civil day having the same designation; that is, twelve hours later than the civil day. The civil year is held to commence at midnight of the 31st of December of the year preceding; the astronomical year commences at noon of the 1st of January of the civil year.

5. **Relation between Time and Longitude.**—At any given instant the mean solar time at two stations differs by an amount proportional to their difference of longitude, the time at the eastern station being the later.

CORRESPONDING DIFFERENCES.

Longitude.	Time.	Longitude.	Time.
15″	1 second.	75°	5 hours.
1′	4 seconds.	90	6 "
15′	1 minute.	105	7 "
1°	4 minutes.	120	8 "
15°	1 hour.	135	9 "
30	2 hours.	150	10 "
45	3 "	165	11 "
60	4 "	180	12 "

To show the exact date of any event, the meridian at which the time is reckoned must be specified.

It is customary for civil and commercial purposes to reckon time at all places throughout Britain as for the meridian of Greenwich; local mean solar time being found for scientific purposes, when required, by calculation.

At stations close to the two sides of the meridian of 180° there is necessarily a difference of a whole day in the dates corresponding to the same real instant, the date at the western side of that meridian being the later. The position of the meridian of 180° is purely arbitrary, depending on the position assumed for the meridian of 0°, which is different in each different nation.

6. **Divisions of the Year.**—Intervals in days from the beginning of the first day of January to the beginning of the first day of each of the other calendar months:—

* The rules in brackets are an improvement proposed by Sir John Herschel, which cannot come into operation until A.D. 4000.

	Common Year.	Leap Year.		Common Year.	Leap Year.
January,...	0	0	July,...........	181	182
February,..	31	31	August,........	212	213
March,......	59	60	September,....	243	244
April, ...	90	91	October,.......	273	274
May,........	120	121	November,....	304	305
June, ...	151	152	December,....	334	335

A so-called "lunar" month is four weeks, or twenty-eight days.*

Section III.—Measures of Length.

1. The British **Standard Yard** is the distance, at the temperature of 62° Fahrenheit, between two marks on a certain bar which is kept in the office of the Exchequer, at Westminster.†

2. The French **Metre** is $\frac{443296}{864000}$ of the distance, at the temperature of 13° Réaumur (see pages 105, 106), between the ends of a certain bar, called the "Toise of Peru" (see pages 93, 94), and is approximately one ten-millionth part of the distance from one of the earth's poles to the equator.‡ The use of this measure, and others founded on it, is lawful in Britain, and a copy of the standard metre is kept in the Exchequer office.

3. **British Measures of Length.—**

		Inches.		Feet.		Yards.		Statute Miles.		Metres.
Inch §	=	1	=	$\frac{1}{12}$	=	$\frac{1}{36}$	=	$\frac{1}{63360}$	=	0·02539977
Hand		4		$\frac{1}{3}$		$\frac{1}{9}$		$\frac{1}{15840}$		0·10159907
Foot		12		1		$\frac{1}{3}$		$\frac{1}{5280}$		0·30479721
Yard		36		3		1		$\frac{1}{1760}$		0·91439180
Chain		792		66		22		$\frac{1}{80}$		20·11662
Furlong		7,920		660		220		$\frac{1}{8}$		201·1662
Mile		63,360		5,280		1,760		1		1,609·3296

The Inch is subdivided—

By artificers, sometimes into 12ths, or *lines*, but more commonly into binary divisions, as halves, quarters, 8ths, 16ths and 32ds.

By mechanical engineers, into decimal divisions, as 10ths, 100ths, 1,000ths, and 10,000ths.

* A mean lunation, or real lunar month, is approximately 29½, or more exactly, 29·53059 mean solar days; 235 lunations nearly = 19 years,—a period called a *lunar* or *Metonic cycle.*

† See "Weights and Measures Act," 1855. Official copies of the standard yard are kept at the Royal Mint, London, the Royal Observatory, Greenwich, the Rooms of the Royal Society of London, and the Palace of Westminster.

‡ The distance from the pole to the equator is not exactly the same on different meridians. (see page 117).

§ An inch is almost exactly one 500,500,000th part of the earth's polar axis.

The HAND is used for heights of horses and girths of spars.

The FOOT is subdivided decimally by civil engineers.

The YARD, in CLOTH MEASURE, is subdivided binarily, into halves, quarters, half-quarters, and *nails*, or 16ths of a yard. An English ELL is $1\frac{1}{4}$ yard, or 45 inches.

The CHAIN, in LAND MEASURE, is subdivided into 4 *poles* or *perches* (each of $5\frac{1}{2}$ yards) and 100 *links* (each of 7·92 inches).

A FATHOM is two yards.

The GEOGRAPHICAL, NAUTICAL, or SEA MILE, or KNOT, depends on the dimensions of the earth, which are known approximately only. The following are estimates of its value :—

	Feet nearly.	Statute Mile nearly.	Metres nearly.
Mean length of one minute of longitude at the equator; being the nautical mile by Admiralty Regulation,......	6,086	1·1527	1,855
Mean length of one minute of latitude,......................	6,076	1·1508	1,852

A LEAGUE is three nautical miles.

The nautical mile is sometimes subdivided into 10 *cables* and 1,000 *fathoms*; the fathom thus obtained being about one-80th part longer than the common fathom.

4. **French Metrical Measures of Length.—**

	Metres.		British Measures.	
Millimetre,...........	0·001	=	0·03937043	inch.
Centimetre,..........	0·01			
Decimetre,............	0·1			
Metre, (= $\frac{443296}{864000}$ Toise),	1	=	3·2808693	feet.
Decametre,...........	10			
Hectometre,.........	100			
Kilometre,............	1,000	=	0·6213768	mile.
Myriametre,.........	10,000			

The French *nœud* = British nautical mile.

Log. feet in a metre = 0·5159889356.

5. **Old Scottish and Irish Measures of Length.—**

The IRISH PERCH = 7 yards = $\frac{14}{11}$ imperial perch.

The IRISH MILE = 320 Irish perches = 2,240 yards = $\frac{14}{11}$ statute mile.

The SCOTTISH INCH = 1·0162 imperial inch.

The SCOTTISH ELL = 37 Scottish inches = 37·06 imperial inches = 3·0883 imperial feet.

The SCOTTISH FALL = 6 Scottish ells = 18·53 imperial feet.

The Scottish Mile = 320 falls = 1,920 ells = 5,929·6 imperial feet = 1·123 statute mile.

Each of those miles was divided into 8 furlongs, and 80 chains.

As to Scottish measures, see Buchanan's *Weights and Measures*, Edinburgh, 1829.

6. **Various Measures of Length.—**

	British Measures.	Metres.
United States, as in Britain.		
India—		
Hath or haut (cubit), ...	18 inches.	0·4572
Coss (mile) = 4,000 cubits,...	6,000 feet. = 1·136 stat. mile.	1,828·8
Russia—		
Foot = 12 inches, ...	1 foot.	0·3048
Sashen or sagène, ...	7 feet.	2·1336
Verst (500 sashen), ...	3,500 „	1,066·8
Prussia, Denmark, Norway—		
Foot = 12 inches, ...	1·02972 foot.	0·31385
Ruthe (rod) = 12 feet, ...	12·35664 feet.	3·7662
Mile = 24,000 feet, ...	24,713·28 „ = 4·6806 stat. miles.	7,532·4
Austria—		
Foot = 12 inches, ...	1·03713 foot.	0·31611
Klafter = 6 feet, ...	6·22278 feet.	1·89666
Mile = 24,000 feet, ...	24,891·12 „ =4·7142 stat. miles.	7,586·64
German geographical mile, ...	4 geographical miles.	7,408 nearly.
German sea-mile, ...	1 geographical mile.	1,852 nearly.
Sweden—		
Foot = 12 inches, ...	0·97410 foot.	0·2969
Fathom = 3 ells = 6 feet, ...	5·8446 feet.	1·7814
Mile = 6,000 fathoms, ...	35,067·6 „ = 6·6116 stat. miles.	10,688·5
Netherlands—		
Palm, ...	3·937043 inches.	0·1
El, ...	3·2808693 feet.	1·0
Myle, ...	3,280·8693 „ =0·6213768 stat.mil.	1,000
Belgium, Italy, Portugal, Spain—French Metric Measures.		
China—		
Chih (foot), ...	1·054 foot.	0·32125
Chang = 10 chih, ...	10·54 feet.	3·2125
Li = 180 chang, ...	1,897 feet. = 0·3593 stat. mile.	578·25
Old French foot = 12 inches = 144 lines, ...	1·0657556 foot.	0·32483939
Old French Toise = 6 feet, ...	6·3945335 feet.	1·94903632

Log. feet in a toise, 0·8058088656.

For the measures of length used in various States of Germany, see *der Ingenieur*, by Dr. Julius Weisbach.

SECTION IV.—MEASURES OF AREA.

1. British Measures of Area.—

	Sq. Inches.	Sq. Feet.	Sq. Metres.
USED IN SCIENCE AND IN ENGINEERING—			
Sq. inch (decimally subdivided),	1	$\frac{1}{144}$	0·000645148
1 foot × 1 inch,	12	$\frac{1}{12}$	0·007741775
Square foot (decimally or duodecimally subdivided),	144	1	0·0929013
Square yard,	1,296	9	0·836112
	Sq. Yards.		
Square mile,	3,097,600	27,878,400	2,589,941
LAND MEASURE—			
Perch,	$30\frac{1}{4}$	$272\frac{1}{4}$	25·292
Sq. chain (= 10,000 sq. links), ...	484	4,356	404·678
Rood = 40 perches,	1,210	10,890	1,011·696
Acre = 4 roods = 10 sq. chains,	4,840	43,560	4,046·782
USED IN THE ARTS—			
Square (of roofing or flooring),	...	100	9·29013
Rood (face of masonry),	36	324	30·1
Rod (face of brickwork),	...	272	25·269

2. French Metric Measures of Area.—

Science and Engineering.	Land.	Square Metres.	British Measures.
Sq. millimetre,...	......	0·000001	=0·00155003 sq. inch.
Sq. centimetre,..	......	0·0001	0·155003 sq. inch.
Sq. decimetre,...	......	0·01	15·5003 sq. inches.
	Milliare,	0·1	1·07641 sq. foot.
Sq. metre,.....=	Centiare,	1·0	10·7641 sq. feet.
	Deciare,	10	107·641 sq. feet.
Sq. decametre,=	Are,	100	1,076·41 sq. feet.
	Decare,	1,000	10,764·1 sq. feet.
Sq. hectometre,=	Hectare,	10,000	107,641 sq. feet=2·4711 acres.

3. Old Scottish and Irish Land Measures.—Irish acre = 4 roods = 160 perches = 70,560 square feet = $\frac{196}{121}$, or 1·6198 imperial acre. Scottish acre = 4 roods = 160 falls = 54,937 square feet = 1·2612 imperial acre.

4. Various Measures of Area.—

	British Measures.	Square Metres.
UNITED STATES, as in Britain.		
RUSSIA—		
Square foot = 144 square in.,.	1 square foot.	0·0929013
Square sashen = 49 square ft.,	49 square feet.	4·55217
Dessatine = 2,400 sq. sashen,.	117,600 " =2·69977 acres.	10,925

VARIOUS MEASURES OF AREA—*continued.*

	British Measures	Square Metres.
PRUSSIA, DENMARK, NORWAY—		
Square foot = 144 square in.,.	1·06033 sq. foot.	0·09850
Square ruthe = 144 square ft.,	152·6875 sq. feet.	141·85
Morgen = 180 square ruthen,	27,483·75 ,, = 0·63094 acre.	2,553·3
AUSTRIA—		
Square ft. = 144 square in.,...	1·07564 sq. foot.	0·09993
Square klafter = 36 square ft.,	38·723 sq. feet.	3·5975
Joch = 1,600 square klafter,...	61,957 ,, = 1·47366 acre.	5,756
SWEDEN—		
Square ft. = 144 square in.,...	0·94887 sq. foot.	0·08815
Tunnland = 56,000 square ft.,	53,136·72 sq. feet. = 1·21977 acre.	4,936·4
NETHERLANDS—		
Square el,........................	10·7641 sq. feet.	1·00000
Bunder = 10,000 square el,....	107,641 ,, = 2·4711 acres.	10,000
BELGIUM, ITALY, PORTUGAL, SPAIN—French Metric Measures.		
Old French square foot = 144 square inches,..................	1·1358 sq. foot.	0·10552

SECTION V.—SOLID MEASURES.

1. British Solid Measures.—

	Cubic Inches.	Cubic ft.	Cubic Metres.
Cubic inch (subdivided decimally),.....	1	$\frac{1}{1728}$	0·000016387
1 foot × 1 inch × 1 inch, ...	12	$\frac{1}{144}$	0·00019664
1 foot × 1 foot × 1 inch,....................	144	$\frac{1}{12}$	0·0023597
Cubic foot (subdivided decimally or Duodecimally,...........................	1,728	1	0·0283161
Cubic yard,....................................	46,656	27	0·764534
Load of hewn timber,......................	......	50	1·4158
Rood of masonry (=36 square yards face × 2 feet thick), ...	Cubic yards. 24	648	18·35
Rod of brickwork (= 272 square feet face × 13½ inches thick),...............	11⅓	306	8·665
Ton of displacement of a ship,...........	......	35	0·9910624
Ton registered of internal capacity of a ship,..	......	100	2·83161
Ton, shipbuilders' old measurement,...	......	94	2·6617

2.—**French Metric Solid Measures.**—

Science and Engineering.	Trade.	Cubic Metres.	British Measures.
Cubic millimetre,	...	0·000000001	0·0000610254 cubic in.
Cubic centimetre,	...	0·000001	0·0610254 ,,
Cubic decimetre =	Millistere,	0·001	61·0254 ,,
	Centistere,	0·01	610·254 ,,
	Decistere,	0·1	6,102·54 ,, = 3·53156 cubic feet.
Cubic metre..... =	Stere,	1·0	35·3156 ,,
	Decastere,	10	353·156 ,,
	Hectostere,	100	3,531·56 ,,
Cubic decametre =	Kilostere,	1,000	35,315·6 ,,

3. **Various Solid Measures.**—

	British Cubic Feet.	Cubic Metres.
UNITED STATES, as in Britain.		
RUSSIA, cubic foot, ...	1·	0·0283161
PRUSSIA, DENMARK, NORWAY, cubic ft.,	1·09184	0·03092
AUSTRIA, cubic foot,........................	1·11557	0·03159
SWEDEN, cubic foot,..........................	0·9243	0·02617
NETHERLANDS, cubic el,....................	35·3156	1·00000
BELGIUM, ITALY, PORTUGAL, SPAIN—French metric measures.		
Old French cubic foot,......................	1·2105	0·03428

NORWAY, last (of ship's displacement) = $2\frac{1}{4}$ British tons, nearly.

SECTION VI.—MEASURES OF WEIGHT.

1. The **Standard Pound Avoirdupois** is the weight, at the temperature of 62° Fahrenheit, and under the atmospheric pressure of 30 inches of mercury, in the latitude of London, and at or near the level of the sea, of a certain piece of platinum which is kept in the Exchequer Office at Westminster.

2. The **Standard Kilogramme** is the weight, at the temperature of the maximum density of water (about 4° Centigrade), and under the atmospheric pressure of 760 millimetres of mercury, in the latitude of Paris, of a certain piece of platinum which is kept in the French Archives. The use of weights founded on this standard is lawful in Britain, and a copy of it is kept in the Exchequer Office.*

In the tables of the following articles the relative values of the pound avoirdupois and kilogramme are taken from Professor Miller's paper "On the Standard Pound" in the *Philosophical Transactions* for 1856.

* The kilogramme was at first intended to be the weight of a cubic decimetre of pure water at its maximum density; but it is in fact somewhat greater.

3. British Measures of Weight.—

	Grains.	Lbs. Avoirdupois.	Grammes.
Avoirdupois Weight—			
Dram,	27·34375	0·00390625	1·7718463
Ounce = 16 drams,	437·5	0·0625	28·3495408
Pound = 16 ounces,	7,000	1	453·5926525
Stone,	Ton. 0·00625	14	6,350·297135
Quarter = 2 stone,	0·0125	28	12,700·59427
Cental,	......	100	45,359·26525
Hundredweight = 8 stone	0·05	112	50,802·37708
Ton = 20 cwt,	1	2,240	1,016,047·5416
Troy and Apothecaries' Weight—	Grains.		
Grain,	1	$\frac{1}{7000}$	0·06479895
Scruple (Apoth.),	20	0·00285714	1·295979
Pennyweight (Troy),	24	0·003428571	1·5551748
Drachm (Apoth.) = 3 scruples,	60	0·00857143	3·887937
Ounce = 20 dwt. = 8 drachms,	480	0·06857143	31·103496
Pound = 12 oz.,	5,760	0·82285714	373·241952
Diamond Weight—			
Diamond grain,	0·8	$\frac{1}{8750}$	0·05183916
Carat = 4 diamond grains,	3·2	$\frac{1}{2100}$	0·20735664

4. French Metric Measures of Weight.—

	Grammes.		British Measures.
Milligramme,	0·001		...
Centigramme,	0·01		...
Decigramme,	0·1		...
Gramme,	1·0	=	15·43234874 grains.
Decagramme,	10		...
Hectogramme,	100		...
Kilogramme,	1,000	=	2·20462125 lbs. avoirdupois.
Myriagramme,	10,000		...
Quintal,	100,000		...
Tonneau (in shipbuilding) or millier,	1,000,000	=	0·9842059 ton.

5. Various Measures of Weight.—

	British Measures.	Grammes.
United States, as in Britain, with the following exception:—		
Quintal,	100 lbs.	45,359·26525
Russia—		
Pound = 32 loth = 96 solotnik,	0·90283	409·52
Berkowrtz = 10 pud = 400 pounds,	361·132	163,808·
German Zollverein, Denmark, Norway—		
Pound,	1·10231	500·
Centner = 100 pounds,	110·231	50,000·
Austria—		
Pound = 32 loth,	1·2346	560·012
Centner = 100 pounds,	123·46	56,001·2

VARIOUS MEASURES OF WEIGHT—*continued*.

	British Measures.	Grammes.
SWEDEN—		
Skalpund = 32 loth,	0·9377	425·3395
Skeppund = 400 skalpund,	375·08	170,135·8
NETHERLANDS—		
Pond = 10 Oncen = 100 Looden = 1,000 Wigtjes,	2·20462	1,000·
BELGIUM, ITALY, SPAIN, PORTUGAL— French Metric Measures.		
CHINA—		
Gin or Catty = 16 tael or lyang,	$1\frac{1}{3}$ lb. avoir.	604·79
Picul = 100 catties,	$133\frac{1}{3}$,,	60,479

SECTION VII.—MEASURES OF CAPACITY.

1. The **Standard Gallon** is the volume of 10 lbs. avoirdupois of pure water, at the temperature of 62° Fahrenheit, and under the atmospheric pressure of 30 inches of mercury. At that temperature the volume of water is 1·001118 times its minimum volume.

2. The **Standard Litre** is the volume of a kilogramme of pure water, at its temperature of maximum density (about 4° Centigrade), and under the atmospheric pressure of 760 millimetres of mercury. It was originally intended to be a cubic decimetre, but is actually somewhat greater.

3. **British Measures of Capacity.**—

			Gallons.	British Solid Measure, nearly.	Litres.
Gill,			0·03125	8·660 cub. inches.	0·141907
Pint	=	4 gills,	0·125	34·640 ,,	0·567628
Quart	=	2 pints,	0·25	69·280, ,,	1·135255
Pottle	=	2 quarts,	0·5	138·5615 ,,	2·27051
Gallon	=	2 pottles,	1·0	277·123 ,, * = 0·160372 cub. foot.	4·54102
Peck	=	2 gallons,	2	0·320744 ,,	9·08204
Bushel	=	4 pecks,	8	1·282976 ,,	36·32816
Quarter	=	8 bushels,	64	10·263808 cub. feet.	290·62528

A *tun* of ale = 2 butts = 4 hogsheads = 216 gallons = 980·86 litres.

A *ton* of sea-water = 35 cubic feet = $218\frac{1}{4}$ gallons nearly = 991·04 litres.

APOTHECARIES' FLUID MEASURE.—		Cubic Inches.	Litres.
Minim	=	0·00376	0·0000616
Fluid drachm	= 60 minims,	0·2256	0·003697
Fluid ounce	= 8 fluid drachms,	1·8047	0·029572
Pint	= 16 fluid ounces,	28·8750	0·473154
Gallon	= 8 pints,	231·0000	3·785235

* This is the correct volume of 10 lbs. of pure water at 62° Fahr., and is therefore the true value of a gallon in cubic inches. In an Act of Parliament, now partly repealed, that volume is stated to be 277·274 cubic inches.

4. **French Metric Measures of Capacity.—**

	Litres.		Cubic Inches.	Gallons.
Millilitre,	0·001		...	...
Centilitre,	0·01		...	...
Decilitre,	0·1		...	...
Litre,	1·	=	61·027	0·220215
Decalitre,	10		...	...
Hectolitre,	100		...	...
Kilolitre,	1,000		...	...
Myrialitre,	10,000		...	...

5. **Various Measures of Capacity.—**

	Gallons.	Litres.
UNITED STATES—		
Gallon = 231 cubic inches,	0·833565	3·785235
RUSSIA—		
Vedro = 10 kruschki = 750·568 cubic inches =	2·70843	12·299
PRUSSIA—		
Quart or Viertel (= 64 Prussian cubic inches),	0·25215	1·145
Oxhoft = 1¼ ohm = 3 eimer = 6 anker = 180 quart,	45·387	206·1
Tonne = 4 scheffel = 64 metzen = 192 viertel,	48·413	219·84
AUSTRIA—		
Maass = 40 seidel = 80 pfiff = 0·0448 Austrian cubic foot,	0·3116	1·415
Eimer = 40 maass,	12·464	56·6
SWEDEN—		
Kann (= 0·1 Swedish cubic foot),	0·57635	2·617
Åm = 60 kannar,	34·581	157·02
NETHERLANDS—		
Kan (subdivided decimally),	0·220215	1
Old Scottish gallon = 8 pints = 16 chopins = 32 mutchkins = 128 gills,	3·0651	13·9187

SECTION VIII.—MEASURES OF VALUE.

1. The **Fineness of Gold and Silver Coins** means the proportion of the precious metal which they contain, and is generally expressed in thousandths of their total weight. The fineness of gold coins is also expressed in *carats*, or 24ths of their total weight.

The fineness of British gold coins is 22 carats, or 0·916⅔; of British silver coins, 0·925; and of the coins of most other nations, 0·900.

2. The **Pound Sterling** is the value of the pure gold in a sovereign, viz.,	113·001	grains.
The alloy in a sovereign consists of copper,	10·273	,,
Full weight of a sovereign,	123·274	,,
Fineness, 22 carats = 0·916⅔.		
Least legal tender weight,	122·75	,,
Current weight, or least weight received at par at the Bank of England,	122·5	,,

3. The **Franc** is the value of 4·5 grammes of pure silver; which being alloyed with 0·5 gramme of copper, the full weight of the coin is 5 grammes. The fineness is 0·900. The Italian **Lira** is equal to the franc in weight, fineness, and value.

4. The German **Union Dollar** (Vereinsthaler) is the value of $\frac{1}{30}$ of a Zollpfund $\left(= \frac{1}{60}\right.$ of a kilogramme, or 257·2 grains$\left.\right)$ of pure silver, to which is added $\frac{1}{9}$ of its weight of alloy, the fineness being 0·900.

5. The **Comparative Value** of moneys in different countries fluctuates with the *rate of exchange*, and cannot be stated exactly. A conventional estimate of the average comparative value of the moneys of two countries is called *par*. A few rates of exchange at par are given in the following table. For further information, reference may be made to M'Culloch's *Commercial Dictionary*, and Kelly's *Universal Cambist*.

	£ Sterling.	Francs.
British *Pound sterling* = 20 *shillings* = 240 *pence* = 960 *farthings*,......	1·00000	25·220
French and Belgian *Franc* = 100 *centimes* = Italian *lira*, ...	0·03965	1·000
American *Dollar* = 100 *cents*, ...	0·20548	5·182
Russian *Ruble* = 100 *kopeks*,...........	0·15625	3·941
German *Vereinsthaler* (Union Dollar), = Prussian *thaler* = 30 *silbergroschen* = 360 *pfennige*,............	0·14493	3·655
Austrian *Gulden* (Florin) = $\frac{2}{3}$ *vereinsthaler* = 100 *neukreutzer*,........	0·09662	2·437
South German *Gulden* (Florin) = $\frac{4}{7}$ *vereinsthaler* = 60 *kreutzer* = 240 *pfennige*, ...	0·08282	2·089
Netherlandish *Gulden*, *Guilder* (or Florin) = 100 *cents*, ...	0·0833̇3	2·102
Danish *Rigsbankdaler* = 96 *skilling*, ...	0·10984	2·770
Norwegian *Speciesdaler* = 120 *skilling*,................................	0·21968	5·540
Swedish *Riksdaler* = 100 *öre* (*speciesdaler* = 4 *riksdaler*),.................	0·05479	1·382
Portuguese *Milreis* = 1,000 *reis*,.......	0·2354	5·937
Spanish *Duro* (Dollar) = 20 *reales*,....	0·2083	5·254
British Indian *Rupee* = 16 *annas* = 192 *pice* (*lac* = 100,000 *rupees*),...	0·0927	2·338

SECTION IX.—MEASURES OF SPEED, HEAVINESS, PRESSURE, WORK, AND POWER.

1. **Speed** or **Velocity** of advance is expressed in units of length per unit of time.

Comparison of Different Measures of Velocity.

	Miles per hour.		Feet per second.		Feet per minute.		Feet per hour.
	1	=	$1\cdot4\dot{6}$	=	88	=	5280·
	$0\cdot68\dot{1}\dot{8}$	=	1·	=	60	=	3600
	$0\cdot011\dot{3}\dot{6}$	=	$0\cdot01\dot{6}$	=	1	=	60
	$0\cdot00018\dot{9}\dot{3}$	=	$0\cdot0002\dot{7}$	=	$0\cdot01\dot{6}$	=	1
1 nautical mile per hour, or "knot,"......	= 1·1508	=	1·688	=	101·275	=	$6076\frac{1}{2}$

The units of time being the same in all civilized countries, the proportions amongst their units of velocity are the same with those amongst their linear measures.

2. **Speed of Turning,** or **Angular Velocity,** is expressed in turns per second, per minute, or per hour, or in circular measure per second.

To convert turns into circular measure, multiply by 6·2832
To convert circular measure into turns, multiply by 0·159155

Comparison of Different Measures of Angular Velocity.

Circular Measure per second.	Turns per second.	Turns per minute.	Turns per hour.
1	0·159155	9·5493	572·958
6·2832	1	60	3600
0·10472	$0\cdot01666\dot{6}$	1	60
0·001745	$0\cdot0002\dot{7}7$	$0\cdot0166\dot{6}$	1

3. **Heaviness** is expressed in units of weight per unit of volume; as pounds to the cubic foot, or kilogrammes to the cubic metre. (See Section XI.) **Specific Gravity** is the ratio of the heaviness of a given substance to the heaviness of pure water, at a standard temperature, which in Britain is 62° Fahr., and in France the temperature of the maximum density of water. To convert specific gravity, as estimated in Britain, into heaviness in lbs. to the cubic foot, multiply by 62·355.

In metric measures the specific gravity of a substance is equal to its heaviness in kilogrammes to the litre (or cubic decimetre very nearly).

4. **The Intensity of Pressure** is expressed in units of weight on the unit of area, as pounds on the square inch, or kilogrammes on the square metre; or by the height of a column of some fluid; or in *atmospheres*, the unit in this case being the average pressure of the atmosphere at the level of the sea.

The following table gives a comparison of various units in which the intensities of pressures are commonly expressed.

	Pounds on the square foot.	Pounds on the square inch.
One pound on the square inch,....	144	1
One pound on the square foot,.....	1	$\frac{1}{144}$
One inch of mercury (that is, weight of a column of mercury, at 32° Fahr., one inch high),............	70·7275	0·491163
One foot of water (at 39°·1 Fahr.),	62·425	0·4335
One inch of water,....................	5·2021	0·036125
One atmosphere, of 29·922 inches of mercury, or 760 millimetres,	2,116·3	14·7
One foot of air, at 32° Fahr., and under the pressure of one atmosphere, ...	0·080728	0·0005606
One kilogramme on the square metre,..................................	0·20481	0·0014223
One kilogramme on the square millimetre,........................	204,813	1,422·3
One millimetre of mercury,.........	2·7847	0·01934

Comparison of Heads of Water in Feet, with Pressures in Various Units.

One foot of water at 52°·3 Fahr. =	62·4	lbs. on the square foot.
" "	0·4333	lb. on the square inch.
" "	0·0295	atmosphere.
" "	0·8823	inch of mercury at 32°.
" "	773·	feet of air at 32°, and one atmosphere.
One lb. on the square foot,.........	0·016026	foot of water at 52°·3 Fahr.
One lb. on the square inch,..........	2·308	feet of water.
One atmosphere of 29·922 inches of mercury,........................	33·9	" "
One inch of mercury at 32°,........	1·1334	" "
One foot of air at 32°, and one atmosphere,.........................	0·001294	" "
One foot of average sea water,.....	1·026	foot of pure water.

5. **Work** is expressed in units of weight lifted through an unit of height; as in lbs. lifted one foot, called *foot-pounds;* or

kilogrammes lifted one metre, called *kilogrammetres.* (See Section XI. of this part.)

A kilogrammetre is 7·23308 foot-pounds.
A foot-pound is 0·138254 kilogrammetre.

6. **Power** is expressed in units of work done in an unit of time; as in foot-pounds per second, per minute, or per hour; or in conventional units called *horse-power.*

One *Horse-Power,* British measure, = 550 ft.-lbs. per second = 33,000 ft.-lbs. per minute = 1,980,000 ft.-lbs. per hour.

One "*Force de Cheval,*" French measure, = 75 kilogrammetres per second = 542½ ft.-lbs. per second nearly = 0·9863 British horse-power.

One British horse-power = 1·0139 force de cheval.

7. The **Statical Moment** of a given weight relatively to a given vertical plane is the product of the weight into its horizontal distance from that plane, and is expressed in the same sort of units with work.

Comparison of Measures of Statical Moment.

						Kilogrammetres.
Inch-lb. =						0·011521
12 =	1 Ft.-lb. =					0·138254
112 =	9⅓ =	1 Inch-cwt. =				1·29037
1,344 =	112 =	12 =	1 Foot-cwt. =			15·4844
2,240 =	186⅔ =	20 =	1⅔ =	1 Inch-ton =		25·8074
26,880 =	2,240 =	240 =	20 =	12 =	1 Foot-ton =	309·689

8. **Absolute Units of Force.**—The "Absolute Unit of Force" is a term used to denote the force which, acting on an unit of mass for an unit of time, produces an unit of velocity.

The unit of time employed is always a second.

The unit of velocity is in Britain one foot per second; in France one metre per second.

The unit of mass is the mass of so much matter as weighs one unit of weight near the level of the sea, and in some definite latitude.

In Britain the latitude chosen is that of London; in France, that of Paris.

In Britain the unit of weight chosen is sometimes a grain, sometimes a pound avoirdupois; and it is equal to 32·187 of the corresponding absolute units of force.

In France the unit of weight chosen is a gramme, and it is equal to 9·8087 of the corresponding absolute units of force.

The proportions borne to each other by the absolute units of force in different countries are nearly the same with those of the units of work (see Article 5 of this Section), and would be exactly

the same but for the variation of the force of gravity in the latitude. Gravity is about 1·00017 times greater in London than in Paris.

SECTION X.—MEASURES OF HEAT.

1. **Temperature, or, Intensity of Heat.—**

STANDARD POINTS—	Corresponding Degrees on Scale. Fahrenheit.	Centigrade.	Réaumur.
Boiling point of water under one atmosphere,	212°	100°	80°
Melting point of ice,.........	32°	0°	0°
(Absolute zero; known by theory only,.........	about — 461°·2	— 274°	— 219°·2)

9° Fahrenheit = 5° Centigrade = 4° Réaumur.

$$\text{Temp. Fahr.} = \frac{9}{5}\text{ Temp. Cent.} + 32^\circ$$

$$= \frac{9}{4}\text{ Temp. Réaum.} + 32^\circ$$

$$\text{Temp. Cent.} = \frac{5}{9}(\text{Temp. Fahr.} - 32^\circ) = \frac{5}{4}\text{ Temp. Réaum.}$$

$$\text{Temp. Réaum.} = \frac{4}{9}(\text{Temp. Fahr.} - 32^\circ) = \frac{4}{5}\text{ Temp. Cent.}$$

2. **Quantities of Heat** are expressed in units of weight of water heated one degree; as in pounds of water heated one degree of Fahr. (the British unit of heat): or in kilogrammes of water heated one degree Centigrade (the French unit of heat).

One French unit of heat (called *Calorie*) = 3·96832 British units.
One British unit of heat = 0·251996 French units.

Quantities of heat are sometimes also expressed in *units of evaporation;* that is, units of weight of water evaporated under the pressure of one atmosphere.

Heat which evaporates one lb. of water under one atmosphere,... .... = 966·1 British units of heat.
Heat which evaporates one kilogramme of water,......... = 536·7 French units of heat.

Comparative Table of Scales of Temperature.

Fahr.	Cent.	Réaum.	Fahr.	Cent.	Réaum.	Fahr.	Cent.	Réaum.
−58	−50	−40	311	155	124	680	360	288
−49	−45	−36	320	160	128	689	365	292
−40	−40	−32	329	165	132	698	370	296
−31	−35	−28	338	170	136	707	375	300
−22	−30	−24	347	175	140	716	380	304
−13	−25	−20	356	180	144	725	385	308
− 4	−20	−16	365	185	148	734	390	312
+ 5	−15	−12	374	190	152	743	395	316
14	−10	− 8	383	195	156	752	400	320
23	− 5	− 4	392	200	160	761	405	324
32	0	0	401	205	164	770	410	328
41	+ 5	+ 4	410	210	168	779	415	332
50	10	8	419	215	172	788	420	336
59	15	12	428	220	176	797	425	340
68	20	16	437	225	180	806	430	344
77	25	20	446	230	184	815	435	348
86	30	24	455	235	188	824	440	352
95	35	28	464	240	192	833	445	356
104	40	32	473	245	196	842	450	360
113	45	36	482	250	200	851	455	364
122	50	40	491	255	204	860	460	368
131	55	44	500	260	208	869	465	372
140	60	48	509	265	212	878	470	376
149	65	52	518	270	216	887	475	380
158	70	56	527	275	220	896	480	384
167	75	60	536	280	224	905	485	388
176	80	64	545	285	228	914	490	392
185	85	68	554	290	232	923	495	396
194	90	72	563	295	236	932	500	400
203	95	76	572	300	240	941	505	404
212	100	80	581	305	244	950	510	408
221	105	84	590	310	248	959	515	412
230	110	88	599	315	252	968	520	416
239	115	92	608	320	256	977	525	420
248	120	96	617	325	260	986	530	424
257	125	100	626	330	264	995	535	428
266	130	104	635	335	268	1004	540	432
275	135	108	644	340	272	1013	545	436
284	140	112	653	345	276	1022	550	440
293	145	116	662	350	280	1031	555	444
302	150	120	671	355	284	1040	560	448

SECTION XI.—TABLES OF MULTIPLIERS FOR CONVERTING MEASURES.

1. Comparison of Binary, Decimal, and Duodecimal Fractions.—

Halves.	4ths.	8ths.	16ths.	32ds.	Decimals.	12ths.	6ths.	4ths.	3ds.	Halves.
				1	·03125					
			1	2	·06250					
					·08333	1				
				3	·09375					
		1	2	4	·12500					
				5	·15625					
					·16667	2	1			
			3	6	·18750					
				7	·21875					
	1	2	4	8	·25000	3		1		
				9	·28125					
			5	10	·31250					
					·33333	4	2		1	
				11	·34375					
		3	6	12	·37500					
				13	·40625					
					·41667	5				
			7	14	·43750					
				15	·46875					
1	2	4	8	16	·50000	6	3	2		1
				17	·53125					
			9	18	·56250					
					·58333	7				
				19	·59375					
		5	10	20	·62500					
				21	·65625					
					·66667	8	4		2	
			11	22	·68750					
				23	·71875					
	3	6	12	24	·75000	9		3		
				25	·78125					
			13	26	·81250					
					·83333	10	5			
				27	·84375					
		7	14	28	·87500					
				29	·90625					
					·91667	11				
			15	30	·93750					
				31	·96875					
2	4	8	16	32	1·00000	12	6	4	3	2

The values, in decimals, of the binary fractions are exact. Those of duodecimal fractions which are not also binary fractions, are approximate only.

2. Multipliers for Converting British Measures.—

	A.—Links into Feet.	B.—Feet into Links.	C.—Square Links into Square Feet.	D.—Square Feet into Square Links.	
1	0·66	1·51515	0·4356	2·2957	1
2	1·32	3·03030	0·8712	4·5914	2
3	1·98	4·54545	1·3068	6·8871	3
4	2·64	6·06061	1·7424	9·1827	4
5	3·30	7·57576	2·1780	11·4784	5
6	3·96	9·09091	2·6136	13·7741	6
7	4·62	10·60606	3·0492	16·0698	7
8	5·28	12·12121	3·4848	18·3655	8
9	5·94	13·63636	3·9204	20·6612	9
10	6·60	15·15152	4·3560	22·9568	10

	E.—Mean Geographical Miles into Statute Miles.	F.—Statute Miles into Mean Geographical Miles.	G.—Tons into Lbs.	H.—Lbs. into Tons.	
1	1·151	0·869	2,240	·0004464	1
2	2·302	1·738	4,480	·0008929	2
3	3·452	2·607	6,720	·0013393	3
4	4·603	3·476	8,960	·0017857	4
5	5·754	4·345	11,200	·0022321	5
6	6·905	5·214	13,440	·0026786	6
7	8·056	6·083	15,680	·0031250	7
8	9·207	6·952	17,920	·0035714	8
9	10·357	7·821	20,160	·0040179	9
10	11·508	8·690	22,400	·0044643	10

	I.—Tons Displacement into Cubic Feet.	J.—Cubic Feet into Tons Displacement.	K.—Lbs. on the Square Inch into Lbs. on the Square Foot.	L.—Lbs. on the Square Foot into Lbs. on the Square Inch.	
1	35	·02857	144	·00694	1
2	70	·05714	288	·01389	2
3	105	·08571	432	·02083	3
4	140	·11429	576	·02778	4
5	175	·14286	720	·03472	5
6	210	·17143	864	·04167	6
7	245	·20000	1,008	·04861	7
8	280	·22857	1,152	·05556	8
9	315	·25714	1,296	·06250	9
10	350	·28571	1,440	·06944	10

	M.—Lbs. Avoir. into Grains.	N.—Grains into Lbs. Avoir.	O.—Cubic Feet into Gallons.	P.—Gallons into Cubic Feet.	
1	7,000	0·000142857	6·2355	0·16037	1
2	14,000	0·000285714	12·4710	0·32074	2
3	21,000	0·000428571	18·7065	0·48112	3
4	28,000	0·000571429	24·9420	0·64149	4
5	35,000	0·000714286	31·1775	0·80186	5
6	42,000	0·000857143	37·4130	0·96223	6
7	49,000	0·001000000	43·6485	1·12260	7
8	56,000	0·001142857	49·8840	1·28298	8
9	63,000	0·001285714	46·1195	1·44335	9
10	70,000	0·001428571	62·3550	1·60372	10

Q.—Values of Decimal Fractions of a Pound Sterling in Shillings and Pence.

£		s.	d.	£		s.	d.	£		s.	d.
·001	=	0	0·24	·01	=	0	2·4	·1	=	2	0
·002		0	0·48	·02		0	4·8	·2		4	0
·003		0	0·72	·03		0	7·2	·3		6	0
·004		0	0·96	·04		0	9·6	·4		8	0
·005		0	1·20	·05		1	0·0	·5		10	0
·006		0	1·44	·06		1	2·4	·6		12	0
·007		0	1·68	·07		1	4·8	·7		14	0
·008		0	1·92	·08		1	7·2	·8		16	0
·009		0	2·16	·09		1	9·6	·9		18	0

R.—Values of Farthings, Pence, and Shillings in Decimal Fractions of a Pound.

Farthings.	£	Shillings.	£
1	·0010417	1	·05
2	·0020833	2	·10
3	·0031250	3	·15
Pence.		4	·20
1	·004167	5	·25
1½	·006250	6	·30
2	·008333	7	·35
3	·012500	8	·40
4	·016667	9	·45
4½	·018750	10	·50
5	·020833	11	·55
6	·025000	12	·60
7	·029167	13	·65
7½	·031250	14	·70
8	·033333	15	·75
9	·037500	16	·80
10	·041667	17	·85
10½	·043750	18	·90
11	·045833	19	·95

V.—Comparative Table of French and British Measures.

	No.	Log.	Log.	No.	
Grains in a gramme,...	15.43235	1.188432	$\bar{2}.811568$	0.064799	Gramme in a grain.
Pounds avoird. in a kilogramme,	2.20462	0.343334	$\bar{1}.656666$	0.453593	Kilog. in a lb. avoirdupois.
Ton in a tonne,....................	0.984206	$\bar{1}.993086$	0.006914	1.01605	Tonnes in a ton.
Feet in a mètre, ...	3.2808693	0.515989	$\bar{1}.484011$	0.3047972 1	Mètres in a foot.
Inch in a millimètre,............	0.03937043	$\bar{2}.595170$	1.404830	25.39977	Millimètres in an inch.
Mile in a kilomètre, ...	0.621377	$\bar{1}.793355$	0.206645	1.60933	Kilomètres in a mile.
Square feet in a square mètre,..	10.7641	1.031978	$\bar{2}.968022$	0.0929013	Square mètre in a square foot.
Square inch in a square millimètre, ...	0.00155003	$\bar{3}.190340$	2.809660	645.148	Square millim. in a square inch.
Cubic feet in a cubic mètre,.....	35.3156	1.547967	$\bar{2}.452033$	0.0283161	Cubic mètre in a cubic foot.
Foot-pounds in a kilogrammètre,	7.23308	0.859323	$\bar{1}.140677$	0.138254	Kilogrammètre in a foot-pound.
Pounds-to-the-foot in a kilogramme-to-the-mètre,......	0.671963	$\bar{1}.827345$	0.172655	1.48818	Kilogrammes-to-the-mètre in a pound-to-the-foot.
Pounds-to-the-square-foot in a kilogramme-to-the-square-mètre, ...	0.204813	$\bar{1}.311356$	0.688644	4.88252	Kilogrammes-to-the-square-mètre in a pound-to-the-square-foot.
Pounds-to-the-square-inch in a kilog.-to-the-square-millimètre,.......................	1422.31	3.152994	$\bar{4}.847006$	0.000703083	Kilog.-to-the-square-millimètre in a pound-to-the-square-inch.
Pounds-to-the-cubic-foot in a kilogramme-to-the-cubic-mètre,........................	0.062426	$\bar{2}.795367$	1.204633	16.019	Kilogrammes-to-the-cubic-mètre in a pound-to-the-cubic-foot.
Fahrenheit-degrees in a centigrade-degree,.................	1.8	0.255273	$\bar{1}.744727$	0.55555	Centigrade-degree in a Fahrenheit degree.
British units of heat in a French unit,..................	3.96832	0.598607	$\bar{1}.401393$	0.251996	French units of heat in a British unit.

	No.	Log.	Log.	No.	
Cubic inch in a cubic millimetre,...............	0·000061025	$\bar{5}$·785511	4·214489	16,387	Cubic millimetres in a cubic inch.
Yards in a metre,.............	1·0936231	0·038868	$\bar{1}$·961132	0·91439180	Metre in a yard.
Square yards in a sq. metre,	1·19601	0·077735	$\bar{1}$·922265	0·836112	Square metre in a square yard.
Cubic yards in a cubic metre,	1·30799	0·116603	$\bar{1}$·883397	0·764534	Cubic metre in a cubic yard.
Sq. miles in a sq. kilometre,	0·386109	$\bar{1}$·586710	0·413290	2·589941	Square kilometre in a sq. mile.
Acres in a hectare,............	2·4711	0·392889	$\bar{1}$·607111	0·4046782	Hectare in an acre.
Mean geographical mile in a kilometre, nearly,......	0·54	$\bar{1}$·73236	0·26764	1·852	Kilometres in a mean geographical mile, nearly,
Gallon in a litre,...............	0·220215	$\bar{1}$·342847	0·657153	4·54102	Litres in a gallon.
£ sterling in a franc,.........	0·039651	$\bar{2}$·598255	1·401745	25·22	Francs in a £ sterling.
Shilling in a franc,...........	0·79302	$\bar{1}$·899285	0·100715	1·261	Franc in a shilling.
Penny in a centime,...........	0·09516	$\bar{2}$·978466	1·021534	10·508	Centimes in a penny.
Horse-power in a force de cheval,....................	0·98633	$\bar{1}$·99402	0·00598	1·01386	Force de cheval in a horse-power.
£ per foot in a franc per metre,......................	0·012086	$\bar{2}$·082266	1·917734	82·74	Francs per metre in a £ per foot.
£ per square foot in a franc per square metre,.........	0·0036836	$\bar{3}$·566277	2·433723	271·48	Francs per square metre in a £ per square foot.
£ per cubic foot in a franc per cubic metre,..........	0·00112276	$\bar{3}$·050288	2·949712	890·66	Francs per cubic metre in a £ per cubic foot.
£ per lb. avoirdupois in a franc per kilogramme,...	0·017986	$\bar{2}$·254921	1·745079	55·61	Francs per kilogramme in a £ per lb. avoirdupois.
£ per acre in a franc per hectare,....................	0·016046	$\bar{2}$·205365	1·794635	62·321	Francs per hectare in a £ per acre.
£ per gallon in a franc per litre,......................	0·18006	$\bar{1}$·255408	0·744592	5·5538	Francs per litre in a £ per gallon.

4. Multipliers for Converting British and French Measures.

	A.—Metres into Feet.	B.—Feet into Metres.	C.—Millimetres into Inches.	D.—Inches into Millimetres.	
1	3·2809	0·3048	·03937	25·400	1
2	6·5617	0·6096	·07874	50·800	2
3	9·8426	0·9144	·11811	76·199	3
4	13·1235	1·2192	·15748	101·599	4
5	16·4043	1·5240	·19685	126·999	5
6	19·6852	1·8288	·23622	152·399	6
7	22·9661	2·1336	·27559	177·798	7
8	26·2470	2·4384	·31496	203·198	8
9	29·5278	2·7432	·35433	228·598	9
10	32·8087	3·0480	·39370	253·998	10

	E.—Square Metres into Square Feet.	F.—Square Feet into Square Metres.	G.—Square Millimetres into Square Inches.	H.—Square Inches into Square Millimetres.	
1	10·764	·0929	·0015500	645·15	1
2	21·528	·1858	·0031001	1290·30	2
3	32·292	·2787	·0046501	1935·44	3
4	43·056	·3716	·0062001	2580·59	4
5	53·821	·4645	·0077501	3225·74	5
6	64·585	·5574	·0093002	3870·89	6
7	75·349	·6503	·0108502	4516·04	7
8	86·113	·7432	·0124002	5161·18	8
9	96·877	·8361	·0139503	5806·33	9
10	107·641	·9290	·0155003	6451·48	10

	I.—Cubic Metres into Cubic Feet.	J.—Cubic Feet into Cubic Metres.	K.—Cubic Millimetres into Cubic Inches.	L.—Cubic Inches into Cubic Millimetres.	
1	35·316	·028316	·00006103	16387	1
2	70·631	·056632	·00012205	32773	2
3	105·947	·084948	·00018308	49160	3
4	141·262	·113264	·00024410	65546	4
5	176·578	·141581	·00030513	81933	5
6	211·894	·169897	·00036615	98320	6
7	247·209	·198213	·00042718	114706	7
8	282·525	·226529	·00048820	131093	8
9	317·840	·254845	·00054923	147480	9
10	353·156	·283161	·00061025	163866	10

Multipliers for Converting British and French Measures—*continued.*

	M.—Grammes into Grains.	N.—Grains into Grammes.	O.—Kilogrammes into Lbs.	P.—Lbs. into Kilogrammes.	
1	15·4323	·06480	2·2046	0·4536	1
2	30·8647	·12960	4·4092	0·9072	2
3	46·2970	·19440	6·6139	1·3608	3
4	61·7294	·25920	8·8185	1·8144	4
5	77·1617	·32399	11·0231	2·2680	5
6	92·5941	·38879	13·2277	2·7216	6
7	108·0264	·45359	15·4323	3·1751	7
8	123·4588	·51839	17·6370	3·6287	8
9	138·8911	·58319	19·8416	4·0823	9
10	154·3235	·64799	22·0462	4·5359	10

	Q.—Tonneaux into Tons.	R.—Tons into Tonneaux.	S.—Litres into Gallons.	T.—Gallons into Litres.	
1	0·9842	1·0160	0·2202	4·541	1
2	1·9684	2·0321	0·4404	9·082	2
3	2·9526	3·0481	0·6606	13·623	3
4	3·9368	4·0642	0·8809	18·164	4
5	4·9210	5·0802	1·1011	22·705	5
6	5·9052	6·0963	1·3213	27·246	6
7	6·8894	7·1123	1·5415	31·787	7
8	7·8736	8·1284	1·7617	36·328	8
9	8·8579	9·1444	1·9819	40·869	9
10	9·8421	10·1605	2·2021½	45·410	10

	U.—Kilogrammetres into Foot-Lbs.	V.—Foot-Lbs. into Kilogrammetres.	W.—Kilogrammes on the Square Millimetre into Lbs. on the Square Inch.	X.—Lbs. on the Square Inch into Kilogrammes on the Square Millimetre.	
1	7·233	0·13825	1422	·000703	1
2	14·466	0·27651	2845	·001406	2
3	21·699	0·41476	4267	·002109	3
4	28·932	0·55302	5689	·002812	4
5	36·165	0·69127	7111	·003515	5
6	43·398	0·82952	8534	·004219	6
7	50·632	0·96778	9956	·004922	7
8	57·865	1·10603	11378	·005625	8
9	65·098	1·24429	12801	·006328	9
10	72·331	1·38254	14223	·007031	10

Multipliers for Converting British and French Measures—*continued.*

	Y.—Kilometres into Miles.	Z.—Miles into Kilometres.	AA.—Hectares into Acres.	BB.—Acres into Hectares.	
1	0·6214	1·6093	2·471	0·4047	1
2	1·2428	3·2186	4·942	0·8094	2
3	1·8641	4·8280	7·413	1·2140	3
4	2·4855	6·4373	9·884	1·6187	4
5	3·1069	8·0467	12·356	2·0234	5
6	3·7283	9·6560	14·827	2·4281	6
7	4·3496	11·2653	17·298	2·8328	7
8	4·9710	12·8747	19·769	3·2375	8
9	5·5924	14·4840	22·240	3·6421	9
10	6·2138	16·0933	24·711	4·0468	10

	CC.—Francs into £.	DD.—£ into Francs.	EE.—Francs into Pence.	FF.—Pence into Francs.	
1	·03965	25·22	9·516	0·10508	1
2	·07930	50·44	19·033	0·21017	2
3	·11895	75·66	28·549	0·31525	3
4	·15860	100·88	38·065	0·42033	4
5	·19826	126·10	47·581	0·52542	5
6	·23791	151·32	57·098	0·63050	6
7	·27756	176·54	66·614	0·73558	7
8	·31721	201·76	76·130	0·84067	8
9	·35686	226·98	85·646	0·94575	9
10	·39651	252·20	95·163	1·05083	10

5. Conversion of Velocities.

	A.—Miles per Hour into Feet per Second.	B.—Feet per Second into Miles per Hour.	C.—Knots into Feet per Second.	D.—Feet per Second into Knots.	
1	1·467	0·682	1·688	0·592	1
2	2·933	1·364	3·376	1·185	2
3	4·400	2·045	5·064	1·777	3
4	5·867	2·727	6·752	2·370	4
5	7·333	3·409	8·439	2·962	5
6	8·800	4·091	10·127	3·555	6
7	10·267	4·773	11·815	4·147	7
8	11·733	5·455	13·503	4·740	8
9	13·200	6·136	15·191	5·332	9
10	14·667	6·818	16·879	5·925	10

CONVERSION OF VELOCITIES—*continued.*

	E.—Knots into Metres per Second.	F.—Metres per Second into Knots.	Angular Velocity. G.—Turns per Second into Circular Measure.	Angular Velocity. H.—Circular Measure into Turns per Second.	
1	0·5144	1·944	6·28	0·159	1
2	1·0288	3·888	12·57	0·318	2
3	1·5432	5·832	18·85	0·477	3
4	2·0576	7·776	25·13	0·637	4
5	2·5720	9·720	31·42	0·796	5
6	3·0864	11·664	37·70	0·955	6
7	3·6008	13·608	43·98	1·114	7
8	4·1152	15·552	50·27	1·273	8
9	4·6296	17·496	56·55	1·432	9
10	5·1440	19·440	62·83	1·592	10

6. CONVERSION OF PRESSURES IN ATMOSPHERES.

Atmospheres.	Lbs. on the Square Inch.	Lbs. on the Square Foot.	Kilogrammes on the Square Metre.	Millimetres of Mercury.	Inches of Mercury.	Feet of Water.
1	14·7	2116	10333	760	29·922	33·9
2	29·4	4233	20666	1520	59·844	67·8
3	44·1	6349	30999	2280	89·765	101·7
4	58·8	8465	41332	3040	119·687	135·6
5	73·5	10581	51665	3800	149·609	169·5
6	88·2	12698	61998	4560	179·531	203·4
7	102·9	14814	72331	5320	209·453	237·3
8	117·6	16930	82664	6080	239·374	271·2
9	132·3	19047	92997	6840	269·296	305·1
10	147·0	21163	103330	7600	299·218	339·0

PART III.

RULES IN ENGINEERING GEODESY.

SECTION I.—RULES DEPENDING ON THE DIMENSIONS AND FIGURE OF THE EARTH.

1. **Earth's Principal Dimensions** (as calculated at the British Ordnance Survey Office, and published in 1866.)—Longitude of the earth's greater equatorial axis, about 15° 34′ east of Greenwich. Longitude of the earth's lesser equatorial axis, about 105° 34′ east of Greenwich.

	Feet.	Metres.
Greater equatorial axis,............	41,852,700	12,756,588
Lesser equatorial axis, ...	41,839,944	12,752,701
Mean equatorial diameter,.........	41,846,322	12,754,644
Polar axis,.........................	41,706,858	12,712,136
Mean between mean equatorial diameter and polar axis,......	41,776,590	12,733,390

In the present state of our knowledge, calculations of the earth's dimensions are doubtful beyond the fifth figure.

2. **Minute of Latitude.**—Length on the earth's surface corresponding to a minute of the *mean meridian*;

$$\text{in feet} = 6076 - 31 \cos \cdot 2 \text{ latitude of middle of arc;}$$
$$\text{in metres} = 1852 - 9{\cdot}4 \cos \cdot 2 \text{ latitude of middle of arc;}$$

(observing that cosines of obtuse angles have their signs reversed.) These formulæ are correct, for any meridian, to the nearest foot, and to the nearest $\frac{3}{10}$ of a metre.

3. **Minute of Prime Vertical** (being the great circle perpendicular to the meridian),

$$\text{in feet} = \frac{12214 + \text{length of minute of meridian}}{3};$$

$$\text{in metres} = \frac{3723 + \text{length of minute of meridian}}{3}.$$

4. **Minute of Longitude.**—For its length multiply the length of a minute of the prime vertical by the cosine of the latitude.

5. **Explanation of Table.**—The following table gives the results of the three preceding rules in feet, correct to the nearest foot, for latitudes at intervals of one degree, from 0° to 90°:—

Lat.	Min. Long.	Min. pr. v.	Min. Lat.	Min. Lat.	Min. pr. v.	Min. Long.	Lat.
0°	6086	6086	6045	6107	6107	0	90°
1	6085	6086	6045	6107	6107	107	89
2	6083	6086	6045	6107	6107	213	88
3	6078	6086	6045	6107	6107	320	87
4	6071	6086	6045	6107	6107	426	86
5	6063	6086	6045	6107	6107	532	85
6	6053	6087	6046	6106	6107	638	84
7	6041	6087	6046	6106	6107	744	83
8	6027	6087	6046	6106	6107	850	82
9	6012	6087	6047	6105	6106	955	81
10	5994	6087	6047	6105	6106	1060	80
11	5975	6087	6047	6105	6106	1165	79
12	5954	6087	6048	6104	6106	1270	78
13	5931	6087	6048	6104	6106	1374	77
14	5907	6088	6049	6103	6106	1477	76
15	5880	6088	6049	6103	6106	1580	75
16	5852	6088	6050	6102	6105	1683	74
17	5822	6088	6050	6102	6105	1785	73
18	5790	6088	6051	6101	6105	1887	72
19	5757	6089	6052	6100	6105	1988	71
20	5721	6089	6052	6100	6105	2088	70
21	5684	6089	6053	6099	6104	2188	69
22	5646	6089	6054	6098	6104	2287	68
23	5605	6089	6054	6098	6104	2385	67
24	5563	6090	6055	6097	6104	2483	66
25	5519	6090	6056	6096	6103	2579	65
26	5474	6090	6057	6095	6103	2675	64
27	5427	6091	6058	6094	6103	2771	63
28	5378	6091	6059	6093	6102	2865	62
29	5327	6091	6060	6092	6102	2958	61
30	5275	6092	6061	6091	6102	3051	60
31	5222	6092	6061	6091	6102	3142	59
32	5166	6092	6062	6090	6101	3233	58
33	5109	6092	6063	6089	6101	3323	57
34	5051	6093	6064	6088	6101	3413	56
35	4991	6093	6065	6087	6100	3499	55
36	4930	6093	6066	6086	6100	3586	54
37	4867	6094	6067	6085	6100	3671	53
38	4802	6094	6068	6084	6099	3755	52
39	4736	6095	6070	6082	6099	3838	51
40	4669	6095	6071	6081	6098	3920	50
41	4600	6095	6072	6080	6098	4001	49
42	4530	6096	6073	6079	6098	4080	48
43	4458	6096	6074	6078	6097	4158	47
44	4385	6096	6075	6077	6097	4235	46
45	4311	6097	6076	6076	6097	4311	45

6. **Minute of a Great Circle in any Azimuth.**—*Azimuth* is the angle which a given vertical plane traversing a station makes with the plane of the meridian of that station. Let m denote the length of a minute of the meridian, and p the length of a minute of the prime vertical, at the latitude of the middle of the arc to be measured; then the length required

$$= \frac{p+m}{2} - \frac{p-m}{2} \cdot \cos 2 \text{ azimuth};$$

observing, that when the azimuth exceeds 45°, the second term of the formula is to be added, instead of subtracted.

EXAMPLE I.—In latitude 60°, required the length in feet of one minute of a great circle on the earth's surface whose azimuth is 30°.

$$\frac{p+m}{2} = \frac{6102 + 6091}{2} = \frac{12193}{2} = \quad 6096{\cdot}5 \text{ feet.}$$

$$\frac{p-m}{2} = \quad \frac{11}{2} = 5{\cdot}5 \text{ feet.}$$

$$\times \cos 60° = \quad 0{\cdot}5$$

Product to be subtracted,................ 2·75

Length required, to the nearest foot,... 6094 feet.

EXAMPLE II.—In the same latitude, let the azimuth be 60°; then 60° × 2 = 120°, an obtuse angle, whose cosine is = − cos (180° − 120°) = − cos 60° = − 0·5.

$\frac{p+m}{2}$ as before, ...6096·5 feet.

$\left(\frac{p-m}{2} = 5{\cdot}5\right) \times 0{\cdot}5$ (to be added) = 2·75

Length required, to the nearest foot, 6099 feet.

6A. **Contained Arc.**—Divide the distance between two stations by the length of a minute on the great circle through them; the quotient will be the contained arc in minutes.

7. **To find the True Azimuth of a Station-Line.**

I. *By the Two greatest Elongations of a Circumpolar Star.*—Observe the greatest and least horizontal angles made by a star near the pole with the station-line when the star is at its greatest distances east and west of the pole, and take the mean of those angles, which is the true azimuth of the station-line. In the northern hemisphere the Pole-star, α Ursæ Minoris, is the best.

This method is seldom practicable with an ordinary theodolite, as in general one of the observations must be made by daylight.

II. *By equal Altitudes of a Star.*—The theodolite being at a

station in the station-line chosen, measure the horizontal angle from the station-line to any star which is not near the highest or lowest point of its apparent daily course, and take also the altitude of that star. Leave the vertical circle clamped, and let the instrument remain undisturbed until the star is approaching the same altitude at the other side of its apparent circular course. Then, without moving the vertical circle, direct the telescope towards the star, clamp the vernier-plate, and by the aid of its tangent-screw follow the star in azimuth with the cross wires until it arrives exactly at its former altitude, as is shown by its image coinciding with the cross wires; then measure the horizontal angle between the new direction of the star and the station-line: the mean between the two horizontal angles will be the true azimuth of the station-line.*

In both the preceding processes it is to be understood that the *mean of two horizontal angles* means their *half-sum* when they are at the same side of the station-line, but their *half-difference* when they are at opposite sides.

The second method may be applied to the sun, observing the sun's west limb in the forenoon and east limb in the afternoon, or *vice versâ;* but in that case a correction is required, owing to the sun's change of declination. When the sun's declination is changing towards the $\left\{\begin{matrix}\text{north}\\ \text{south}\end{matrix}\right\}$, the approximate direction of the meridian, as found by the method just described, is too far to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$. The correction required is given by the formula,†

$$\frac{\text{change of sun's declination}}{2} \times \sec\cdot\text{latitude} \times \operatorname{cosec}\frac{1}{2}\text{ angular motion of sun between the observations.}$$

III. *By One greatest Elongation of a Circumpolar Star.*—To use this method, the declination of the star, and the latitude of the place, should be known. Then

$$\sin\cdot\text{azimuth of star at greatest elongation} = \cos\cdot\text{declination} \div \cos\cdot\text{latitude};$$

and that azimuth, being added to or subtracted from the horizontal angle between the station-line and the star, when at its greatest elongation (according as the station-line lies to the same side of

* In observing at night with the theodolite, it is necessary to throw, by means of a lamp and a small mirror, enough of light into the tube to make the cross wires visible.

† At the equinoxes, the rate of change of the sun's declination is about 59" per hour; and it varies nearly as the cosine of the sun's right ascension.

the meridian with the star, or to the opposite side) gives the azimuth of the station-line.*

IV. *By observing the Altitude of a Star, and the Horizontal Angle between it and the Station-Line.*—The altitude being corrected for refraction, the azimuth of the star is computed by taking the zenith distance, or complement of that altitude, the polar distance† of the star, and the co-latitude of the place, as the three sides of a spherical triangle; when the azimuth of the star will be the

* The following is a table of the declinations of a few of the more conspicuous stars for the 1st of January, 1865, together with the annual rate at which those declinations are changing, + denoting increase, and — diminution:—

NORTHERN HEMISPHERE.

	Star.	North Declination.			Rate of Annual Variation.
α	Andromedæ,	28°	20′	42″	+ 19″·9
α	Ursæ Minoris (Pole-Star),	88	35	23	+ 19 ·2
α	Arietis,	22	49	21	+ 17 ·2
α	Ceti,	3	33	28	+ 14 ·4
α	Persei,	49	22	39	+ 13 ·2
α	Tauri (Aldebaran),	16	14	6	+ 7 ·6
α	Aurigæ (Capella),	45	51	24	+ 4 ·2
α	Orionis (Betelgeuze),	7	22	43	+ 1 ·1
α	Geminorum (Castor),	32	10	52	— 7 ·4
α	Canis Minoris (Procyon),	5	34	7	— 8 ·9
β	Geminorum (Pollux),	28	20	57	— 8 ·3
α	Leonis (Regulus),	12	37	32	— 17 ·4
α	Ursæ Majoris,	62	28	44	— 19 ·4
η	Ursæ Majoris,	49	59	17	— 18 ·1
α	Bootis (Arcturus),	19	53	12	— 18 ·9
α	Ophiuchi,	12	39	39	— 2 ·9
α	Lyræ (Vega),	38	39	36	+ 3 ·1
α	Aquilæ (Altair),	8	30	51	+ 9 ·2
α	Cygni,	44	47	58	+ 12 ·7
α	Pegasi (Markab),	14	28	46·5	+ 19 ·3

SOUTHERN HEMISPHERE.

	Star.	South Declination.			Rate of Annual Variation.
β	Orionis (Rigel),	8°	21′	38″	— 4″·5
α	Columbæ,	34	8	51	— 2 ·2
α	Argûs (Canopus),	52	37	23	+ 1 ·8
α	Canis Majoris (Sirius),	16	32	1	+ 4 ·6
α	Hydræ,	8	4	31	+ 15 ·4
η	Argûs,	58	58	29	+ 18 ·7
α	Crucis,	62	20	58·5	+ 19 ·9
α	Virginis (Spica),	10	27	21	+ 18 ·9
α	Centauri,	60	16	24	+ 15 ·0
α	Scorpii (Antares),	26	7	46	+ 8 ·4
α	Trianguli Australis,	68	46	27	+ 7 ·4
α	Pavonis,	57	9	49	— 11 ·1
α	Gruis,	47	36	46	— 17 ·2
α	Piscis Australis (Fomalhaut),	30	20	13	— 19 ·0

† The polar distance is the complement of the declination.

angle opposite the side representing the polar distance. The azimuth of the station-line is then to be found as in Method III.

V. *Approximate Method by observing certain Stars.*—In the northern hemisphere a meridian-line may be fixed approximately by observing, with the aid of a plumb-line, the instant when the Pole-star A, and the star *Alioth* (ϵ Ursæ Majoris), appear in the same vertical plane. The Pole-star is marked A in fig. 36.

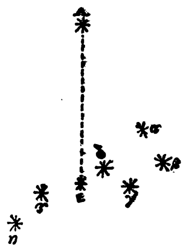


Fig. 36.

8. **Angle between Two Meridians.**—When two points on the earth's surface have the same latitude, but different longitudes, the horizontal angle made by their meridians with each other is found by the following equation:—

$$\sin \frac{1}{2} \text{ horizontal angle} = \sin \frac{1}{2} \text{ difference of long.} \times \sin \cdot \text{lat.}$$

9. **Astronomical Refraction.**—The correction for refraction is always to be subtracted from an altitude. It may be found in seconds approximately by the following formula:—

$$\text{Refraction} = 58'' \times \text{cotan apparent altitude.}$$

For more exact information on the subject, see a paper by the Rev. Dr. Robinson in the *Transactions of the Royal Irish Academy*, vol. xix. Tables of Refraction are given in treatises on Navigation, such as Raper's.

Below about 8° or 10° of altitude the changeable condition of the atmosphere makes the correction for refraction very uncertain.

10. **Dip of the Sea-Horizon,** in seconds = $\sqrt{}$ (height of station in feet) × 57″·4, nearly.

11. **To find the Latitude of a Place.**

METHOD I. *By the Mean Altitude of a Circumpolar Star.*—Take the altitudes of a circumpolar star at its upper and lower culminations (which positions are known by watching for the instants when the altitude is greatest and least). From each of those *apparent* altitudes subtract the correction for refraction; the mean of the *true* altitudes thus found is the latitude of the place.

METHOD II. *By One Meridian Altitude of a Star.*—Observe the meridian altitude of a star by watching for the instant when its altitude is greatest or least, and subtract the corrections for refraction, and also for dip, if necessary. The complement of the true altitude is the *zenith distance.* Find the declination of the star from the *Nautical Almanac* (which is published four years in advance.)

Then if the star is between the zenith and the equator,

$$\text{Latitude} = \text{Zenith distance} + \text{Declination}; \;\ldots\ldots(1.)$$

If the star is between the equator and the horizon,

$$\text{Latitude} = \text{Zenith distance} - \text{Declination}; \quad \ldots\ldots(2.)$$

If the star is between the zenith and the elevated pole,

$$\text{Latitude} = \text{Declination} - \text{Zenith distance}; \quad \ldots\ldots(3.)$$

If the star is between the elevated pole and the horizon,

$$\text{Latitude} = 180^\circ - \text{Declination} - \text{Zenith distance}; \quad \ldots(4.)$$

METHOD III. *By the Sun's Meridian Altitude.*—In this method the final calculation, from the sun's declination, as found in the *Nautical Almanac*, and the *true* altitude of his centre, is the same as in Method II. But besides the correction for refraction and dip, the altitude requires to be further corrected by subtracting or adding the sun's semidiameter, according as his upper or lower limb has been observed, and by adding the sun's parallax, being the angle subtended at the sun by the distance between the earth's centre and the place of observation.

To find the correction for parallax, find the sun's horizontal parallax on the day of observation, from the *Nautical Almanac*, and multiply it by the cosine of the altitude of the sun's centre.

(The mean value of the sun's horizontal parallax is about $8''\cdot6$).

The sun's semidiameter on the day of observation is to be found in the *Nautical Almanac*. It varies from $15'\ 46''$ to $16'\ 18''$.

The calculation may be thus set down algebraically—

$$\left\{\begin{array}{l}\text{True altitude} = \text{apparent altitude} - \text{Dip (if the sea-} \\ \quad\text{horizon has been observed)} - \text{Refraction} \pm \text{sun's} \\ \quad\text{semidiameter} + \text{parallax}; \ldots\ldots\ldots\ldots\ldots\ldots\end{array}\right\} (5.)$$

$$\text{Zenith distance} = 90^\circ - \text{true altitude}, \ldots\ldots\ldots \quad (6.)$$

Latitude (see Equations 1, 2, 3, 4).

Equations 1 and 2 are the most frequently applicable to the sun. Equation 3 is occasionally applicable between the tropics; and Equation 4 relates to observations made at midnight, in summer, in the polar regions.

12. **The Difference of Latitude** of two stations near each other is best found by observing the difference of the meridian altitudes or zenith distances of the same star as seen from the two stations.

13. **To Measure a Base-Line for a Survey Approximately, by Latitudes.**—The stations for the two ends of the base-line should be within sight of each other; not less than about fifty miles apart, if possible, and as nearly as possible in the same meridian.

Take the true azimuth of the base-line by Rule 7; and, if possible, take it from both stations, and take the mean of the results, which will be slightly different.

Take the latitudes of both stations by Rule 11, and the difference of their latitudes by Rule 12. The difference should be taken with the utmost possible precision; the absolute latitudes need not be determined so closely. Take the mean or half-sum of those absolute latitudes.

Multiply the difference of latitude by the secant (or divide by the cosine) of the azimuth; reduce the angle so found to minutes and decimal fractions of a minute; multiply it by the length corresponding to a minute of a great circle in the given mean latitude and azimuth (see Rule 6); the product will be the required length of base, correct to about one-6,000th part of itself.

EXAMPLE.—Suppose the data to be as follows:—

Mean azimuth,..................................	30°
Mean latitude,..................................	60°
Difference of latitude,........................	50′

Then,—

$$\frac{\text{Difference of latitude}}{\text{cos azimuth}} = \frac{50'}{\cdot 86603} = 57'\cdot 735$$

× Length corresponding to one minute, as already computed in Example 1 of Rule 6,..................................	6,094 feet.
Length of base required,..................	351,837 feet.

Which is correct to the nearest 60 feet, or thereabouts.

14. **To Reduce an Elevated or Depressed Base to the Level of the Sea.**—Multiply the base as measured, by its elevation above or depression below the sea-level, and divide by the earth's mean radius; the quotient will be the correction, to be subtracted if the base is elevated, or added if it is depressed. (Earth's mean radius, accurate enough for the present purpose;

20,900,000 feet, or 6,370,000 metres.)

Section II.—Scales for Plans and Sections.

1. Plans.

Ordinary Designation of Scale.	Fraction of real Dimensions.	Use.
(1.) 1 inch to a mile,.........	$\frac{1}{63,360}$	Scale of the smaller ordnance maps of Britain. This scale is well adapted for maps to be used in exploring the country.
(2.) 4 inches to a mile,.......	$\frac{1}{15,840}$	Smallest scale permitted by the standing orders of parliament for the deposited plans of proposed works.
(3.) 6 inches to a mile,.......	$\frac{1}{10,560}$	Scale of the larger ordnance maps of Great Britain and Ireland. This scale, being just large enough to show buildings, roads, and other important objects distinctly in their true forms and proportions, and at the same time small enough to enable the eye of the engineer to embrace the plan of a considerable extent of country at one view, is on the whole the best adapted for the selection of lines for engineering works, and for parliamentary plans and preliminary estimates.
(4.) 6·336 inches to a mile,...	$\frac{1}{10,000}$	Decimal scale possessing the same advantages.
(5.) 400 feet to an inch,......	$\frac{1}{4,800}$	Smallest scale permitted by the standing orders of parliament for "enlarged plans" of buildings and of land within the curtilage of buildings.
(6.) 6 chains to an inch,......	$\frac{1}{4,752}$	Scale answering the same purpose.
(7.) 15·84 inches to a mile,...	$\frac{1}{4,000}$	Scales well suited for the working surveys and land plans of great engineering works, and for enlarged parliamentary plans.
(8.) 5 chains to an inch, or 16 inches to a mile,	$\frac{1}{3,960}$	(Scale 8 is that prescribed in the standing orders of parliament for "cross sections" of proposed railways, showing alterations of roads.)
(9.) 25·344 inches to a mile,	$\frac{1}{2,500}$	Scale of plans of part of the ordnance survey of Britain, from which the maps beforementioned are reduced. Well adapted for land plans of engineering works and plans of estates.
(10.) 200 feet to an inch,......	$\frac{1}{2,400}$	Scale suited for similar purposes. Smallest scale prescribed by law for land or contract plans in Ireland.

Ordinary Designation of Scale.	Fraction of real Dimensions.	Use.
(11.) 3 chains to an inch,......	$\frac{1}{2,376}$	Scale of the Tithe Commissioners' plans. Suited for the same purposes as the above.
(12.) 100 feet to an inch,......	$\frac{1}{1,200}$	Scale suited for plans of towns, when not very intricate.
(13.) 88 feet to an inch, or 60 inches to a mile,	$\frac{1}{1,056}$	Scale of ordnance plans of the less intricately built towns.
(14.) 63·36 inches to a mile,...	$\frac{1}{1,000}$	Decimal Scale having the same properties.
(15.) 44 feet to an inch, or 120 inches to a mile,	$\frac{1}{528}$	Scale of ordnance plans of the more intricately built towns.
(16.) 126·72 inches to a mile,	$\frac{1}{500}$	Decimal scale having the same properties.
(17.) 30 feet to an inch,.......	$\frac{1}{360}$	Scales for special purposes.
(18.) 20 feet to an inch,.......	$\frac{1}{240}$	
(19.) 10 feet to an inch,.......	$\frac{1}{120}$	
&c.	&c.	

2. Sections.

Ordinary Designation of Vertical Scale.	Fraction of real Height.	Horizontal Scales with which the Vertical Scale is usually combined.	Exaggeration.	Use.
(1.) 100 feet to an inch,	$\frac{1}{1,200}$	$\frac{1}{15,840}$ to $\frac{1}{10,560}$	From 13·2 to 8·8	Smallest scale permitted by the standing orders of parliament for sections of proposed works.
(2.) 40 feet to an inch,	$\frac{1}{480}$	$\frac{1}{4,800}$ to $\frac{1}{3,960}$	10 to 8·25	Smallest scale permitted by the standing orders of parliament for cross sections, showing alterations of roads.
(3.) 30 feet to an inch,	$\frac{1}{360}$	$\frac{1}{3,960}$ to $\frac{1}{2,376}$	11 to 6·6	Scales suitable for working sections.
(4.) 20 feet to an inch,	$\frac{1}{240}$	$\frac{1}{3,960}$ to $\frac{1}{2,376}$	16·5 to 9·9	
&c.	&c.	&c.	&c.	

Vertical sections, on a large scale $\left(\text{say } \frac{1}{100} \text{ or } \frac{1}{120}\right)$, and *without exaggeration*, are required at the sites of special works.

SECTION III.—RULES RELATING TO SURVEYING.

1. **Chaining on a Declivity—Reduction to the Level.** — The correction is always to be subtracted from the distance as measured.

When the angle of inclination has been measured by a "clinometer" or other angular instrument:—Correction in links per chain = 100 × versed sine of inclination.

When the vertical fall in links for each chain of distance on the slope is known:—Correction in links per chain $= 100 - \sqrt{10{,}000 - \text{fall}^2}$.

When the slope is gentle:—Correction in links per chain $= \frac{\text{fall}^2}{200}$ nearly.

1A. **Expansion of Measuring Rods and Chains.**—Increase of length by an elevation of temperature of 100° Cent. = 180° Fahr.:—brass, 0·00216; bronze, 0·00181; copper, 0·00184; wrought iron and steel, 0·0012; cast iron, 0·0011; platinum, 0·0009; glass, 0·0009; dry deal, 0·00043.

2. **To Set Out a Right Angle by the Chain.**—Choose any two numbers; take the sum of their squares, the difference of their squares, and twice their product; those three numbers will be proportional—the first to the hypothenuse, and the other two to the two legs of a right-angled triangle, which is to be set out on the ground.

For example: numbers chosen, 1 and 2; hypothenuse, $2^2 + 1^2 = 5$; legs, $2^2 - 1 = 3$, and $2 \times 2 \times 1 = 4$. This is the most generally useful right-angled triangle. Other examples: 13, 12, 5; 25, 24, 7; 17, 15, 8; 29, 21, 20; &c.

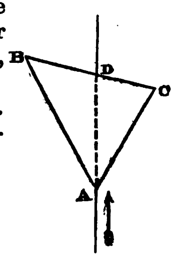


Fig. 37.

3. **Tie-Line.**—In a chained triangle, A B C, fig. 37, to find the length of a tie-line, A D. By calculation,

$$AD = \sqrt{\left\{\frac{AB^2 \cdot CD + AC^2 \cdot BD}{BC} - BD \cdot CD\right\}}$$

or by construction, draw the triangle and measure A D on paper. The measurement of A D on the ground is a check on the accuracy of the measurement of A B, B C, C A.

4. To Measure Gaps in Station-Lines by the Chain alone.

Case I.—*When the obstacle can be chained round.*

Rule I. (see fig. 37.)—A and D being marks in the station-line at the nearer and further sides of the obstacle, set out a triangle, A B C, of any form and size that will conveniently enclose the obstacle, subject only to the conditions, that B and C are to be ranged in one straight line with D, and that the angles at B and C are neither to be very acute nor very obtuse. Measure with the chain the lengths A B, A C, B D, D C, and find the length of A D as a tie-line (Article 3.)

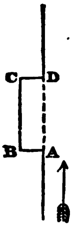


Fig. 38.

Rule II. (see fig. 38.)—Let A and D be marks at the nearer and further sides of the obstacle respectively. Range A B, D C at right angles to the station-line; make those perpendiculars equal to each other, and of any length that may be requisite in order to chain past the obstacle along B C, which will be parallel and equal to A D, the distance required.

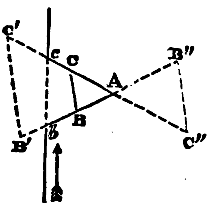


Fig. 39.

Rule III. (see fig. 39.)—Let b and c be points in the station-line at the nearer and further side of the obstacle respectively. From a convenient station, A, chain the lines A b, A c, being two sides of the triangle A b c: connect those lines by a line, B C, in any position which will form a well-conditioned triangle, A B C, of as large a size as is practicable: measure its three sides. Then the inaccessible distance is given by the formula,

$$b\,c = \sqrt{\left\{ \mathrm{A}\,b^2 + \mathrm{A}\,c^2 - \frac{(\mathrm{A}\,b + \mathrm{A}\,c)^2 - (\mathrm{A}\,b - \mathrm{A}\,c)^2}{(\mathrm{A\,B} + \mathrm{A\,C})^2 - (\mathrm{A\,B} - \mathrm{A\,C})^2} \cdot (\mathrm{A\,B}^2 + \mathrm{A\,C}^2 - \mathrm{B\,C}^2) \right\}}$$

The same formula applies to such positions of the connecting line as B′ C″ and B″ C″ as well as to B C.

If A B and A C can be laid off so as to be respectively proportional to A b and A c, the triangles A B C and A b c become similar, B C is parallel to b c, and the inaccessible distance is simply

$$b\,c = \mathrm{B\,C} \cdot \frac{\mathrm{A}\,b}{\mathrm{A\,B}}$$

In this method, as well as in the two preceding, the inaccessible distance may be found by plotting.

Case II.—*When it is impossible to chain round the obstacle.*

RULE IV. (see fig. 40.)—Let b and c be marks in the station-line at the nearer and further side of the gap respectively. On the nearer side of the obstacle, range the stations A and B in a straight line with c, making the angle $b\,c\,\mathrm{B}$ greater than 30°, and place them so that the intersecting lines $\mathrm{A}\,b$, $\mathrm{B}\,a$, connecting them with two points, a and b, in the station-line, shall form a pair of triangles, $a\,b\,\mathrm{C}$, $\mathrm{A\,B\,C}$, with no angle less than 30°. Measure the sides of those triangles, and compute the inaccessible distance $b\,c$ as follows:

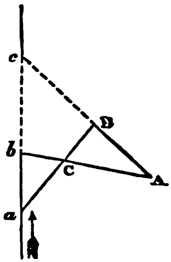


Fig. 40.

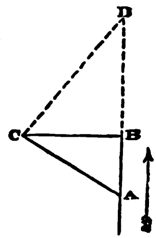
$$b\,c = \frac{a\,b \cdot \mathrm{A}\,b \cdot \mathrm{B\,C}}{\mathrm{C\,A} \cdot a\,\mathrm{B} - \mathrm{A}\,b \cdot \mathrm{B\,C}}.$$

As a check upon the position thus found for the point c, compute also the inaccessible distance $\mathrm{B}\,c$ as follows:

$$\mathrm{B}\,c = \frac{\mathrm{A\,B} \cdot a\,\mathrm{B} \cdot b\,\mathrm{C}}{\mathrm{C}\,a \cdot \mathrm{A}\,b - a\,\mathrm{B} \cdot b\,\mathrm{C}}.$$

This problem is solved graphically by plotting the figure $a\,b\,c\,\mathrm{A\,B\,C}\,a$, and producing $a\,b$ and A B till they intersect in c.

RULE V. (see fig. 41.)—When the inaccessible distance B D does not much exceed three or four chains. At B set out B C perpendicular to the station-line, and of a length such as to make the angle at D not less than 30°. At C range C A perpendicular to C D, cutting the station-line in A. Measure A B, B C; then



$$\mathrm{B\,D} = \frac{\mathrm{B\,C}^2}{\mathrm{A\,B}}.$$

Fig. 41.

When *angular instruments* are used, a gap in a station-line is measured by making it one side of a triangle, of which the angles and another side are given.

5. **Measuring Areas of Land.**—Almost all areas of land are made up of parallelograms, trapezoids, and triangles (see Rules at page 63), with the addition or subtraction of strips contained between straight station-lines and irregular boundaries (see Rules for "Any Plane Area," pp. 64 to 67.) For Land Measures, see p. 95.

6. **References to Rules of Trigonometry.**—The following are the rules of trigonometry chiefly used in surveying by angles:—

For *Plane Triangles;* 1, 2, page 53; and sometimes 3 and 4, pp. 53, 54; and 6, page 55.

For *Triangles so large as to be sensibly spherical;* the rule for spherical excess, page 55; and the approximate rules, page 58.

The three angles of every triangle should be measured, if possible, as a check upon accuracy.

7. **Reduction of Angles to the Centre of the Station.**—When the theodolite cannot be planted exactly at a station in a trigonometrical survey, but has to be placed at a short distance to one side of it, the angle actually measured between two objects is reduced to the angle which would have been measured had the theodolite been exactly at the station, by a correction which is calculated approximately as follows:—

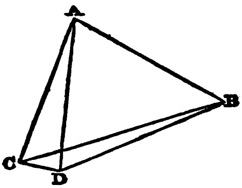


Fig. 42.

In fig. 42, let C be the station, D the position of the theodolite, A and B two objects; A D B the horizontal angle between them as measured at D; A C B the required horizontal angle at the station C.

Measure C D, and the angle A D C; calculate A C and C B approximately as if A C B were equal to A D B; then

$$A\,C\,B = A\,D\,B - 206264''.8\ C\,D \left\{ \frac{\sin A\,D\,C}{A\,C} - \frac{\sin B\,D\,C}{B\,C} \right\}$$

The above formula gives the correction in seconds when D lies to the *right* of both C A and C B. When it lies to the left of C B, sin B D C changes its sign; when to the left of C A, sin A D C changes its sign.

8. **Reduction of Sextant-Angles to the Level.**—To find with a reflecting instrument the horizontal angle between two objects that are not at the same level with the observer's eye. For an approximate method, set up a vertical pole in a line with each object, and measure the horizontal angle between the poles. For an accurate method, measure the angle between the objects themselves, and to take also the angle of altitude or depression of each. Find the *zenith distance* of each object by subtracting its altitude from, or adding its depression to, 90°.

In fig. 43, let O represent the observer's station; O B, O C the directions of the objects; B O C the angle between them; O D E a horizontal plane; D O B and E O C the altitudes of the objects; O A a vertical line, and A D E a spherical surface.

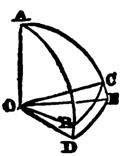


Fig. 43.

Then, in the spherical triangle A B C, the three sides are given—viz., A B and A C, the zenith distances, and B C, the angle between the objects; and the horizontal projection of that angle, being equal to the angle A, may be computed by the proper formula. (See page 57.)

9. **Determining Stations Afloat.**—In fig. 44, let D be the station afloat whose position is to be determined; and A, B, C, three known fixed objects, or landmarks, which ought *not* to be in or near the circumference of one circle traversing D. With a sextant (or, better still, with two sextants) measure the angles A D B, B D C; if practicable also, with a third sextant, measure the angle A D C = A D B + B D C, as a check on the accuracy of those angles. Then to plot the position of D, let A, B, and C be shown on the plan. From A draw A E, making the angle C A E = C D B: from C draw C E, making the angle A C E = A D B, and cutting A E in E: through the three points A, C, E describe a circle: through E and B draw a straight line cutting the circle in D; D will be the required station on the plan.

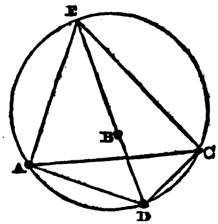


Fig. 44.

Or otherwise,—On a piece of tracing paper draw three straight lines radiating from one point, so as to make with each other angles equal to A D B and B D C. Lay it on the plan, and shift it about till the three lines traverse A, B, and C respectively; the point from which they diverge being pricked through on the plan, will give the position of D.

In the instrument called the *station-pointer*, three straight arms turning about one centre, and set to make any given angles with each other by means of a graduated arc, answer the purpose of the three lines on the tracing paper.

SECTION IV.—RULES RELATING TO LEVELLING AND SOUNDING.

1. **Correction for Curvature and Refraction.**—The correction for the earth's curvature, to be *subtracted* from the reading of a levelling-staff, is found as follows: Divide the square of the distance from the level to the staff by the earth's diameter (41,800,000 feet nearly, or 12,740,000 metres nearly).

Or otherwise,—Take two-thirds of the square of the distance in statute miles for the correction in feet.

The correction for refraction, to be *added* to the reading, is very variable and uncertain. On an average it may be taken at *one-sixth* of the correction for curvature.

Correction for curvature and refraction combined, to be *subtracted* from the reading on the staff,—average value about

$$= \frac{5}{6} \cdot \frac{\text{Distance}^2}{\text{Earth's diam.}} = 0{\cdot}56 \text{ foot} \times (\text{distance in statute miles})^2.$$

2. **Levelling by Angles.**—This process is approximate only.

Rule I.—Find the distance between the two objects whose difference of level is required.

Measure the angle of altitude of the higher object as seen from the lower, and (at the same instant, if possible) the angle of depression of the lower object as seen from the higher. (These are called *reciprocal angles.*) Take the half sum of those angles, and by its tangent multiply the horizontal distance between the objects: the product will be their difference of level.

Rule II.—When one angle only can be taken, it must be corrected for curvature and refraction. The correction for curvature to be added to altitudes and subtracted from depressions is *one-half of the contained arc;* which *contained arc* is computed, in minutes, by dividing the horizontal distance, if in feet, by 6,076, or, if in metres, by 1,852. The correction for refraction is uncertain; but on an average it may be allowed for by diminishing the correction for curvature by *one-sixth* of its amount.

3. **Levelling by the Barometer.** (Approximate only).—Let the quantities observed be denoted as follows:—

Stations.	Heights of Mercurial column.	Temperatures of the Mercury, by "attached" Thermometer.	Temperatures of the Air, by "detached" Thermometer.
Higher,...............	h	t	t'
Lower,................	H	T	T'.

Then, height of the higher station above the lower, for feet and Fahrenheit's scale,

$$= 60360 \left\{ \log. \mathrm{H} - \log. h - \cdot000044\,(\mathrm{T} - t) \right\} \cdot \left(1 + \frac{\mathrm{T}' + t' - 64}{986}\right);$$

and for metres and the Centigrade scale,

$$= 18400 \left\{ \log. \mathrm{H} - \log. h - \cdot00008(\mathrm{T} - t) \right\} \cdot \left(1 + \frac{\mathrm{T}' + t'}{548}\right).$$

Common logarithms are used in both formulæ. (See page 303.)

In the absence of logarithms, for heights not exceeding about 3,000 feet, or 1,000 metres, correct the mercurial column at the higher station as follows:—

$$h' = h\left(1 + \frac{\mathrm{T} - t\ (\mathrm{Fahr.})}{10000}\right) = h \cdot \left(1 + \frac{\mathrm{T} - t\ (\mathrm{Cent.})}{5550}\right);\ \text{then}$$

difference of level for feet and Fahrenheit's scale,

$$= 52428\,\frac{\mathrm{H} - h'}{\mathrm{H} + h'} \cdot \left(1 + \frac{\mathrm{T}' + t' - 64}{986}\right);$$

and for metres and the Centigrade scale,

$$= 15980\,\frac{\mathrm{H} - h'}{\mathrm{H} + h'} \cdot \left(1 + \frac{\mathrm{T}' + t'}{548}\right).$$

4. **Levelling by the Boiling-point of Pure Water.**—Let boiling-point = T. Calculate z as follows: for feet and Fahrenheit's scale,

$$z = 517\ (212^\circ - T) + (212^\circ - T)^2;$$

or for metres and the centigrade scale,

$$z = 284\ (100^\circ - T) + (100^\circ - T)^2;$$

the difference of the values of z at two stations will be their difference of level, nearly.

5. **Reduction of Soundings.**—Take the difference between each sounding and the height of the surface of the water above the datum of the survey at the instant when the sounding was made, as found by a tide register. According as the sounding is the $\left\{\begin{matrix}\text{greater}\\ \text{less}\end{matrix}\right\}$, that difference is the $\left\{\begin{matrix}\text{depth}\\ \text{height}\end{matrix}\right\}$ of the bottom $\left\{\begin{matrix}\text{below}\\ \text{above}\end{matrix}\right\}$ the datum.

In the absence of direct observations of the tide, the height of the surface of the water above the datum may be calculated approximately as follows:—Divide the time before or after high water at which the sounding was taken by the whole duration of the rise or fall of the tide, and multiply the quotient by 180°; this gives the *tidal angle*. Multiply the cosine of the tidal angle by half the total rise of the tide; the product is to be $\left\{\begin{matrix}\text{added to}\\ \text{subtracted from}\end{matrix}\right\}$ the height of the mean tide-level above the datum, according as the tidal angle is $\left\{\begin{matrix}\text{acute}\\ \text{obtuse}\end{matrix}\right\}$. (See page 53, line 2.)

Duration of the rise or fall of tide on an open coast, about 6h. 12m. In narrow channels the duration of the rise is less, and that of the fall greater.

Section V.—Rules relating to Setting Out.

1. **Setting Out Centre Lines of Railway Curves.**

Rule I. (see fig. 45).—To find the radius of a circular arc which shall touch successively three given straight lines, BD, DE, EC. Measure the middle straight line DE, and the *acute* angles at D and E. Then

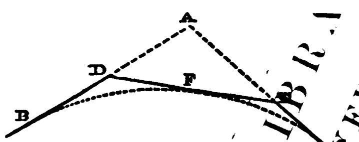


Fig. 45.

$$\text{Radius} = DE \div \left(\tan\frac{D}{2} + \tan\frac{E}{2}\right).$$

RULE II.—To find the points of contact, B, F, C.

$$D\,B = D\,F = \text{radius} \times \tan\frac{D}{2};\quad E\,F = E\,C = \text{radius} \times \tan\frac{E}{2}.$$

RULE III.—To calculate the lengths of the arcs B F and F C.

$$B\,F = \text{radius} \times \text{circular measure of } D.$$
$$F\,C = \text{radius} \times \text{circular measure of } E.$$

(Circular measure = angle in minutes × 0·0002909
= angle in degrees × 0·017453;
see also pages 39 and 41.)

RULE IV.—To calculate the angle subtended at any station in the circumference of a circle by an arc of that circle of a given length; divide the length of the arc by the radius, and multiply the quotient by 1718·873; the product will be the angle at the circumference in minutes: or, otherwise, convert the quotient into minutes of angle at the centre, by Table 4 K, page 39, and divide by 2 for the angle at the circumference.

If the station is at one end of the arc, the angle in question is that between the tangent and the chord of the arc.

RULE V.—To calculate approximately the chord of an arc of a given length in a circle of a given radius; from the length of the arc subtract the cube of that length, divided by 24 times the square of the radius.

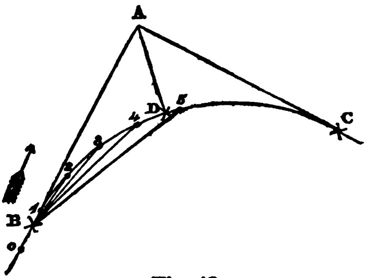


Fig. 46.

RULE VI.—To set out a circular curve of a given radius touching two given straight lines in given points, B, C, fig. 46.

It is convenient (though not always necessary) to find the middle point of the curve. For that purpose, range, by means of the theodolite, the line A D bisecting the angle at A, where the tangents intersect; and lay off the distance,—

$$A\,D = r \cdot \left(\operatorname{cosec}\frac{A}{2} - 1\right);$$

then will D be the middle point of the curve.

The points B and C (and also D, if marked) should be marked by stakes, distinguished in some way from the ordinary stakes, which are driven all along the centre line of the proposed railway at equal distances of one chain, or 100 feet, or some other uniform distance.

Any one of the points B, C, or D will answer as a station for the theodolite in ranging the curve. When the length of the curve exceeds about half a mile, the middle point, D, is the best station as regards accuracy and convenience.

The following is the process of ranging the curve with the theodolite planted at its commencement, B:—

For brevity's sake, the distance between the stakes which mark the centre line of the proposed railway will be called "a chain," whether it is 66 feet, 100 feet, or a greater distance.

Let *o*, in fig. 46, represent the last stake in the portion of the straight line immediately preceding the curve; the distance B 1 from the commencement of the curve to the first stake in it will be the difference between one chain and *o* B. The angle at the circumference subtended by the arc B 1 having been calculated by Rule IV., is to be laid off by the theodolite from the tangent B A, the zero-point of azimuth being directed towards A. The line of collimation will then point in the proper direction for the first stake in the curve, 1; and its proper distance from B being laid off by means of the chain, its position will be determined at once.

The angles at the circumference subtended by B 1 + 1 chain, B 1 + 2 chains, B 1 + 3 chains, &c., being also calculated and laid off from the tangent B A in succession, will respectively give the proper directions for the ensuing stakes, 2, 3, 4, &c., which are at the same time to be placed successively at uniform distances of one chain by means of the chain.

The difference between an arc of one chain and its chord, on any curve which usually occurs on railways, is in general too small to cause any perceptible error in practice, even in a very long distance; but should curves occur of unusually short radii, calculate the proper chord by Rule V., and set it off from each stake to the next, instead of one chain, the length of the arc.

When the curve is ranged with the theodolite at D, or at any other intermediate point in the curve, or at its termination, C, the process is precisely the same, except that the zero-point of azimuth is to be turned towards B instead of A; and that when the chain passes the theodolite station (for example, in going from stake 4 to stake 5 in fig. 49, with the theodolite at D), the telescope is to be turned completely over.

When the inequalities of the ground make it impossible to range the entire curve from the stations B, D, and C, any stake which has already been placed in a commanding position will answer as a station for the theodolite.

The stakes or poles, after having been ranged by the theodolite, should have their positions finally checked and adjusted by the method of offsets, for which see page 137.

RULE VII. (see fig. 47).—To set out a circular curve of a given radius, r, touching two given straight lines, A B, A C, when the point of intersection of those lines, A, is inaccessible.

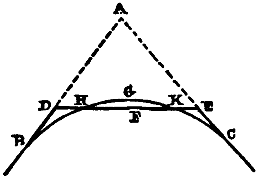


Fig. 47.

Chain a straight line, D E, upon accessible ground, so as to connect the two tangents. The position of the *transversal* D E is arbitrary; but it is convenient so to place it that it will cut the proposed curve in two points, which may be determined, and used as theodolite stations.

Measure the angles A D E, A E D, which may be denoted by D and E. Then the angle at A is

$$A = 180° - D - E;$$

$$AD = DE \cdot \frac{\sin E}{\sin A}; \; AE = DE \cdot \frac{\sin D}{\sin A};$$

$$DB = r \cdot \text{cotan} \frac{A}{2} - AD; \; EC = r \cdot \text{cotan} \frac{A}{2} - AE;$$

and by laying off the distances D B and E C as thus calculated, the ends of the curve B and C are marked, and it can be ranged from either of those stations as in Rule VI.

But it is often convenient to have intermediate points in the curve for theodolite stations; and of those the points of intersection with the transversal H and K, and the point G, midway between these, can be found by the following calculations, in making which a table of squares is useful (page 11):—

Let F be the point on the transversal, midway between H and K.

If $BD = CE$, the point F is at the middle of D E. If B D and C E are unequal, let B D be the greater; then the position of F is given by either of the two following formulæ:—

$$DF = \frac{DE}{2} + \frac{BD^2 - CE^2}{2\,DE}; \; EF = \frac{DE}{2} - \frac{BD^2 - CE^2}{2\,DE}.$$

The points H and K are at equal distances on each side of F, given by the following formula:—

$$FH = FK = \sqrt{(DF^2 - BD^2)} = \sqrt{(EF^2 - CE^2)}.$$

The point G in the curve is found by setting off the ordinate F G perpendicular to D E, of the following length:—

$$FG = r - \sqrt{r^2 - FH^2}.$$

The angles subtended at the *centre* of the curve by the several

arcs between the commencement B and the points H, G, K, C, are as follows:—

$$\left.\begin{array}{ll}\text{Angle subtended at the centre by } BH = D - \text{arc}\cdot\sin\frac{FH}{r}; \\ \text{— — — —} \quad BG = D; \\ \text{— — — —} \quad BK = D + \text{arc}\cdot\sin\frac{FH}{r}; \\ \text{— — — —} \quad BC = D + E;\end{array}\right\}$$

and the length of any one of those arcs may be computed by means of Rule III.

Rule VIII.—To set out a circular curve touching two given straight lines, when part of the curve is inaccessible to the chain.

If the point of intersection of the tangents is accessible, the two ends of the curve are to be determined and marked as in Rule I., and also the middle point of the curve, unless it lies on the inaccessible ground; and the length of the curve is to be computed by Rule III.

If the point of intersection of the tangents is inaccessible, the two ends of the curve, and at least one intermediate point, are to be determined and marked by the aid of a transversal, as in Rule VII., and the lengths of the arcs bounded by those points are to be computed.

A transversal may be useful even when the point of intersection of the tangents is accessible.

Each of the points thus marked will serve either as a theodolite station, or as a station to chain from, or for both purposes; and the stakes lying between the obstacle and the next station beyond it are to be planted by chaining backwards from that station.

Rule IX.—To set out a circular curve by offsets commencing at a given point on a straight line (fig. 48).

Let A be the commencement of the curve; A B the prolongation of the straight line (being a tangent to the curve); and B the end of the chain when laid along that prolongation from the last stake in the straight line. Plant a small pole at B, calculate the offset B C by the formula $BC = \frac{AC^2}{2 \text{ radius}}$; shift the end of the chain, and the pole along with it, sideways from B to C, keeping the chain tight, and leave the pole at C.

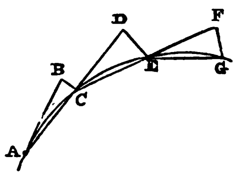


Fig. 48.

Drag the chain onward in the prolongation of A C; range a pole at D in a straight line with A and C, and at one chain's dis-

tance from C; shift the pole and the end of the chain through the offset D E, calculated by the formula, $DE = \frac{CE \cdot AD}{2 \text{ radius}}$.

Drag the chain onward; range a pole at F in a straight line with C and E, and at one chain's distance from E; shift the pole and the end of the chain through the offset F G, calculated by the formula $FG = \frac{CE^2}{\text{radius}}$; leave the pole at G, and repeat the same process for the rest of the curve.

This method is clumsy and tedious as a means of ranging curves; but it is very useful for testing the uniformity of curvature of curves already ranged, and for rectifying the positions of individual stakes to the extent of an inch or two.

RULE X.—To set out a circular curve by successive bisections of arcs.

This is a method to be used only in the absence of angular instruments. It depends on the following relation between the versed sine of an angle B and that of its half:

$$\text{versin}\,\frac{B}{2} = 1 - \sqrt{1 - \frac{\text{versin}\,B}{2}}.$$

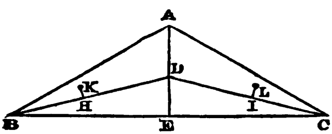


Fig. 49.

To apply this principle, let B A, C A, in fig 49, be the two tangents, and B and C the ends of the curve, so placed that A B and A C shall be equal, but leaving the radius to be found by calculation. Measure the chord B C.

To find the radius, bisect B C in E, measure A E, and make

$$\text{radius} = \frac{AB \cdot BE}{AE}.$$

Calculate the versed sine of the angle A B E = B, which is that subtended at the centre by one-half of the curve, as follows:—

$$\text{versin}\,B = \frac{AB - BE}{AB};$$

and by means of the first formula of the rule (using a table of squares, if one is at hand) calculate the versed sines of $\frac{B}{2}, \frac{B}{4}, \frac{B}{8}$, &c., in succession, observing that versin B enables one intermediate point in the curve to be found, versin $\frac{B}{2}$, three points, versin $\frac{B}{4}$,

seven points; and generally, that versin $\frac{B}{2^n}$ enables $2^{n+1} - 1$ intermediate points in the curve to be found.

From the middle, E, of the chord B C, and perpendicular to it, lay off the offset E D = r versin B; D will be the middle point of the curve.

Chain and bisect the chords B D, D C, and from their middle points, and perpendicular to them, lay off the offsets

$$\text{H K} = \text{I L} = r \text{ versin } \frac{B}{2};$$

K and L will be points in the curve, midway respectively between B and D, and between D and C; and so on until a sufficient number of points have been marked by poles.

Then chain round the curve as ranged by the poles, and drive stakes at equal distances apart.

The uniformity of the curvature may be finally checked by Rule IX.

2. **Cant of Rails of a Curve.**—Divide the square of the greatest ordinary speed of a train by the radius of the curve, and by a divisor whose values are as follows:—

For speed in feet per second and radius in feet,	32;
For speed in miles per hour and radius in feet,	15;
For speed in metres per second and radius in metres,	9·8.

Multiply the quotient by the gauge of the rails; the product will be the cant required, in the same sort of measure with the gauge.

	Ft.	In.		Metres.
British narrow gauge,	4	8½	=	1·435
British broad gauge,	7	0	=	2·134
Irish gauge, . .	5	3	=	1·600

Half of the cant should be given by raising the outer rail above the level of the centre line, and half by depressing the inner rail.

Examples of cant in feet for 40 miles an hour:—

Gauge. Ft.	In.		
4	8½	...	500 ÷ radius in feet.
5	3	...	560 ÷ radius in feet.
7	0	...	747 ÷ radius in feet.

Additional cant for cylindrical wheels at speeds not exceeding 12 miles an hour, 600 feet ÷ radius in feet.

3. **To Ease Changes of Curvature** (*Froude's Method*).

Begin by ranging the centre line as a series of straight lines and circular arcs, by the rules of Article 1 of this Section. Calculate the cant of each curve by the rule of Article 2.

RULE I.—Compute the several *changes of cant* at the junctions of curves with straight lines and with each other, observing that the change of cant between a straight line and a curve is simply the cant of the curve; that if two adjacent curves are curved in the same direction, the change is the difference of cant; and that if they are curved in reverse directions, the change is the sum of the two cants.

Multiply the *greatest change of cant* by 300; the product will be the length of the *curve of adjustment.*

RULE II.—Compute, for each circular arc of the series, the *shift* as follows:—

$$\text{Shift} = (\text{length of curve of adjustment})^2 \div 24 \text{ radius.}$$

Then shift the poles by which a given circular arc is marked inwards (that is, towards the centre of curvature of the arc) through the distance computed by the above formula. For example, in fig. 50, let A B, B C be a pair of consecutive circular arcs, marked

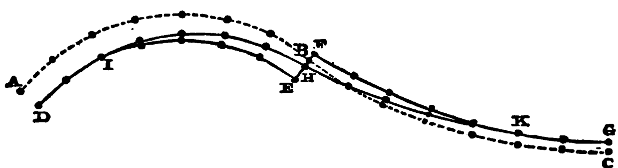


Fig. 50.

by poles, and joining each other at their point of contact, B. Let B E, B F be the *shifts* proper to those two arcs respectively; after all the poles have been shifted, they will mark the arcs D E, F G, having a gap between them at E F, equal to the sum of the two shifts, if the arcs are curved in reverse directions, or the difference of the shifts, if the arcs are curved in the same direction. Straight lines are not to be shifted; so that where a curve joins a straight line, the gap is simply the shift of the curve.

RULE III.—Set out the "*curve of adjustment*" I H K as follows:— For its middle point bisect the gap E F in H. For its ends I and K lay off E I and F K, each equal to half its length, as computed by Rule I. For intermediate points in the division I H lay off ordinates at right angles from a series of points in the circular arc I E, proportional to the cubes of the distances from I; and for intermediate points in the division K H lay off ordinates at right angles from a series of points in the circular arc K F, proportional to the cubes of the distances from K.

Let a denote the length I K of the curve of adjustment;

b, the gap E F, or sum of the shifts;

x, the distance, measured on the circular arc, of any point from I or from K, as the case may be;
the ordinate; then

$$y = \frac{4\,b\,x^3}{a^3}.$$

EXAMPLE.—A curve of 20 chains radius (= 1,320 feet), with cant suited to a speed of 40 miles an hour on a narrow gauge line, is to be connected with a straight line.

Cant (see p. 139) = 500 feet ÷ 1,320 = ·3788 foot;
Length of curve of adjustment, a = ·3788 × 300 = 113·6 feet;
Shift for circular arc = $(113{\cdot}6)^2 \div 24 \times 1{,}320$ = ·407 foot;
(As the arc is to join a straight line, this is also = the gap b.)

$$\text{Formula for ordinates, } y = \frac{4 \times {\cdot}407\,x^3}{(113{\cdot}6)^3} = {\cdot}000{,}001{,}11\,x^3.$$

RULE IV.—To connect a circular arc and a straight line, or two circular arcs, which do not touch or cut each other, by means of a curve of adjustment. Fig. 50 illustrates the case where two arcs curved in reverse directions are to be connected; fig. 51, that in which two arcs curved in the same direction are to be connected.

Find the pair of points at which the arcs or lines to be connected are nearest to each other. This is best done by first finding two pairs of points at which the lines to be connected are at equal distances apart; the pair of points required will be midway between those two pairs of points. Let E and F be the pair of points thus found; measure the *gap* E F, then calculate the *half-length of the curve of adjustment* by means of the following formula, in which r and r' denote the radii of the arcs to be connected:—

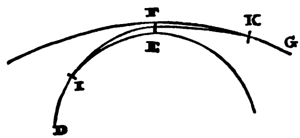


Fig. 51.

$$\mathrm{E\,I} = \mathrm{F\,K} = \sqrt{\left\{6\,\mathrm{E\,F} \div \left(\frac{1}{r} \pm \frac{1}{r'}\right)\right\}};$$

the sign + or − being used in the denominator, according as the directions of curvature are reverse or similar. If one of the lines to be connected is straight, $1 \div r'$ is to be made = 0; so that the formula becomes

$$\mathrm{E\,I} = \mathrm{F\,K} = \sqrt{6\,\mathrm{E\,F} \cdot r}.$$

The curve of adjustment is now to be set out by ordinates, as in Rule III.

4. **Breadth of Formation of a Railway.**—The following are examples:—

SINGLE LINE.	Narrow Gauge. Ft. In.	Irish Gauge. Ft. In.	Broad Gauge. Ft. In.
Clear space outside of rail,	4 0	4 0	4 0
Head of rail,	0 2½	0 2¼	0 2½
Gauge,	4 8½	5 3	7 0
Head of rail,	0 2½	0 2½	0 2½
Clear space outside of rail,	4 0	4 0	4 0
Least breadth of top of ballast; and least width admissible for archways, &c., traversed by the railway,	13 1½	13 8	15 5
Spaces for slopes of ballast, and benches beyond them, on embankments, from / to	3 10½ / 8 10½	4 4	9 2
Total breadth of top of embankments, from / to	17 0 / 22 0	18 0	24 7

DOUBLE LINE.	Narrow Gauge. Ft. In.	Irish Gauge. Ft. In.	Broad Gauge. Ft. In.
Clear space outside of rail,	4 0	4 0	4 0
Head of rail,	0 2½	0 2½	0 2½
Gauge,	4 8½	5 3	7 0
Head of rail,	0 2½	0 2½	0 2½
Middle space (called the "*six feet*,")	6 0	6 0	6 0
Head of rail,	0 2½	0 2½	0 2½
Gauge,	4 8½	5 3	7 0
Head of rail,	0 2½	0 2½	0 2½
Clear space outside of rail,	4 0	4 0	4 0
Least breadth of top of ballast; and least width admissible for archways, &c., traversed by the railway,	24 3	25 4	28 10
Spaces for slopes of ballast and trenches beyond them, on embankments, from / to	3 9 / 8 9	4 8	9 2
Total breadth of top of embankments, from / to	28 0 / 33 0	30 0	38 0

Additional width at bottoms of cuttings, from 0 to 9 feet.

Arches over the railway are seldom made of the minimum spans shown by the foregoing tables, except in the case of tunnels. Bridges over narrow gauge lines are usually of the following spans:

over a single line, from 16 to 18 feet;
over a double line, from 28 to 30 feet.

5. **Breadths of Slopes of Earthwork.**—Let h denote the central depth of the piece of earthwork, whether cutting or embankment;

b, the half-breadth of its base, or formation;

s, the rate of slope of the earthwork; that is, s horizontal to 1 vertical;

r, the rate of sidelong slope of the natural ground, if any; that is, r horizontal to 1 vertical;

B, the required breadth of the slope of the earthwork.

CASE I.—In ground level across, $B = s\,h$.

CASE II.—In ground that slopes away from the base,

$$B = \frac{r\,s}{r - s} \cdot \left(h + \frac{b}{r}\right).$$

CASE III.—In ground that slopes towards the base, but without intersecting it;

$$B = \frac{r\,s}{r + s} \cdot \left(h - \frac{b}{r}\right).$$

CASE IV.—In ground that intersects the base between the centre line and the edge of the earthwork,

$$B = \frac{r\,s}{r - s} \cdot \left(\frac{b}{r} - h\right).$$

SECTION VI.—RULES RELATING TO MENSURATION OF EARTH WORK.

1. **Sectional Areas of Earthwork.**—Figs. 52, 53, and 54 repre-

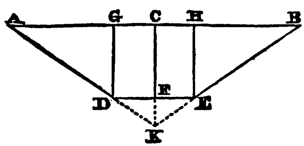


Fig. 52.

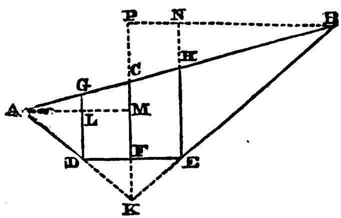
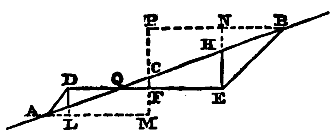


Fig. 53.

sent examples of *cross-sections* of pieces of earthwork, in each of which D E is the base, A B the natural surface, and D A and E B are the slopes.



Figs. 52 and 53 represent cuttings; to represent embankments, conceive them to be turned upside down.

Fig. 54 represents a piece of earthwork, of which one side, Q E B, is in side cutting, and the other, Q D A, in embankment.

The following are the symbols used in the rules:—

Natural slope of the ground, r (horizontal) to 1 (vertical).
Slope of the earthwork, s (horizontal) to 1 (vertical).
Half-breadth of base, $\mathrm{D\,F} = \mathrm{F\,E} = b$.
Central depth, $\mathrm{C\,F} =$ .... h.
Area of cross-section, ... A.

In many measurements of earthwork having sections such as figs. 52 and 53, it is convenient to suppose the slopes produced till they meet at K, and to calculate or measure the following quantity:—

$$\text{Augmented depth, C K} = h + \frac{b}{s} = k.$$

To find k by direct measurement in a longitudinal section of earthwork, draw a line parallel to the formation line of the work, and at the vertical distance $\frac{b}{s}$ below it in cuttings, or above it in embankments. Depths measured from that line to the surface of the ground will be augmented depths.

Rule I.—When the ground is level across;

$$\mathrm{A} = \text{triangle A B K} - \text{triangle D E K} = s\,k^2 - \frac{b^2}{s}$$

Or otherwise,—

Rule Ia.

$$\mathrm{A} = \text{rectangle D G H E} + 2 \text{ triangle A D G} = 2\,b\,h + s\,h^2.$$

Rule II.—When the ground has an uniform sidelong slope, not intersecting the base, as in fig. 53,

$$\mathrm{A} = \text{triangle A B K} - \text{triangle D E K} = \frac{r^2\,s}{r^2 - s^2} \cdot k^2 - \frac{b^2}{s}.$$

Rule III.—To find the augmented depth in ground level across, of a cross-section of earthwork equal to a given cross-section in sidelong sloping ground; take a mean proportional between the augmented depths measured from K vertically to the two edges A and B respectively; that is to say, in fig. 53, parallel to D E, draw A M and B P, cutting the vertical centre line in M and P; then make

$$k' = \sqrt{(\mathrm{K\,M} \cdot \mathrm{K\,P})};$$

and the area may be found by Rule I., as follows:—

$$\mathrm{A} = s\,k'^2 - \frac{b^2}{s} = s \cdot \mathrm{K\,M} \cdot \mathrm{K\,P} - \frac{b^2}{s}.$$

Rule IV.—When the ground has a sidelong slope intersecting the base at Q, in fig. 54. Let A′ be the larger and A″ the smaller division of the cross-section.

$$\mathrm{A}' = \text{triangle Q E B} = \frac{(b + r\,h)^2}{2\,(r - s)};$$

$$A'' = \text{triangle } Q\ D\ A = \frac{(b - r\,h)^2}{2\,(r - s)}.$$

2. **Volumes or Quantities of Earthwork.**—Rule I.—When a series of equidistant cross-sections are given, see p. 72, Article 5; also the rules there referred to, A, B, C, pages 64 to 66.

Rule II.—When the piece of earthwork to be measured is a "prismoid," as shown in page 74, fig. 12, use the rule given in that page below the figure.

The most simple algebraical expression of that rule, as applied to the present case, is as follows:—The prismoidal piece of earth to be measured is to be considered as formed by a wedge of a cross-section such as A B K in fig. 52 or fig. 53, from which is taken away a wedge of uniform cross-section such as D E K.

Let x denote the length of the piece of earth; k_1 and k_2, the values of the *augmented depth* C K at its two ends; then,

$$\text{Volume} = x \cdot \left\{ \frac{r^2\,s}{6(r^2 - s^2)} \cdot \left(k_1^2 + (k_1 + k_2)^2 + k_2^2\right) - \frac{b^2}{s} \right\}$$

$$= x \cdot \left\{ \frac{r^2\,s}{r^2 - s^2} \cdot \left(\frac{(k_1 + k_2)^2}{4} + \frac{(k_1 - k_2)^2}{12}\right) - \frac{b^2}{s}. \right\}$$

The last formula is specially suited for calculation by the aid of a table of squares.

When the ground is level across, the co-efficient of the first term becomes simply $= s$.

The quantity in brackets by which the length x is multiplied is the *mean sectional area.*

If the measurements are in feet, the preceding rules give quantities in cubic feet. To reduce these to cubic yards divide by $3^3 = 27$.

Rule III.—When earthwork on sidelong ground occurs on a sharp curve. By the rules of pages 142, 143, calculate the half-breadths (A L, B N, fig. 53) required for the two slopes; take their difference, and divide it by three times the radius of the curve; the quotient is to be added to or subtracted from 1, according as the greater half-breadth lies from or towards the centre of the curve. The result will be a factor by which the area A B K in fig. 53—that is, the first of the two terms of the formula in Rule II., page 144—is to be multiplied. From the product subtract the area D E K; the remainder will be an area modified for curvature; then proceed as in Rule I. of this Article.

PART IV.

RULES AND TABLES RELATING TO DISTRIBUTED FORCES AND MECHANICAL CENTRES.

1. **Specific Gravity** (as stated at page 102) is the ratio of the weight of a given bulk of a given substance to the weight of the same bulk of pure water at a standard temperature. In Britain the standard temperature is 62° Fahr. = 16°·67 Cent. In France it is the temperature of the maximum density of water = 3°·94 Cent. = 39°·1 Fahr.

In rising from 39°·1 Fahr. to 62° Fahr., pure water expands in the ratio of 1·001118 to 1; but that difference is of no consequence in calculations of specific gravity for engineering purposes.

Rule I.—To find the specific gravity of a solid body that is heavier than water approximately, by experiment. Weigh it in air, and again weigh it immersed in pure water. Divide the weight in air by the loss of weight when immersed (or *buoyancy*); the quotient will be the specific gravity.

Rule II.—When the body is lighter than water, weigh it in air; then load it with a piece of a substance heavier than water, and large enough to make the light body sink, and weigh them in water together. Also weigh the heavy body separately, in air and in water. Subtract the buoyancy of the heavy body from the buoyancy of the two bodies together; the remainder will be the buoyancy of the light body separately; by which its weight in air is to be divided as before.

Rule III.—To find approximately the specific gravity of a liquid; weigh some convenient solid body in air, in pure water, and in the given liquid; divide the buoyancy or loss of weight in the given liquid by the buoyancy in water; the quotient will be the required specific gravity.

Rule IV.—To find approximately the specific gravity of a solid body that is soluble in water; ascertain its buoyancy in some liquid which does not dissolve it, and whose specific gravity is known; divide the weight in air by the buoyancy in that liquid, and multiply the quotient by the specific gravity of the liquid.

The approximate character of all those rules arises from their not taking account of the buoyancy due to the pressure of the air, whether on the body weighed or on the weights; but for ordinary practical purposes the error so occasioned is immaterial.

2. The **Heaviness** of any substance (as stated at page 102) is the weight of an unit of volume of it in units of weight.

In British measures heaviness is most conveniently expressed in *lbs. avoirdupois to the cubic foot;* in French measures, in *kilogrammes to the cubic decimetre.*

RULE V.—Given, the specific gravity of a substance; to find its heaviness; multiply by the heaviness of water.

(In British measures 62·4 lbs. to the cubic foot is near enough for practical purposes; in French measures no calculation is needed, heaviness and specific gravity being identical.)

3. The **Density** of a substance is either the number of units of *mass* in an unit of volume (see page 104), in which case it is equal to the heaviness,—or the ratio of the mass of a given volume of the substance to the mass of an equal volume of water, in which case it is equal to the specific gravity.

In its application to *gases* the term "Density" is often used to denote the ratio of the heaviness of a given gas to that of air, at the same temperature and pressure.

4. The **Bulkiness** of a substance is the number of units of volume which an unit of weight fills; and is the *reciprocal of the heaviness.* (See Table of Reciprocals, page 11.)

In British measures bulkiness is most conveniently expressed in *cubic feet to the lb. avoirdupois;* in French measures, in *cubic decimetres to the kilogramme.*

RULE VI.—Given, the specific gravity of a substance; to find its bulkiness; divide the bulkiness of pure water by the specific gravity of the given substance.

(In British measures 0·01602 cubic foot of pure water to the lb. is near enough for practical purposes; in French measures the bulkiness of pure water is 1.)

5. **Effect of Heat on Bulkiness.**—Rise of temperature produces (with certain exceptions) increase of bulkiness.

RULE VII. (For perfect gases).—Given, the bulkiness of a perfect gas at the temperature of melting ice; to find its bulkiness under the same pressure at any other temperature; multiply by the given temperature, as reckoned from the *absolute zero* (see page 105), and divide by the absolute temperature of melting ice (274° Cent. = 493°·2 Fahr.)

RULE VIII. (Approximate rule for water).—Divide the given temperature by 500° Fahr. or 278° Cent.; divide 500° Fahr. or 278° Cent. by the given *absolute* temperature; multiply the half-sum of the quotients by the least bulkiness of water (0·01602 cubic feet to the lb., or 1 cubic decimetre to the kilogramme); the product will be the required bulkiness nearly enough for practical purposes.

EXAMPLE.—Given, temperature on common scale, 212° Fahr.; that is, 212° + 461°·2 = 673°2 Fahr., absolute.

$\frac{1}{2}\left(\frac{673 \cdot 2}{500} + \frac{500}{673 \cdot 2}\right) = 1 \cdot 045$, ratio in which the bulkiness is increased (the exact ratio is 1·04775, so that the error is about $\left.\frac{1}{400}\right)$;

$$0 \cdot 01602 \times 1 \cdot 045 = 0 \cdot 01675 \text{ cubic foot to the lb.; bulkiness required, nearly}$$

$\frac{62 \cdot 425}{1 \cdot 045} = 59 \cdot 7$ lbs. to the cubic foot; corresponding heaviness, nearly.

The following are the rates of expansion in bulk, in rising from the freezing point (0° Cent. or 32° Fahr.) to the boiling point (100° Cent. or 212° Fahr.) of some materials:—

Perfect gases,	0·365
Air at ordinary pressures,	0·366
Pure water,	0·04775
Sea-water, ordinary,	0·05
Spirit of wine,	0·1112
Mercury,	0·018153
Oil, linseed and olive,	0·08
Brass,	0·0065
Bronze,	0·0054
Copper,	0·0055
Cast iron,	0·0033
Wrought iron and steel,	0·0036
Lead,	0·0057
Tin,	0·0066
Zinc,	0·0058
Brick, common,	0·0106
„ fire,	0·0015
Cement,	0·0042
Glass (average),	0·0027
Slate,	0·0031

6. **Effect of Pressure on Bulkiness of Perfect Gases.**—Given, the bulkiness of a perfect gas at a given temperature and under the absolute pressure of one atmosphere; to find the bulkiness at the same temperature under any other pressure; divide by the absolute pressure in atmospheres (see page 115).

7. **Explanation of the Tables.**—Table I. is a general table of heaviness in lbs. to the cubic foot for gases, liquids, and solids, and of specific gravity for liquids and solids. Table II. gives the heaviness of earth in lbs. to the cubic foot and to the cubic yard.

Table III. gives the heaviness of various kinds of rock in lbs. to the cubic foot, and to the cubic yard; and the bulkiness in cubic feet to the ton. Table IV. gives, for various metals, the weights of a cubic inch (column A); of a bar a foot long and an inch square (column C); of a round rod a foot long and an inch diameter (column B); of a plate a foot square and an inch thick (column D); of a cubic foot (column E); and of a sphere one inch in diameter (column F). To find the weight of one foot of a round rod of a diameter given in inches; multiply the number in column B by the square of the diameter. For the weight of a foot of a cylindrical tube, multiply the number in column B by the difference of the squares of the outside and inside diameters. For the weight of a solid sphere, multiply the number in column F by the cube of the diameter. For the weight of a hollow sphere, multiply the same number by the difference of the cubes of the outside and inside diameters.

I.—General Table of Heaviness and Specific Gravity.

Gases, at 32° Fahr., and under one atmosphere:	Weight of a cubic foot in lb. avoirdupois.
Air,	0·080728
Carbonic acid,	0·12344
Hydrogen,	0·005592
Oxygen,	0·089256
Nitrogen,	0·078596
Steam (ideal),	0·05022
Æther vapour (ideal),	0·2093
Bisulphuret-of-carbon vapour (ideal),	0·2137
Olefiant gas,	0·0795

Liquids at 32° Fahr. (except Water, which is taken at 39°·1 Fahr.):	Weight of a cubic foot in lbs. avoirdupois.	Specific gravity, pure water = 1.
Water, pure, at 39°·1,	62·425	1·000
„ sea, ordinary,	64·05	1·026
Alcohol, pure,	49·38	0·791
„ proof spirit,	57·18	0·916
Æther,	44·70	0·716
Mercury,	848·75	13·596
Naphtha,	52·94	0·848
Oil, linseed,	58·68	0·940
„ olive,	57·12	0·915
„ whale,	57·62	0·923
„ of turpentine,	54·31	0·870
Petroleum,	54·81	0·878

	Weight of a cubic foot in lbs. avoirdupois.	Specific gravity, pure water = 1.
Solid Mineral Substances, non-metallic:		
Basalt,	187·3	3·00
Brick,	125 to 135	2 to 2·167
Brickwork,	112	1·8
Chalk,	117 to 174	1·87 to 2·78
Clay,	120	1·92
Coal, anthracite,	100	1·602
„ bituminous,	77·4 to 89·9	1·24 to 1·44
Coke,	62·43 to 103·6	1·00 to 1·66
Felspar,	162·3	2·6
Flint,	164·2	2·63
Glass, crown, average,	156	2·5
„ flint, „	187	3·0
„ green, „	169	2·7
„ plate, „	169	2·7
Granite,	164 to 172	2·63 to 2·76
Gypsum,	143·6	2·3
Limestone (including marble),	169 to 175	2·7 to 2·8
„ magnesian,	178	2·86
Marl,	100 to 119	1·6 to 1·9
Masonry,	116 to 144	1·85 to 2·3
Mortar,	109	1·75
Mud,	102	1·63
Quartz,	165	2·65
Sand (damp),	118	1·9
„ (dry),	88·6	1·42
Sandstone, average,	144	2·3
„ various kinds,	130 to 157	2·08 to 2·52
Shale,	162	2·6
Slate,	175 to 181	2·8 to 2·9
Trap,	170	2·72
Metals, solid:		
Brass, cast,	487 to 524·4	7·8 to 8·4
„ wire,	533	8·54
Bronze,	524	8·4
Copper, cast,	537	8·6
„ sheet,	549	8·8
„ hammered,	556	8·9
Gold,	1186 to 1224	19 to 19·6
Iron, cast, various,	434 to 456	6·95 to 7·3
„ average,	444	7·11
Iron, wrought, various,	474 to 487	7·6 to 7·8

	Weight of a cubic foot in lbs. avoirdupois.	Specific gravity, pure water = 1.
Metals, solid,—*continued.*		
Iron, wrought, average,.........	480	7·69
Lead,............................	712	11·4
Platinum,........................	1311 to 1373	21 to 22
Silver, ...	655	10·5
Steel,...........................	487 to 493	7·8 to 7·9
Tin,.............................	456 to 468	7·3 to 7·5
Zinc,............................	424 to 449	6·8 to 7·2
Timber: *		
Ash,.............................	47	0·753
Bamboo,..........................	25	0·4
Beech,...........................	43	0·69
Birch,...........................	44·4	0·711
Blue-Gum,........................	52·5	0·843
Box,.............................	60	0·96
Bullet-tree,.....................	65·3	1·046
Cabacalli,.......................	56·2	0·9
Cedar of Lebanon,................	30·4	0·486
Chestnut,........................	33·4	0·535
Cowrie,..........................	36·2	0·579
Ebony, West Indian,..............	74·5	1·193
Elm,.............................	34	0·544
Fir: Red Pine,...................	30 to 44	0·48 to 0·7
,, Spruce,.......................	30 to 44	0·48 to 0·7
,, American Yellow Pine,..	29	0·46
,, Larch,........................	31 to 35	0·5 to 0·56
Greenheart,......................	62·5	1·001
Hawthorn,........................	57	0·91
Hazel,...........................	54	0·86
Holly,...........................	47	0·76
Hornbeam,........................	47	0·76
Laburnum,........................	57	0·92
Lancewood,.......................	42 to 63	0·675 to 1·01
Larch. See "Fir."		
Lignum-Vitæ,.....................	41 to 83	0·65 to 1·33
Locust,..........................	44	0·71
Mahogany, Honduras,..............	35	0·56
,, Spanish,......................	53	0·85
Maple,...........................	49	0·79
Mora,............................	57	0·92

* The Timber in every case is supposed to be dry.

	Weight of a cubic foot in lbs. avoirdupois.	Specific gravity, pure water = 1.
TIMBER,—*continued.*		
Oak, European,	43 to 62	0·69 to 0·99
„ American, Red,	54	0·87
Poon,	36	0·58
Saul,	60	0·96
Sycamore,	37	0·59
Teak, Indian,	41 to 55	0·66 to 0·88
„ African,	61	0·98
Tonka,	62 to 66	0·99 to 1·06
Water-Gum,	62·5	1·001
Willow,	25	0·4
Yew,	50	0·8

II.—Heaviness of Earth.

		Cubic Foot.			Cubic Yard.	
Chalk,	from	117 to 174	lbs.	from	3160 to 4730	lbs.
Clay,	„	120 to 135	„	„	3240 to 3645	„
Gravel and Shingle,	„	90 to 110	„	„	2430 to 2970	„
Marl,	„	100 to 119	„	„	2700 to 3210	„
Mud,		102	„	„	2750	„
Sand, dry,		89	„	„	2400	„
„ damp,		118	„	„	3190	„
Shale,		162	„	„	4370	„

III.—Heaviness and Bulkiness of Rock.

	Lbs. in one Cubic Foot.	Lbs. in one Cubic Yard.	Cubic Feet to a Ton.
Basalt,	187	5060	12
Chalk,	117 to 174	3160 to 4730	19·1 to 12·9
Felspar,	162	4370	13·8
Flint,	164	4430	13·6
Granite,	164 to 172	4430 to 4640	13·6 to 13
Limestone,	169 to 175	4560 to 4720	13·2 to 12·8
„ magnesian,	178	4810	12·6
Quartz,	165	4450	13·6
Sandstone, average,	144	3890	15·6
„ different kinds,	130 to 157	3510 to 4240	17·2 to 14·3
Shale,	162	4370	13·8
Slate (Clay),	175 to 181	4720 to 4890	12·8 to 12·4
Trap,	170	4590	13·2

IV.—Cubes, Rods, Plates, Bars, and Spheres.

	A.	B.	C.	D.	E.	F.
	Cubic Inch.	Round Rod, 1 ft. long 1 in. diam.	Square Bar, 1 ft. × 1 in. × 1 in.	Plate, 1 ft. × 1 ft. × 1 in.	Cubic foot.	Sphere, 1 inch diam.
	lbs.	lbs.	lbs.	lbs.	lbs.	
Brass, cast, average,...	0·298	2·81	3·58	43·0	516	0·156
„ wire,............	0·308	2·91	3·70	44·4	533	0·162
Bronze,.................	0·303	2·86	3·64	43·7	524	0·159
Copper, sheet,..........	0·318	2·99	3·81	45·75	549	0·166
„ hammered,...	0·322	3·03	3·86	46·3	556	0·168
Iron, cast, average,...	0·257	2·42	3·08	37·0	444	0·134
Iron, wrought, average,	0·278	2·62	3·33	40·0	480	0·146
Lead,....................	0·412	3·88	4·94	59·3	712	0·216
Steel, average,.........	0·283	2·67	3·40	40·8	490	0·148
Tin, average,............	0·267	2·52	3·21	38·5	462	0·140
Zinc, average,..........	0·252	2·38	3·03	36·3	436	0·132

8. **Centre of Gravity—Moment of Weight.**—Rule I.—The centre of gravity of a body of uniform heaviness is its centre of magnitude. (See pages 81 to 88.)

Rule II.—To find the moment of a body's weight relatively to a given *plane of moments;* multiply the weight by the perpendicular distance of the body's centre of gravity from the given plane.

Note.—In comparing together or combining the moments of weights which lie some at one side and some at the other side of a plane of moments, those moments are to be distinguished into positive and negative, according to the sides of the plane at which the weights lie.

Rule III.—To find the common centre of gravity of a set of detached bodies; find their several moments relatively to a convenient fixed plane; find the *resultant* of those moments by adding together, separately, the positive and negative moments, and taking the difference between the two sums, which will be positive or negative according as the positive or negative sum is the greater. Divide that resultant moment by the total weight; the quotient will be the perpendicular distance of the common centre of gravity from the fixed plane; and its positive or negative sign will show at which side of the plane that centre lies. If necessary, repeat the same process for a second and a third fixed plane, so as to determine the position of the required centre completely. The two or three planes (as the case may be) are usually taken perpendicular to each other.

Rule IV.—To find the centre of gravity of a body consisting of parts of unequal heaviness; find separately the centres of those parts, and treat them as detached weights by Rule III.

9. **Moment of Inertia and Radius of Gyration.**—RULE I.—To find the moment of inertia of a body about a given axis; conceive the body divided into an indefinite number of small parts; multiply the mass (or weight) of each part by the square of its perpendicular distance from the axis; the limit towards which the sum of all the products approximates as the parts become smaller and more numerous will be the required moment of inertia.

RULE II.—Given, the moment of inertia of a body about an axis traversing its centre of gravity in a given direction; to find its moment of inertia about another axis parallel to the first; multiply the mass (or weight) of the body by the square of the perpendicular distance between the two axes, and to the product add the given moment of inertia.

RULE III.—Given, the separate moments of inertia of a set of bodies about parallel axes traversing their several centres of gravity; required, the combined moment of inertia of those bodies about a common axis parallel to their separate axes; multiply the mass (or weight) of each body by the square of the perpendicular distance of its centre of gravity from the common axis; add together all the products, and all the separate moments of inertia; the sum will be the combined moment of inertia.

RULE IV.—To find the square of the radius of gyration of a body about a given axis; divide the moment of inertia of the body about the given axis by the mass (or weight) of the body.

RULE V.—Given, the square of the radius of gyration of a body about an axis traversing its centre of gravity in a given direction; to find the square of the radius of gyration of the same body about another axis parallel to the first; to the given square add the square of the perpendicular distance between the two axes.

10.—TABLE OF SQUARES OF RADII OF GYRATION.

BODY.	AXIS.	RADIUS.²
I. Sphere of radius r, ...	Diameter	$\frac{2r^2}{5}$
II. Spheroid of revolution—polar semi-axis a, equatorial radius r, ...	Polar axis	$\frac{2r^2}{5}$
III. Ellipsoid—semi-axes a, b, c, ...	Axis, $2a$	$\frac{b^2+c^2}{5}$
IV. Spherical shell—external radius r, internal r', ...	Diameter	$\frac{2(r^5-r'^5)}{5(r^3-r'^3)}$
V. Spherical shell, insensibly thin—radius r, thickness dr, ...	Diameter	$\frac{2r^2}{3}$

Body.	Axis.	Radius.²
VI. Circular cylinder—length $2a$, radius r,	Longitudinal axis, $2a$	$\frac{r^2}{2}$
VII. Elliptic cylinder—length $2a$, transverse semi-axes b, c,....................	Longitudinal axis, $2a$	$\frac{b^2+c^2}{4}$
VIII. Hollow circular cylinder—length $2a$, external radius r, internal r',.........	Longitudinal axis, $2a$	$\frac{r^2+r'^2}{2}$
IX. Hollow circular cylinder, insensibly thin—length $2a$, radius r, thickness dr,....................................	Longitudinal axis, $2a$	r^2
X. Circular cylinder—length $2a$, radius r,..	Transverse diameter	$\frac{r^2}{4}+\frac{a^2}{3}$
XI. Elliptic cylinder—length $2a$, transverse semi-axes b, c,....................	Transverse axis, $2b$	$\frac{c^2}{4}+\frac{a^2}{3}$
XII. Hollow circular cylinder—length $2a$, external radius r, internal r',.........	Transverse diameter	$\frac{r^2+r'^2}{4}+\frac{a^2}{3}$
XIII. Hollow circular cylinder, insensibly thin—radius r, thickness dr,........	Transverse diameter	$\frac{r^2}{2}+\frac{a^2}{3}$
XIV. Rectangular prism—dimensions $2a$, $2b$, $2c$,..	Axis, $2a$	$\frac{b^2+c^2}{3}$
XV. Rhombic prism—length $2a$, diagonals $2b$, $2c$,..	Axis, $2a$	$\frac{b^2+c^2}{6}$
XVI. Rhombic prism, as above,...............	Diagonal, $2b$	$\frac{c^2}{6}+\frac{a^2}{3}$

11. **Centre of Percussion—Equivalent Simple Pendulum.**—RULE I.—To find the centre of percussion of a given body turning about a given axis.

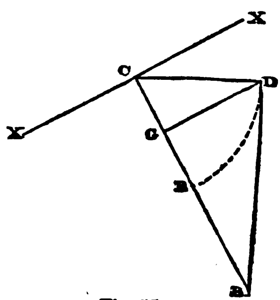


Fig. 55.

In fig. 55, let X X be the given axis, and G the centre of gravity of the body. From G let fall G C perpendicular to X X. Through G draw G D parallel to X X, and equal to the radius of gyration of the body about the axis G D. Join C D. Then will C E = C D = $\sqrt{\text{G D}^2 + \text{C G}^2}$ = the radius of gyration of the body about X X. From D draw D B perpen-

dicular to C D, cutting C G produced in B. Then will B be the centre of percussion of the body for the axis X X.

To find B by calculation; make $G\,B = \frac{G\,D^2}{G\,C}$.

C is the centre of percussion for an axis traversing B parallel to X X.

Rule II.—To convert the body into an "equivalent simple pendulum" for the axis X X, or for an axis through B parallel to X X; divide the mass of the body into two parts inversely proportional to G C and G B, and conceive those parts to be concentrated at C and B respectively, and rigidly connected together.

(Let W be the whole mass, and C and B the two parts; then $C = \frac{W \cdot G\,B}{C\,B}$; $B = \frac{W \cdot G\,C}{C\,B}$.)

(The "equivalent simple pendulum" has the same weight with the given body, and also the same moment of weight, and the same moment of inertia, with the given body, relatively to an axis in the given direction X X, traversing either C or B.)

12. **Equivalent Ring, or Equivalent Fly-wheel.**—When the given axis traverses the centre of gravity, G, there is no centre of percussion. The moment of the body's weight is nothing, and its moment of inertia is the same as if its whole mass were concentrated in a ring of a radius equal to the radius of gyration of the body. That ring may be called the "equivalent ring," or "equivalent fly-wheel."

13. The **Centre of Pressure** in a plane surface is the point traversed by the resultant of a pressure that is exerted at that surface.

Rule.—Conceive that upon the pressed surface as a base, there stands a prismatic solid of a height at each point of that surface proportional to the intensity of the pressure (page 103); the point in the pressed surface at the foot of a perpendicular from the centre of magnitude of the solid (pages 81 to 88) will be the centre of pressure.

The following are particular cases:—

I. **Uniform Pressure.**—When the intensity is uniform, the centre of pressure is at the *centre of magnitude* of the pressed surface. (See page 83.)

II. **Uniformly Varying Pressure.**—When the intensity of the pressure varies simply as the perpendicular distance from a given axis, the centre of pressure is at the *centre of percussion* of the pressed surface, relatively to that axis (see page 155); the surface being regarded as a thin plate of uniform thickness and heaviness.

EXAMPLES OF CENTRES OF UNIFORMLY-VARYING PRESSURE.

In each of the following examples the greatest perpendicular distance of any point of the pressed surface from the axis is denoted by h; and that of the centre of pressure from the axis by k.

FIGURE OF PRESSED SURFACE.	AXIS.	$k =$
Parallelogram, ...	One edge.	$\frac{2}{3}h.$
Triangle, ...	One edge.	$\frac{1}{2}h.$
Triangle, ...	Through an angle, and parallel to the opposite edge.	$\frac{3}{4}h.$
Semicircle or semi-ellipse,.....	Diameter.	$0{\cdot}58905h.$
Circle or ellipse,..................	Tangent.	$\frac{5}{8}h.$
Hollow rectangle,— outer dimensions, $b \times h$,..... inner dimensions, $b' \times h'$,....	One edge of the outer boundary.	$\frac{h}{2}+\frac{bh^3-b'h'^3}{6h\,(bh-b'h')}.$
Hollow square, $h^2 - h'^2$,......	Do.	$\frac{2h}{3}+\frac{h'^2}{6h}.$
Hollow ellipse,— outer dimensions, $b \times h$,..... inner dimensions, $b' \times h'$,...	Tangent to the outer boundary.	$\frac{h}{2}+\frac{bh^3-b'h'^3}{8h\,(bh-b'h')}.$
Hollow circle,— outer diameter, h,........... inner diameter, h'...........	Do.	$\frac{5h}{8}+\frac{h'^2}{8h}.$

14. The **Centre of Buoyancy** of a solid wholly or partly immersed in a liquid is the centre of gravity of the mass of liquid displaced. The resultant pressure of the liquid on the solid is equal to the weight of liquid displaced, and is exerted vertically upwards through the centre of buoyancy.

PART V.

RULES RELATING TO THE BALANCE AND STABILITY OF STRUCTURES.

SECTION I.—COMPOSITION AND RESOLUTION OF FORCES.

1. The **Resultant of a Distributed Force.**—RULE I.—To find the resultant of a body's weight; find the centre of gravity of the body (as in page 153); the resultant will be a single force equal to the weight, acting vertically downwards through the centre of gravity.

RULE II.—To find the resultant of a pressure; find the centre of pressure (as in page 156); the resultant will be a single force equal in amount to the pressure, and acting in the same direction and through the centre of pressure. (The *amount* of the pressure is equal to the area of the pressed surface, multiplied by the *mean intensity* of the pressure, and is also equal to the weight of the imaginary prismatic solid mentioned in page 156, Article 13.) The mean intensity of an *uniformly varying* pressure is its intensity at the *centre of magnitude* of the pressed surface. (See page 49.)

2. **Resultant of Forces acting through one Point.**—RULE III.—If the forces act *along one line*, all in the same direction, their resultant is equal to their sum; if some act in one direction and some in the contrary direction, the resultant is their *algebraical sum;* that is to say, add together separately the forces which act in the two contrary directions respectively; the difference of the two sums will be the amount of the resultant, and its direction will be the same with that of the forces whose sum is the greater.

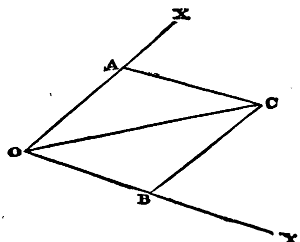


Fig. 56.

RULE IV.—If the forces act along *two lines*, O X, O Y (fig. 56), lay off O A and O B along those lines, to represent the magnitudes of the given forces; through A draw A C parallel to O B; through B draw B C parallel to O A, and cutting A C in C; join O C; the diagonal O C will represent the resultant required, in direction and magnitude.

Formula for finding the magnitude of O C by calculation:

$$O\,C = \sqrt{\left\{ O\,A^2 + O\,B^2 + 2\,O\,A \cdot O\,B \cos A\,O\,B. \right\}}$$

Formulæ for finding the direction of O C by calculation:

$$\sin A\,O\,C = \sin A\,O\,B \cdot \frac{O\,B}{O\,C};\ \sin B\,O\,C = \sin A\,O\,B \cdot \frac{O\,A}{O\,C}.$$

Rule V.—Given, the directions of three forces which balance each other, acting in one plane and through one point; construct a triangle whose sides make the same angles with each other that the directions of the forces do; the proportions of the forces to each other will be the same with those of the corresponding sides of that triangle.

To solve the same question by calculation; let A, B, C, stand for the magnitudes of the three forces; A O B, B O C, C O A, for the angles between their directions; then

$$\sin B\,O\,C : \sin C\,O\,A : \sin A\,O\,B :: A : B : C.$$

Each of those three forces is equal and opposite to the resultant of the other two.

Rule VI.—To find the resultant of any number (F_1, F_2, F_3, &c., fig. 57) of forces in different directions, acting through one point, O. Commence at the point of application, and construct a chain of lines representing the forces in magnitude, and parallel to them in direction, (O A = and || F_1, A B = and || F_2, B C = and || F_3, &c.) Let D be the end of that chain; join O D, this will represent the required resultant; and a force (F_5) equal and opposite to O D will balance the given forces.

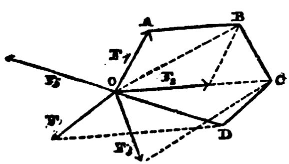


Fig. 57.

(This rule is applicable whether the forces act in one plane or in different planes.)

3. **Resolution of a Force into Inclined Components.**—A single force may be resolved into two inclined components in the same plane acting through the same point, or into three inclined components acting through the same point but not in the same plane.

Rule VII. *Two Components.*—In fig 56, page 158, let O C be the given force, and O X and O Y the directions of the required components. Through C draw C A parallel to O Y, cutting O X in A, and C B parallel to O X, cutting O Y in B; O A and O B will be the required components; and two forces respectively

equal to and directly opposed to these will balance O C. For the proportionate magnitudes of the components, see Article 2 of this section, Rule V., page 159.

Rule VIII. *Two Rectangular Components.*—When the directions of the required components are perpendicular to each other, let R denote the resultant, or force to be resolved; X and Y the required components, α and β the angles which they make respectively with R. Then

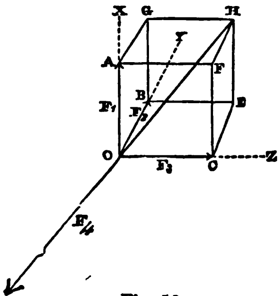


Fig. 58.

$$\alpha + \beta = 90^\circ; \ X = R\cos\alpha = R\sin\beta;$$
$$Y = R\cos\beta = R\sin\alpha;$$
$$X^2 + Y^2 = R^2.$$

Observe that cosines of obtuse angles are negative. (See page 53, line 2.)

Rule IX. *Three Components.*—In fig. 58, let $\overline{OH}$ represent the given force which it is required to resolve into three component forces, acting in the lines O X, O Y, O Z, which cut O H in one point O.

Through H draw three planes parallel respectively to the planes Y O Z, Z O X, X O Y, and cutting respectively O X in A, O Y in B, O Z in C. Then will $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, represent the component forces required.

Rule X. *Three Rectangular Components.*—When the directions of the three required components are perpendicular to each other, let R denote the resultant, or force to be resolved, X, Y, Z, the required components, and α, β, γ, the angles which they respectively make with R. Then

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1; \ X = R\cos\alpha;$$
$$Y = R\cos\beta; \ Z = R\cos\gamma; \ X^2 + Y^2 + Z^2 = R^2.$$

Observe that cosines of obtuse angles are negative. (See page 53, line 2.)

4. **Resultant of any Number of Inclined Forces Acting through one Point.**—To solve the same question by calculation that is solved in Rule VI. by construction.

Rule XI. (When the forces act in *one plane.*)—Assume any two directions at right angles to each other as axes; resolve each force into two components (X, Y) along those axes; take the resultants of those components along the two axes separately (Σ X, Σ Y); these will be the *rectangular components of the resultant* R *of all the forces*; that is to say,

$$R = \sqrt{\left\{(\Sigma X)^2 + (\Sigma Y)^2\right\}};$$

and if α be the angle which R makes with X,

$$\cos \alpha = \frac{\Sigma X}{R}; \ \sin \alpha = \frac{\Sigma Y}{R}.$$

RULE XII. (When the forces act in different planes).—Assume any three directions at right angles to each other as axes; resolve each force into three components (X, Y, Z) along those axes; take the resultants of the components along the three axes separately (ΣX, ΣY, ΣZ); these will be the *rectangular components of the resultant of all the forces;* and its magnitude and direction will be given by the following equations:—

$$R = \sqrt{\left\{ (\Sigma X)^2 + (\Sigma Y)^2 + (\Sigma Z)^2 \right\}}.$$

$$\cos \alpha = \frac{\Sigma X}{R}; \ \cos \beta = \frac{\Sigma Y}{R}; \ \cos \gamma = \frac{\Sigma Z}{R}.$$

5. **Couples.**—In fig. 59, let F, F, represent a couple of equal, parallel, and opposite forces, applied to a rigid body, and not acting in the same line; L, the perpendicular distance between their lines of action; then F is the *force* of the couple, L the *arm*, *span*, or *leverage;* and the product force × leverage = F L, is the *statical moment* of the couple, which is right or left-handed according as the couple tends to turn the rigid body, as seen by the spectator, with or against the hands of a watch. (For measures of statical moment, see page 104, Article 7.) Couples of the same moment, acting in the same direction, and in the same plane or in parallel planes, are *equivalent* to each other.

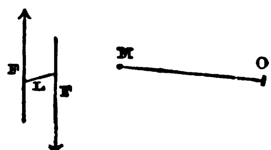


Fig. 59.

RULE XIII.—To find the resultant moment of any number of couples acting on a rigid body in the same plane, or in parallel planes. Take the sums of the right-handed and left-handed moments separately; the difference between those sums will be the resultant moment, which will be right-handed or left-handed according to the direction of the moments whose sum is the greater.

RULE XIV.—To represent the moment of a couple by a single line. Upon any line perpendicular to the plane of the couple, set off a length proportional to the moment (O M, fig. 59), in such a direction that to a spectator looking from O towards M, the couple shall seem right-handed. The line O M is called the *axis* of the couple.

Couples as represented by their axes are compounded and resolved like single forces, by Rules I. to XII. of this section.

RULE XV.—To find the resultant of a single force, F, applied to a rigid body at O, and a couple, M, acting on the same body in the same or in a parallel plane. Conceive the force, F, to be shifted in that plane, parallel to itself, to the left if the couple is right-handed, to the right if the couple is left-handed, through a distance, O A, found by dividing M by F. The shifted single force, F acting through A, will be the resultant required.

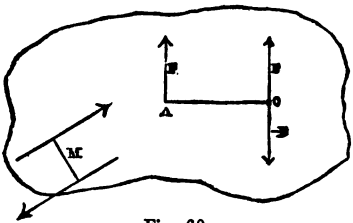


Fig. 60.

(The combination of a single force with a couple acting in a plane perpendicular to the line of action of the force cannot be further simplified.)

RULE XVI.—To resolve a single force into a single force acting in a different but parallel line, and a couple. In fig. 61, let F be the given force acting in the line E D, and B a given point not in E D. Through B conceive a pair of equal and contrary forces to act in a line parallel to E D; viz., + F′ equal to F and in the same direction; and − F′ equal to F and in the contrary direction; also, let fall B A perpendicular to E D. Then the original force F acting through A, is resolved into the equal and parallel force F′ acting through B, and the couple of forces F and — F′, with the arm A B and moment F × A B; which couple is right or left-handed according as B lies to the right or left of F, relatively to a spectator looking in the direction towards which F acts.

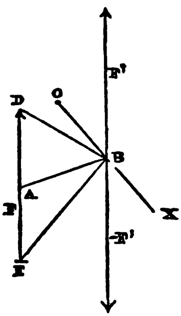


Fig. 61.

F × A B is called *the moment of the force* F *relatively to the point* B; or relatively to the *axis* O X traversing B in a direction perpendicular to the plane of F and A B; or relatively to a *plane* traversing B perpendicularly to A B.

6. **Parallel Forces.**—RULE XVII.—To find the resultant of two parallel forces. The resultant is in the same plane with, and parallel to, the components. It is their sum or difference according as they act in the same or contrary directions; and in the latter case its direction is that of the greater component. To find its line of action by construction, proceed as follows:—Fig. 62 representing the case in which the components act in the same direction, fig. 63 that in which they act in contrary directions. Let A D and B E be the components. Join A E and B D, cutting each other in F. In B D (produced in fig. 63), take B G = D F.

Through G draw a line parallel to the components; this will be the line of action of the resultant. To find its magnitude by con-

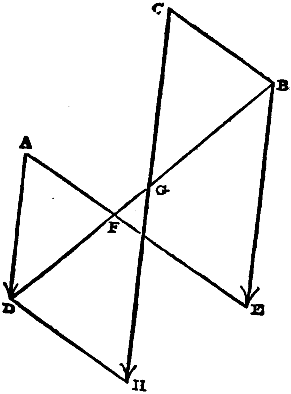


Fig. 62

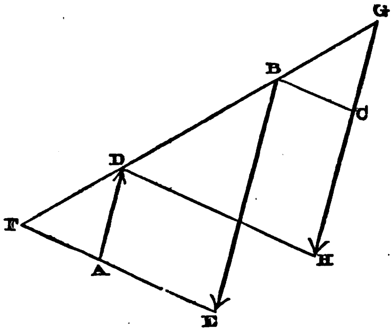


Fig. 63.

struction: parallel to A E, draw B C and D H, cutting the line of action of the resultant in C and H; C H will represent the resultant required; and a force equal and opposite to C H will balance A D and B E.

To find the line of action of the resultant by calculation; make either

$$B\,G = \frac{A\,D \cdot D\,B}{C\,H};\ \text{or}\ D\,G = \frac{B\,E \cdot D\,B}{C\,H}.$$

RULE XVIII.—When the two given parallel forces are opposite and equal, they form a couple, and have no single resultant.

RULE XIX.—To find the relative proportions of three parallel forces which balance each other, acting in one plane; their lines of action being given. Across the three lines of action, in any convenient position, draw a straight line A C B, fig. 64, and measure the distances between the points where it cuts the lines of action. Then each force will be proportional to the distance between the lines of action of the other two. The direction of the middle force C is contrary to that of the other two forces, A and B.

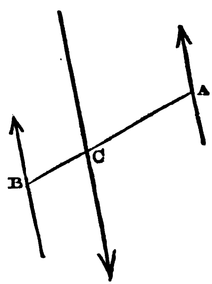


Fig. 64.

In symbols, let A, B, and C, be the forces; then,

$$A + B + C = 0;\ AB : BC : CA :: C : A : B.$$

Each of the three forces is equal and opposite to the resultant of the other two; and each pair of forces are equal and opposite to the components of the third. Hence this rule serves to resolve a given force into two parallel components, acting in given lines in the same plane.

RULE XX.—To find the relative proportions of four parallel forces which balance each other, not acting in one plane; their lines of action being given. Conceive a plane to cross the lines of action in any convenient position; and in fig. 65 or fig. 66, let A, B, C, D, represent the points where the four lines of action cut the plane. Draw the six straight lines joining those four points by pairs. Then the force which acts through each point will be proportional to the area of the triangle formed by the other three points.

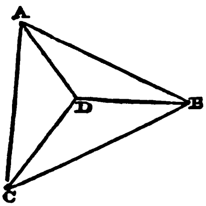


Fig. 65.

In fig. 65, the directions of the forces at A, B, and C, are the same, and are contrary to that of the force at D. In fig. 66 the forces at A and D act in one direction, and those at B and C in the contrary direction.

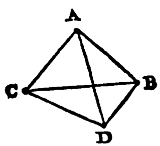


Fig. 66.

In symbols,

$$A + B + C + D = 0;$$

$$BCD : CDA : DAB : ABC :: A : B : C : D.$$

Each of the four forces is equal and opposite to the resultant of the other three; and each set of three forces are equal and opposite to the components of the fourth. Hence the rule serves to resolve a force into three parallel components not acting in one plane.

RULE XXI.—To find the resultant of any number of parallel forces.

Case I.—When the parallel forces act all in one direction, the magnitude of their resultant is their sum. Consider the parallel forces as detached weights, and find the position of the common centre of gravity of those weights by Part IV., Article 9, Rule III., (page 153); the line of action of the resultant will pass through that centre.

Case II.—When the parallel forces act in two contrary directions. Find separately, as in Case I., the magnitudes and lines of action of the resultants of the forces which act in the two contrary directions respectively; if those two resultants are unequal, find the final resultant by Rule XVII.; if they are equal, they form a couple, and have no single force as a resultant.

SECTION II.—FRAMES, CHAINS, AND LINEAR RIBS.

1. **Triangular and Polygonal Frames.**—A frame consists of bars connected together at their ends by joints which offer no sensible resistance to the turning of one bar into a different angular position relatively to the next, the resistance to such turning being given by the fixing of the farther ends of the bars alone. The point in a given joint about which such turning would take place is called the *centre of resistance* of the joint; the straight line joining the centres of resistance at the ends of a bar is called the *line of resistance* of that bar. A bar is called a *strut*, or a *tie*, according as a thrust or a pull is exerted along its line of resistance. A figure showing the centres of resistance and lines of resistance alone may be called the *skeleton diagram* of a frame. When a joint is spoken of as a point, its centre of resistance is meant; when a bar is spoken of as a line, its line of resistance is meant.

When the balance and stability of a frame alone are in question, and not its strength, the load may be treated as if concentrated at the centre of resistance; and if not actually so concentrated, the following rule is to be used:—

RULE I.—Given, the actual load distributed over a frame, whether arising from external forces or from its own weight, and the distribution of that load; to find the equivalent load concentrated at the centres of resistance of the joints. By the rules of the preceding section, and of Part IV., find the resultant of the load on each bar; then, by Rule XIX. of the preceding section (page 163), resolve each such resultant into two parallel components acting through the centres of resistance at the ends of that bar; then take the resultants of those components for each joint separately; those resultants will form the equivalent load required.

RULE II.—Given, the load on a frame, and the line or lines of resistance of its supports; to find the supporting force or forces, commence by finding the resultant of the whole load by the rules of the preceding section, and of Part IV.

Case I.—If there is but one support, its line of resistance must coincide with the line of action of the resultant of the whole load; and the supporting force must be equal and opposite to that resultant.

Case II.—When there are two supports, their lines of resistance must be in the same plane with the line of action of the resultant load, and must either be parallel to it, or, if inclined, cut it in one point. If parallel, use Rule XIX. (page 163), or, if inclined, use Rule VII. (page 159) of the preceding section to resolve the resultant load into two components acting along the lines of resistance

of the supports; the two supporting forces will be equal and opposite to those components.

Case III.—When there are three supports, their lines of resistance must be either parallel to the line of action of the resultant load, or must cut it in one point. If parallel, use Rule XX. (page 164), or, if inclined, use Rule IX. (page 160) of the preceding section, to resolve the resultant load into three components acting along the lines of resistance of the supports; the three supporting forces will be equal and opposite to those components.

Remark.—In all the following rules, those components of a distributed load which, as found by Rule I., rest directly on the supports of the frame, are understood to be left out of account; and the *supporting forces* are supposed to be determined *exclusive* of such parts of them as are required in order to sustain such direct loads on the supports.

Rule III.—To distinguish struts from ties. In fig. 67, let A C and B C be the lines of resistance of two bars of a frame meeting at the joint C. Produce those lines beyond C, as shown by C D, C E; and draw a line to represent the direction of the load at C. Then, if that direction lies between A C produced and B C produced, as at 1, both bars are ties; if between A C produced and C B, as at 2, A C is a tie and B C a strut; if between C A and C B, as at 3, both bars are struts; if between C A and B C produced, as at 4, A C is a strut and B C a tie.

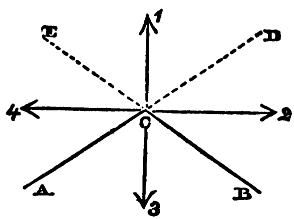


Fig. 67.

Remark as to stability and instability.—A tie is stable, even although one or both ends are moveable. A strut is unstable, unless both ends are fixed. A frame composed altogether of ties is stable even although flexible. A frame containing struts must be stiffened, so as to fix their positions.

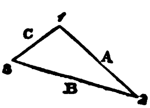


Fig. 68.

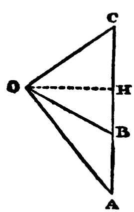


Fig. 69.

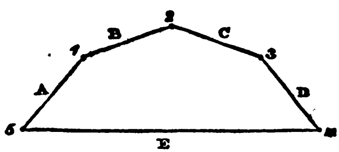


Fig. 70.

Rule IV.—Given, in a *triangular frame*, loaded and supported vertically, the skeleton diagram (fig. 68), to find the relative pro-

portions of the forces acting in the frame. Let A, B, C, be the three bars, 1, 2, 3, the three joints. Construct the *diagram of forces*, fig. 69, as follows:—From any point, O, draw O A, O B, O C, parallel to the lines of resistance A, B, C, respectively; then across those three lines draw the vertical line A B C. Then the required proportions are as follows:—

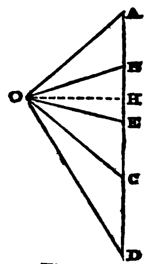


Fig. 71.

Load on	Supporting Forces at		Stress Along		
1 :	2 :	3 :	A :	B :	C
: : C A :	A B :	B C :	O A :	O B :	O C;

and from these proportions, if any one of the six forces is given, the other five may be found.

From O, perpendicular to A B C, draw O H. This will represent the *horizontal stress* of the frame, which is the same in each bar. To find this and the other forces by calculation from the load C A, let a, b, c, be the angles of slope of the three lines of resistance; then

$$O\,H = \frac{C\,A}{\tan c \pm \tan a}.$$

$$A\,B = O\,H \cdot (\tan a \mp \tan b);\ B\,C = O\,H \cdot (\tan b \pm \tan c).$$

The sign $\left\{ \begin{matrix} + \\ - \end{matrix} \right\}$ is to be used when the two inclinations are in $\left\} \begin{matrix} \text{opposite directions} \\ \text{the same direction.} \end{matrix} \right.$

$$O\,A = O\,H \cdot \sec a;\ O\,B = O\,H \cdot \sec b;\ O\,C = O\,H \cdot \sec c.$$

RULE V.—Given, in a *polygonal frame, loaded and supported vertically*, the skeleton diagram, fig. 70, to find the relative proportions of the forces. Let A, B, C, D, E be the bars; 1, 2, 3, 4, 5, the joints, of which 1, 2, 3 are loaded, and 4, 5, supported. Construct the *diagram of forces*, fig. 71, as follows:—From any point, O, draw radiating lines, O A, O B, O C, &c., parallel respectively to the lines of resistance A, B, C, &c., in fig. 70. Then draw a vertical line, A D, across the radiating lines. Then, taking the whole length, A D, to represent the whole load, the several parts into which that length is cut by the lines O B, O C, &c., will represent the parts of the load which must rest on the several loaded joints in order that the frame may be balanced. For example, B C in fig. 71 represents the part of the load to be applied at the joint 2 in fig. 70, where the bars B and C meet. Also, the parts D E and E A into which A D is divided by the line A E, parallel to the bar E, which connects the points of support, 4 and 5, in fig. 70, represent the supporting forces at those points respectively. The lengths of the radiating lines O A, &c., represent the stresses along the lines of resistance to which they are respectively parallel.

From O let fall on A D the perpendicular O H. This will represent the *horizontal stress* of the frame.

REMARKS.—By omitting from the skeleton diagram, fig. 70, the bar E, which connects the points of support, the frame becomes an *open frame*, in which case the supporting forces become identical with the stresses along the outer bars, A and D, and are represented by D O and O A in fig. 71. The obliquity of those forces renders *abutments* necessary at 4 and 5, and not mere vertical supports.

The frame shown in fig. 70 consists chiefly of struts, and is therefore unstable unless their ends are fixed by means of suitable stays. If the same figure be inverted, the bars which were struts become ties, and the frame is stable, although flexible.

RULE VI.—Given, in a vertically-loaded polygonal frame, the load and its distribution, and the inclinations of the two outer bars, A and D, fig. 70; to find the inclinations of the remaining loaded bars, in order that they may be balanced. In fig. 71 draw a vertical line, A D, to represent the whole load, and divide it into parts, A B, B C, &c., to represent the parts of that load which are to be supported at the several loaded joints. From the ends of that line draw A O and D O parallel to the lines of resistance of the two outer bars, and cutting each other in O; then draw radiating lines, O B, O C, &c., from O to the points of division of A D; these will be parallel to the lines of resistance whose inclinations are required.

RULE VII.—Given, in a polygonal frame, vertically loaded, the total load and the inclinations of the lines of resistance of the two outer bars; to find the horizontal stress, divide the load by the *sum* of the tangents of those inclinations, if they are contrary, or by the *difference* of those tangents, if the inclinations are similar.

RULE VIII.—Given, the skeleton diagram of a vertically-loaded polygonal frame and the horizontal stress; to find how much of the load is supported between any two bars, multiply the horizontal stress by the *difference* of the tangents of the inclinations of the lines of resistance of those bars, if they slope the same way, or by the *sum* of those tangents, if the lines of resistance slope contrary ways.

RULE IX.—From the same data, to find the stress along a given bar; multiply the horizontal stress by the secant of the inclination of the line of resistance of that bar.

2. **Braced Frames—Method of Triangles.**—When the external forces applied to a frame, although balancing each other as an entire system, are distributed in a manner not consistent with the equilibrium of each bar separately; then, in the diagram of forces, upon attempting to construct a scale of loads having its points of division on the radiating lines, as in fig. 71, gaps will be left in

that scale. The lines necessary to fill up those gaps will indicate the forces to be supplied by means of the resistance of *braces.* These may be either struts or ties, connecting two or more joints together.

The resistance of a brace introduces a pair of equal and opposite forces, acting along the line of resistance of the brace, upon the pair of joints which it connects.

3. **Method of Sections Applied to Frames.**—When a vertically-loaded braced frame is so designed that a vertical cross-section of it at any point cuts not more than three lines of resistance, the *method of sections* may be applied as follows:—The upper and lower bars, as 1 3, 3 5, &c., and 0 2, 2 4, 4 6, &c., in fig. 72, may be called the *stringers*, and the intermediate bars, 0 1, 1 2, 2 3, &c., the *braces.*

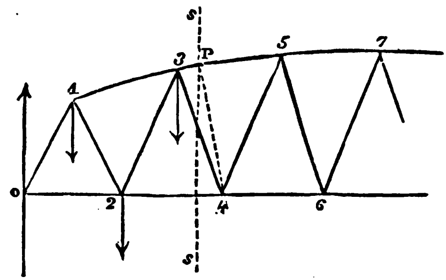


Fig. 72.

RULE I.—Given the skeleton diagram, and the load at each of the joints (1, 2, 3, &c.), to find the stress exerted along any one of the stringers (as 3 5). Find the supporting forces by Rule II. of the last Article (page 165). Then conceive the frame divided into two parts, by a section traversing the joint that is opposite the stringer under consideration (for example, the joint 4, opposite the stringer 3 5). Take the resultant moment relatively to the joint 4 (see preceding section) of all the external forces which act on one of those parts. (That is to say, in the present example, take the moment of the supporting force at the joint O, by multiplying it by its horizontal distance from 4; and from that moment subtract the moments of the several parts of the load which act at 1, 2, and 3.) From the joint (4) opposite the stringer in question, let fall a perpendicular (4 P) on the line of resistance of the stringer (3 5); divide the resultant moment by the length of that perpendicular; the quotient will be the stress along the stringer in question. To find whether that stress is thrust or tension, consider in which direction the resultant moment tends to turn the part of the frame on which it acts about the joint (4); the stress will be of the kind which resists that tendency. (In the example the stress is thrust for the upper stringers, tension for the lower.)

RULE II.—To find the vertical component of the stress along a

stringer, multiply the whole stress by the difference of level of the ends of the stringer, and divide by the length of the stringer.

If the stringer is horizontal, its stress has no vertical component.

The stress of each stringer having been found, the next step is as follows:—

Rule III.—In the same case, to find the stress along any one of the braces (for example, 3 4). Conceive the frame to be divided into two parts by a vertical section, S S, traversing the brace in question. Take the resultant of all the external forces which act on one of those divisions. (That is to say, in the example shown, from the supporting force at the joint O subtract the loads at the joints 1, 2, 3.) With that resultant combine the vertical components (if any) of the stresses along the two stringers cut by the section (in this case 3 5 and 2 4). The vertical component of the required stress on the brace will be equal and opposite to the final resultant found by the preceding processes, and being multiplied by the ratio in which the length of the brace is greater than the difference of level of its ends, will give the whole stress along the brace.

4. **Loaded Chains.**—Rule I.—Given the figure of a loaded chain, C B A D; to find the position of the resultant load on any part of it, A B, and the relative proportions of the forces acting on that part of the chain. Draw tangents, A P and B P, to the chain at the two ends of the part in question, cutting each other in P; the line of action of the resultant load on the part A B traverses the point P. Also, construct a triangle (such as B P X), with its three sides parallel respectively to the two tangents and the resultant load: those three sides will bear to each other the relative proportions of the tensions at A and B, and the load supported between A and B.

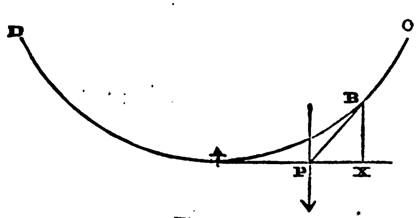


Fig. 73.

Rule II.—Given, in a *vertically*-loaded chain, the total load, and the figure in which the chain hangs; to find the distribution of the load, and the tension at any point of the chain. Construct the diagram of forces, fig. 74, as follows:—Draw a vertical straight line, C D, to represent the total load, and from its ends draw C O and D O, parallel to two tangents at the points of support of the chain, and meeting in O; those lines will represent the tensions on the chain at its point of support.

Let A, in fig. 73, be the lowest point of the chain. In fig. 74 draw the horizontal line O A; this will represent the horizontal

component of the tension of the chain at every point, and if O B be parallel to a tangent to the chain at any point B, A B, in fig. 74, will represent the portion of the load supported between A and B, and O B the tension at B.

RULE III.—Given, in a vertically loaded chain, the load and its distribution; the points of suspension, C and C′ (fig. 75), which points are supposed at the same level, and the horizontal tangent, H H′, at the lowest point of the chain; to construct the figure in which the chain will hang. By Rule XXI. of the preceding section (page 164), find the resultant load, R; then by Rule XIX. of the same section (page 163), find the vertical components, P and P′, of the two supporting forces (equal and opposite to two parallel components of R through C and C′). Then, from the known distribution of the load, find the position of a vertical line, A F, dividing the total load, R = P + P′, into two parts equal to the adjacent supporting forces, P and P′ respectively; the point A, where

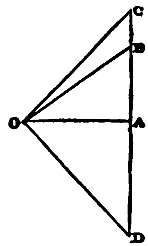


Fig. 74.

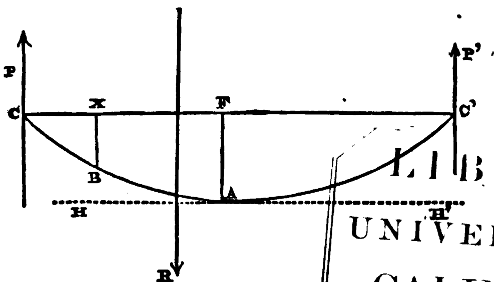


Fig. 75.

that vertical line cuts the horizontal tangent H H′, will be the *lowest point of the chain*. Next, to find the horizontal tension;

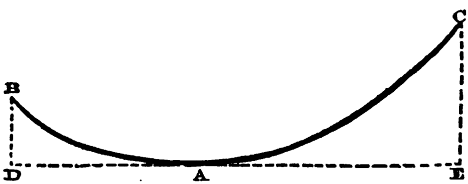


Fig. 76.

conceive the chain divided into two parts by a vertical plane through A F; take the resultant moment, relatively to that plane,

of all the vertical forces which act on one of those parts: for example, of the supporting force P, and of those parts of the load which hang between C and A; divide that moment by the *greatest depression*, F A; the quotient will be the horizontal tension. Lastly, to find the depression, X B, of any other point, B, of the chain below the level of the points of support; conceive the chain to be divided into two parts by a vertical plane through X B; find the resultant moment, relatively to that plane, of all the vertical forces which act on one of those parts; that is, of the supporting force P, and of those parts of the load which hang between C and B; divide that resultant moment by the horizontal tension; the quotient will be the required depression, X B.

The resultant tensions at the points of support are, respectively, $\sqrt{(H^2 + P^2)}$ and $\sqrt{(H^2 + P'^2)}$, where H denotes the horizontal tension.

A balanced chain, being inverted, gives the *curve of equilibrium* for a rib loaded in the same manner with the chain. The tensions in the chain became thrusts in the rib.

5. **Chain, with Load Uniform over the Span.**—The assumption that the load is uniformly distributed over the span of a chain is, in most cases of suspension bridges, near enough to the truth for practical purposes. In fig. 76 let B A C be a chain so loaded; A, its lowest point; D A E, a horizontal tangent at that point; B and C, the points of support; B D and C E, vertical lines through them. The curve B A C is a common parabola, with its vertex at A. Let D E = a; B D = y_1; C E = y_2; A D = x_1; A E = x_2; so that $x_1 + x_2 = a$.

RULE I.—Given, the elevations, y_1, y_2, of the two points of support of the chain above its lowest point, and also the horizontal distance, or span, a, between those points of support; it is required to find the horizontal distances, x_1, x_2, of the lowest point from the two points of support: also the focal distance, m, of the parabola.

$$x_1 = a \cdot \frac{\sqrt{y_1}}{\sqrt{y_1} + \sqrt{y_2}}; \; x_2 = a \cdot \frac{\sqrt{y_2}}{\sqrt{y_1} + \sqrt{y_2}}.$$

$$m = \frac{a^2}{4\, y_1 + 4\, y_2 + 8 \sqrt{y_1\, y_2}}.$$

When the points of support are at the same level,

$$y_1 = y_2; \; x_1 = x_2 = \frac{a}{2}; \; m = \frac{a^2}{16\, y_1}.$$

In the latter case the height $y_1 = y_2$ is called the *depression*.

RULE II.—Given, the same data, to find the inclinations, i_1, i_2, of the chain at the points of support.

$$\tan i_1 = \frac{2\,y_1}{x_1} = \frac{2\,y_1 + 2\sqrt{y_1\,y_2}}{a};\ \tan i_2 = \frac{2\,y_2}{x_2} = \frac{2\,y_2 + 2\sqrt{y_1\,y_2}}{a};$$

when the points of support are at the same level,

$$y_1 = y_2;\ \tan i_1 = \tan i_2 = \frac{4\,y_1}{a}.$$

Rule III.—Given, the same data, and the load, p, per unit of length: required the horizontal tension, H, and the tensions, R_1, R_2, at the points of support.

$$H = 2\,p\,m = \frac{p\,a^2}{2\,y_1 + 2\,y_2 + 4\sqrt{y_1\,y_2}};$$

$$R_1 = H\sqrt{\left(1 + \frac{4\,y_1^2}{x_1^2}\right)};\ R_2 = H\sqrt{\left(1 + \frac{4\,y_2^2}{x_2^2}\right)}.$$

When the points of support are at the same level, or that $y_1 = y_2$, those equations become

$$H = \frac{p\,a^2}{8\,y_1};\ R_1 = R_2 = H\sqrt{\left(1 + \frac{16\,y_1^2}{a^2}\right)}.$$

Rule IV.—Given, the same data as in Problem First, to find the length of the chain.

Calculate the lengths of the arcs A B $= s_1$, and A C $= s_2$, by the rules of page 79, Article 5, and add them together.

Rule V.—Given, the same data, to find, approximately, the small elongation of the chain $d\,(s_1 + s_2)$ required to produce a given small depression, $d\,y$, of the lowest point A, and conversely.

$$\frac{d\,(s_1 + s_2)}{d\,y} = \frac{4}{3}\left(\frac{y_1}{x_1} + \frac{y_2}{x_2}\right).$$

When $y_1 = y_2$, this equation becomes

$$\frac{2\,d\,s_1}{d\,y} = \frac{16\,y_1}{3\,a}.$$

These formulæ serve to compute the depression which the middle point of a suspension bridge undergoes in consequence of a given elongation of the cable or chain, whether caused by heat or by tension.

Rule VI.—To find the pressure on the top of each pier. If the chain passes over a curved plate on the top of the pier called a *saddle*, on which it is free to slide, the tensions of the portions of the chain or cable on either side of the saddle will be sensibly

equal; and in order that those tensions may compose a vertical pressure on the pier, their inclinations must be equal and opposite. Let i be the common value of those inclinations; R the common value of the two tensions; then the vertical pressure on the pier is

$$V = 2\,R \sin i = 2\,H \tan i = 2\,p\,x;$$

that is, twice the weight of the portion of the bridge between the pier and the lowest point, A, of the chain.

But if the two divisions of the chain which meet at the top of a given pier are made fast to a truck, which is supported by rollers on a horizontal cast-iron platform on the top of the pier, let i, i', be the inclinations of the two divisions of the chain or cable in opposite directions, and R, R′, their tensions; then

$$R = H \sec i;\ R' = H \sec i';$$

$$V = R \sin i + R' \sin i' = H (\tan i + \tan i').$$

6. The **Catenary** is the curve in which an uniform chain hangs, when loaded with its own weight only, or with a load everywhere proportional to its own weight. (See fig. 22, page 80, and its explanation.)

Rule I.—Given, in fig. 77, the catenary A B, and its directrix O X, and the weight of an unit of length of the chain; to find the horizontal tension. Multiply the parameter O A by the weight of an unit of length of chain.

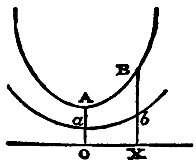


Fig. 77.

Rule II.—To find the tension at any point, B, of the chain. Multiply the height of the ordinate X B from the directrix to the given point, by the weight of an unit of length of chain.

7. A **Catenarian Rib** is of the figure of a catenary inverted, the directrix being above the curve, and the curve concave downwards. To represent it, conceive fig. 77 to be turned upside down. It is the form of equilibrium for an arched rib loaded in such a manner that the load on any arc, A B, is proportional to the *area*, O A B X, of the *spandril*, or space between the rib and its directrix.

Rule I.—Given, a catenarian rib and its directrix, and the weight of load corresponding to an unit of area of spandril; to find the horizontal thrust. Multiply the square of the parameter O A by the load per unit of area.

Rule II.—To find the thrust at any point, B, of the rib. Multiply together the parameter O A, the ordinate X B, and the load per unit of area.

A **Transformed Catenarian Rib** is a curve such as $a\,b$ in fig. 77 (still supposed to be turned upside down), which curve is so related

to the common catenary, A B, that the ordinates drawn to it from the directrix, O X, of both curves, such as Oa and Xb, bear everywhere a constant proportion to the corresponding ordinates, such as O A and X B, of the common catenary; or, in symbols,

$$\frac{y'}{y} = \frac{\mathrm{X}\,b}{\mathrm{X\,B}} = \frac{\mathrm{O}\,a}{\mathrm{O\,A}} = \frac{a}{m} = \text{constant.}$$

A transformed catenary is a form of equilibrium for an arched rib loaded in such a manner that the load on any arc, a b, is proportional to the area of spandril, O a b X.

Rule III.—Given, in a transformed catenary, the least ordinate, O $a = a$; any other ordinate, X $b = y'$; and the half-span, or distance between them, O X $= x$; to find the parameter, O A $= m$, of the corresponding common catenary. Use the following formula:

$$m = x \div \text{hyp. log.} \left(\frac{y'}{a} + \sqrt{\frac{y'^2}{a^2} - 1}\right).$$

(For hyperbolic logarithms, see page 38. For squares and square roots, see page 11.)

Rule IV.—In a transformed catenarian rib under a given load per unit of area of spandril, to find the horizontal thrust; multiply the square of the parameter A O (found by Rule III.) by the load per unit of area of spandril.

Rule V.—To find the thrust along the rib at any point, B; let H denote the horizontal thrust; P, the vertical load corresponding to the area of spandril, O a b X; T, the required thrust; then $\mathrm{T} = \sqrt{(\mathrm{H}^2 + \mathrm{P}^2)}$.

Rule VI.—To find the radius of curvature of a transformed catenary at its vertex or *crown*, a; divide the square of the parameter, O A, by the least ordinate, Oa.

(The radius of curvature of a common catenary at its vertex, A, is equal to the parameter, O A.)

Table for Catenarian Ribs.

$\frac{x}{m}$	$\frac{y}{a}$	$\frac{m\,dy}{a\,dx}$	$\frac{x}{m}$	$\frac{y}{a}$	$\frac{m\,dy}{a\,dx}$
0	1·0000	·0000	1·6	2·5774	2·3755
0·2	1·0200	·2013	1·8	3·1074	2·9421
0·4	1·0810	·4107	2·0	3·7622	3·6269
0·6	1·1854	·6366	2·2	4·5679	4·4571
0·8	1·3373	·8880	2·4	5·5569	5·4662
1·0	1·5431	1·1752	2·6	6·7690	6·6947
1·2	1·8106	1·5094	2·8	8·2526	8·1918
1·4	2·1509	1·9043	3·0	10·0676	10·0178

To interpolate the ordinate, $y \pm v$, corresponding to an intermediate half-span, $x \pm u$, when $\frac{y}{a}$ corresponds to $\frac{x}{m}$ in the table; make

$$\frac{y \pm v}{a} = \frac{y}{a}\left(1 + \frac{u^2}{2\,m^2} + \frac{u^4}{24\,m^4}\right) \pm \frac{m\,d\,y}{a\,d\,x}\left(\frac{u}{m} + \frac{u^3}{6\,m^3}\right).$$

This computation is to be performed by addition to the number next below in the table, or by subtraction from the number next above, according as the intermediate half-span lies nearer to the one next below it or to that next above it.

8. **Uniformly Pressed Hoops.**—The stress on a hoop is tension if it is pressed from within; thrust if it is pressed from without. If the pressure is uniform, of equal intensity in all directions, and normal to the hoop, the form of equilibrium of the hoop is a circle. If the pressure is compounded of two uniform pressures in directions at right-angles to each other, of different intensities, that form is an ellipse.

RULE I.—To find the stress on a circular hoop; multiply the pressure per unit of length of the hoop by the radius of the hoop.

RULE II.—To find the ratio of the greater and lesser axes of an equilibrated elliptic hoop, subjected to two uniform pressures of different intensities in directions perpendicular to each other; extract the square root of the ratio of the intensities of the pressures. The greater axis will lie in the direction of the more intense pressure.

RULE III.—To find the stress on an equilibrated elliptic hoop at the end of one of its axes; multiply half the length of that axis by the pressure per unit of length in a direction perpendicular to it.

9. A **Hydrostatic Rib** is adapted to bear a pressure which, like that of a liquid, is everywhere normal to the rib, and which, at

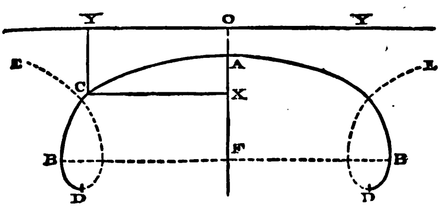


Fig. 78.

any point, C, has an intensity proportional to the depth of spandril, C Y, between the rib and its horizontal directrix, Y O Y. The radius of curvature at each point, such as C, is inversely proportional to the depth of spandril, C Y.

The total thrust at every point of a hydrostatic rib is uniform, and is equal to the load on the half-rib A B.

In what follows, the rib is supposed to spring vertically from its abutments at B, B.

RULE I.—Given, the half-span, $F\,B = c$, and the rise, $F\,A = a$, of a hydrostatic rib; to find the proper depth of load at the crown, $A\,O = h$, *approximately*.

$$\text{Make } b = c + \frac{c^2}{30\,a}; \text{ then } h = \frac{a^4}{b^3 - a^3}, \text{ nearly.}$$

RULE II.—To find the area of spandril corresponding to the uniform thrust along the rib; call that area $\frac{T}{w}$, in which T represents the thrust, and w the load per unit of area of spandril; then

$$\frac{T}{w} = \frac{a^2}{2} + a\,h.$$

RULE III.—To calculate the thrust; being also the load on the half-rib.

$$T = w\left(\frac{a^2}{2} + a\,h\right).$$

RULE IV.—To find the radius of curvature at a point where the depth of spandril, $Y\,C = x$, is given; divide the area found by Rule II. by the depth of spandril; that is to say, let r be the radius of curvature at C; then

$$r = \frac{a^2 + 2\,a\,h}{2\,x}$$

The radii of curvature at the crown, A, and springing, B, are as follows:

$$\text{At A, } r_0 = \frac{a^2 + 2\,a\,h}{2\,h}; \text{ at B, } r_1 = \frac{a^2 + 2\,a\,h}{2\,(a + h)}.$$

A sufficient number of radii having been computed, the figure of the rib may be constructed to any required degree of approximation by drawing a series of short circular arcs.

RULE V.—To draw, approximately, the figure of a hydrostatic rib with three radii only. By RULE IV., find the radii of curvature, r_0, r_1, at the crown and springing. From the crown, A, draw vertically $A\,C = r_0$; and from the springing, B, draw horizontally $B\,D = r_1$. C and D will be the centres of curvature for the crown and springing respectively.

About D, with the radius $D\,E = F\,A - B\,D$, describe a circular arc, and about C, with the radius $C\,E = C\,F$, describe another

circular arc; let E be the point of intersection of those arcs; this will be a third centre; and two more centres will be similarly situated to D and E with respect to the other half-rib.

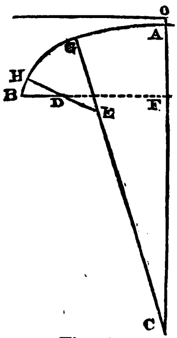


Fig. 79.

Then about C, with the radius C A, draw the circular arc A G till it cuts C E produced in G; about E, with the radius E G = F A, draw the circular arc G H till it cuts E D produced in H; about D, with the radius D B, draw the circular arc H B. This completes one half-rib, and the other is drawn in the same manner.

The curve thus drawn falls a little beyond the true hydrostatic rib at G, and a little within it at H.

10. A **Rib of any Figure**, under a *vertical load distributed in any manner*, being given, it is always possible to determine a system of horizontal pressures, which, being applied to that rib, will keep it in equilibrio.

RULE I.—To find the total horizontal pressure against the rib below a given point. In fig. 80 let C be any point in the rib, and A its crown.

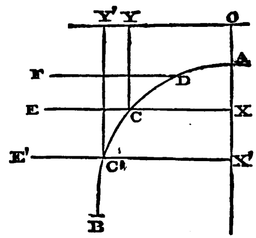


Fig. 80.

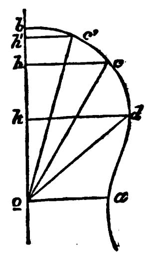


Fig. 81.

In the diagram of forces, fig. 81, draw $o\,c$ parallel to a tangent to the rib at C. Draw the vertical line $o\,b$ as a *scale of loads*, on which take $o\,h = \mathrm{P}$ to represent the vertical load supported on the arc A C. Through h draw the horizontal line $h\,c$, cutting $o\,c$ in c; then $o\,c = \mathrm{T}$ will be the thrust along the rib at C, and $h\,c = \mathrm{H}$, the horizontal component of that thrust, will be the *total horizontal pressure which must be exerted against* C B, *the part of the rib below* C.

At the crown, A, the preceding Rule fails; and the following is to be used.

RULE II.—To find the thrust at the crown of the rib; multiply the radius of curvature at the crown by the vertical load per lineal unit of span there.

RULE III.—To find the horizontal pressure required in a given layer of the spandril.

Let C′ (fig. 80) be a point in the rib a short way below C. In the diagram of forces (fig. 81) draw $o\,c'$ parallel to a tangent to the rib at C′; on the vertical scale of loads take $o\,h'$, vertical load on the arc A C′; draw the horizontal line $h'\,c'$ cutting $o\,c'$ in c'. Then $o\,c' = \mathrm{T}'$ is the thrust along the rib at C′; and $h'\,c' = \mathrm{H}'$, the horizontal component of that thrust, is the horizontal pressure which must be exerted against the part of the rib below C′. H being, as before, the horizontal component of the thrust at C, the difference H — H′ will represent the horizontal pressure required to be exerted through the horizontal layer C E E′ C′.

If H *diminishes in going downwards*, as in the example given, *pressure from without* is required through the layer. Through those layers at which H *increases in going downwards*, either *tension from without*, or *pressure from within*, is required to keep the rib in equilibrio.

RULE IV.—To find the greatest horizontal thrust, and the "point of rupture," and "angle of rupture."

Through o, in fig. 81, draw a number of radiating lines, such as $o\,c$, $o\,c'$, &c., parallel to the rib at various points, as C, C′, &c., and find, as in Rules I. and III., the lengths of those lines so as to represent the thrust along the rib at the several points C, C′, &c. The length of the horizontal line, $o\,a$, representing the thrust at the crown, is to be calculated as in Rule II. Through the points a, c, c', &c., thus found, draw a curve. Find the point, d, in that curve which is farthest from the scale of loads, $o\,b$; then the horizontal line $d\,k = \mathrm{H}_0$ will represent the maximum horizontal thrust.

Join $o\,d$, and find the point, D, in fig. 80, at which the rib is parallel to $o\,d$; this is the "point of rupture," or point at which the horizontal thrust attains a maximum; and the "angle of rupture" is the inclination of the rib at that point, or $\angle\, d\,o\,a$, in fig. 81.

The horizontal plane D F is the upper boundary of that part of the spandril which exerts the maximum horizontal pressure H_0.

SECTION III.—STABILITY OF MASONRY.

1. **Pressure of Earth and Water against Walls.**—RULE I.—The *Centre of Pressure* of a rectangular vertical plane pressed by a mass of water or of earth is at $\frac{2}{3}$ of the total depth down from the upper surface of the water or earth.

RULE II.—The *Direction of the Pressure* against a vertical plane is, for water or a bank of earth in horizontal layers, horizontal;

for a bank of earth in uniformly sloping layers, it is sensibly parallel to the slope.

RULE III.—To find the amount of the pressure of water against each foot in breadth of a vertical plane; multiply the half-square of the total depth by the heaviness of water (62·4 lbs. to the cubic foot).

For the *heaviness of earth*, see page 152.

The following is a table of natural slopes of earth; but the natural slope of earth in engineering works ought, as far as practicable, to be ascertained by observation on the spot:—

EARTH.		Angle of Repose. ϕ	Co-efficient of Friction. f	Customary designation of Natural Slope: $1 \div f$ to 1.
Dry sand, clay, and mixed earth,	from	37°	0·75	1·33 to 1
	to	21°	0·38	2·63 to 1
Damp clay,		45°	1·00	1 to 1
Wet clay,	from	17°	0·31	3·23 to 1
	to	14°	0·25	4 to 1
Shingle and gravel,	from	48°	1·11	0·9 to 1
	to	35°	0·70	1·43 to 1
Peat,	from	45°	1·0	1 to 1
	to	14°	0·25	4 to 1

The most frequent slopes of earthwork are those called $1\frac{1}{2}$ to 1, and 2 to 1, corresponding respectively to the co-efficients of friction 0·67 and 0·5, and to the angles of repose $33\frac{1}{2}°$ and $26\frac{1}{2}°$, nearly.

RULE IV.—To find the amount of the pressure of a bank of earth laid in plane parallel layers, against each foot in breadth of a vertical plane; multiply the half-square of the total depth by the heaviness of the earth; then multiply the product by a ratio found as follows:—

In fig. 82, from one point, O, draw two straight lines, O M X and O R, making with each other the angle M O R = ϕ, the angle of repose, or natural slope of the earth. About any convenient point, M, in one of those straight lines, describe a semicircle, Y R X, touching the other straight line in R. (This may be done by describing the dotted semicircle M R O, so as to find the point R.) Then

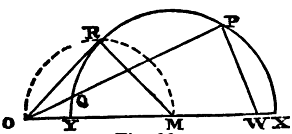


Fig. 82.

Case I.—If the bank is in horizontal layers, the required ratio is

$$\frac{\text{O Y}}{\text{O X}} \left(= \frac{1 - \sin \phi}{1 + \sin \phi} \right).$$

Case II.—If the bank is in layers sloping at the natural slope, the required ratio is

$$\frac{\mathrm{O\,R}}{\mathrm{O\,M}} \left(= \cos \phi\right).$$

Case III.—If the bank consists of layers sloping at any less angle; draw O Q P, making the angle M O P = the actual slope of the bank; from P draw P W perpendicular to O P; then the required ratio is

$$\frac{\mathrm{O\,Q}}{\mathrm{O\,W}} \left(= \cos \theta \cdot \frac{\cos \theta - \sqrt{(\cos^2 \theta - \cos^2 \phi)}}{\cos \theta + \sqrt{(\cos^2 \theta - \cos^2 \phi)}},\right.$$

$$\left.\text{in which } \theta = \angle \mathrm{M\,O\,P}\right).$$

2. Load on Ordinary Foundations.—

		Tons on the Square Foot.
First Class:	rock, moderately hard; strong as the strongest red brick, ...	9·0
„	rock of the strength of good concrete,	3·0
„	rock; very soft, ...	1·8
Second Class:	firm earth; hard clay; clean dry gravel; clean sharp sand, prevented from spreading sideways,	from 1 to 1·5

Third Class: soft or loose earth; let ϕ be the angle of repose; then;

Rule I.*—To find the least weight of earth to be displaced by the foundation of a building when the load is uniformly distributed; multiply the total load (above and below ground) by

$$\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)^2.$$

Rule II.*—When the load produces an uniformly-varying pressure, to find how far the centre of pressure may safely deviate from the centre of figure of the base of the foundation; find the centre of percussion of the base relatively to the edge where the pressure is to be least (see pages 156, 157), and multiply the distance of that centre of percussion from the centre of figure of base by

$$\frac{2 \sin \phi}{1 + \sin^2 \phi}.$$

For a *rock foundation* the value of this multiplier is 1.

Rule III.*—In the case referred to in Rule II.*, to find the least weight of earth to be displaced by the foundation; multiply the total load by

$$\frac{(1 - \sin \phi)^2}{1 + \sin^2 \phi}.$$

RULE IV.*—In the case referred to in Rule I.*, and when the *load above ground* alone is given; to find the least weight of earth to be displaced by the foundation; let w be the heaviness of the earth, and w_1 the mean heaviness of the materials with which the excavation is to be filled (including voids, if any); then *divide* the load above ground by

$$\left(\frac{1 + \sin \varphi}{1 - \sin \varphi}\right)^2 - \frac{w_1}{w}.$$

RULE V.—To find the depth of a foundation; divide the weight of earth to be displaced by the heaviness of that earth and the area of base.

Least depth to escape injurious effects of frost = from 3 feet to 6 feet according to climate.

TABLE OF FUNCTIONS OF ANGLES OF REPOSE.

φ	15°	20°	25°	30°	35°	40°	45°
$\frac{1 + \sin \varphi}{1 - \sin \varphi}$	1·700	2·039	2·464	3·000	3·690	4·599	5·826
$\frac{1 - \sin \varphi}{1 + \sin \varphi}$	0·588	0·490	0·406	0·333	0·271	0·217	0·172
$\left(\frac{1 + \sin \varphi}{1 - \sin \varphi}\right)^2$	2·890	4·159	6·070	9·000	13·619	21·152	33·94
$\left(\frac{1 - \sin \varphi}{1 + \sin \varphi}\right)^2$	0·346	0·240	0·165	0·111	0·073	0·047	0·0295
$\frac{1 + \sin^2 \varphi}{(1 - \sin \varphi)^2}$	1·945	2·579	3·535	5·000	7·310	11·076	17·47
$\frac{(1 - \sin \varphi)^2}{1 + \sin^2 \varphi}$	0·514	0·387	0·283	0·200	0·137	0·090	0·057
$\frac{2 \sin \varphi}{1 + \sin^2 \varphi}$	0·486	0·612	0·717	0·800	0·863	0·910	0·943
$\tan \varphi$	0·268	0·364	0·466	0·577	0·700	0·839	1·000
$\cot \varphi$	3·732	2·747	2·145	1·732	1·428	1·192	1·000

3. **Load on Piled Foundations.**—Ordinary working loads on the heads of piles:—On piles driven till they reach firm ground, 0·45 ton on the square inch; on piles standing in soft ground, by friction, 0·09; ordinary values of greatest load which piles will bear without sinking further, from 0·9 to 1·35 tons on the square inch area of head

The following are rules applicable to pile-driving:—

Let P be the greatest load which a pile is to bear without sinking farther (in tons);
W, the weight of the ram used for driving it (in tons);
h, the height from which the ram falls (in feet);
l, the length of the pile (in feet);
x, the depth it is driven by the last blow (in fractions of a foot);
S, its sectional area (in square inches);
E, its modulus of elasticity.

(Approximate values of E in tons on the square inch—elm, 400 to 600; alder, about 500; beech, about 600; sycamore, about 500; teak and saul, about 1,000; greenheart, 500 to 600.)

RULE VI.—Given, all the above quantities except x; then

$$x = \frac{\mathrm{W}h}{\mathrm{P}} - \frac{\mathrm{P}l}{4\,\mathrm{ES}}.$$

The pile must be driven until the additional depth gained by each blow, of the energy W h, becomes not greater than x, as given by the above rule.

RULE VII.—Given, all the above quantities except W h, the energy required for the final blow; then

$$\mathrm{W}h = \frac{\mathrm{P}^2 l}{4\,\mathrm{ES}} + \mathrm{P}x.$$

RULE VIII.—Given, all the above quantities except P; then

$$\mathrm{P} = \sqrt{\left(\frac{4\,\mathrm{ESW}h}{l} + \frac{4\,\mathrm{E}^2\mathrm{S}^2x^2}{l^2}\right)} - \frac{2\,\mathrm{ES}x}{l}.$$

4. The **Load Supported by a Screw Pile** in practice ranges from 3 times to 7 times the weight of the earth which lies directly above the screw-blade.

5. **Horizontal Resistance of Earth.**—Let R denote the resistance opposed by a stratum of earth to the pushing or dragging of a rectangular plane surface through it horizontally; w, the heaviness of the earth; ϕ, its angle of repose; b, the breadth of the surface; x, the depth to which its lower edge is buried; x', the depth to which its upper edge is buried; x_0, the depth of the resultant of the resistance below the upper surface of the earth.

RULE IX.—To find the resistance;

$$\mathrm{R} = \frac{4\,w \sin \phi}{\cos^2 \phi} \cdot \frac{x^2 - x'^2}{2}$$

RULE X.—To find the position of the resultant;

$$x_0 = \frac{2\,(x^3 - x'^3)}{3\,(x^2 - x'^2)}.$$

6. **Pressure of Wind.**—RULE XI.—To estimate the greatest probable amount of the pressure of wind against a chimney or tower; if the edifice is square, take the area of its vertical cross-section; or if round, take half that area; and multiply by the greatest known pressure of the wind in the neighbourhood against an unit of area of a vertical plane surface, as measured by the anemometer. (In Britain that pressure is about 55 lbs. on the square foot.)

RULE XII.—To find the position of the resultant of that pressure; find the centre of magnitude of the vertical cross-section. (See page 83.) If the edifice is pyramidal or conical, divide the difference of the outside diameter at the base and top by 3 times their sum; subtract the quotient from 1; multiply the remainder by half the height of the edifice; the product will be the height of the resultant pressure above the base.

RULE XIII.—To find the moment of the pressure of the wind; multiply its amount by the height of its resultant above the base.

The calculations described in the above rules should be made not only for the whole chimney or tower from the base upwards, but for the part above each bed-joint where the thickness of the masonry or brickwork diminishes.

7. **Stability of Abutments** (Including buttresses, abutments and piers of arches, retaining and reservoir walls.)

RULE XIV.—To find the greatest deviation of the centre of pressure from the centre of figure at any bed-joint, consistently with *stability of position* (that is, safety against overturning). This may be called the *limiting position* of the centre of pressure.

Case I. *Abutments and Piers of Arches.*—Take as an axis the edge of the bed-joint in question from which the centre of pressure is to deviate farthest; the required position of the centre of pressure will be the centre of percussion of the bed-joint corresponding to that axis. (See pages 156, 157.) The rules and table in those pages give the distance of the centre of pressure from the farthest edge of the bed-joint, from which subtracting the distance from that edge to the centre of figure of the bed-joint (usually half the whole thickness of the abutment), there remains the deviation required.

Case II. *Retaining Walls.*—Greatest deviation of the centre of pressure from the centre of figure, as fixed by practical experience = from 0·3 to 0·375 of the whole thickness of the wall at the given bed-joint.

RULE XV.—Given, the load on a bed-joint and the position of the centre of pressure; to find approximately the intensity of the pressure at the edge to which the centre of pressure is nearest; in

Case I. of Rule I. divide *twice* the load by the area of the bed; in Case II. multiply the breadth of the bed by *once-and-a-half* the distance of the centre of pressure from the nearest edge of the bed, and with the product as a divisor, divide the load; the quotient will be the required intensity.

The intensity of pressure thus found ought not to exceed *one-eighth* of the pressure which crushes the material of the building.

RULE XVI.—To calculate the *moment of stability of an abutment* at a given bed-joint; multiply the weight of the mass of material above the bed-joint by the horizontal distance of a vertical line, through the centre of gravity of that mass, from the limiting position of the centre of pressure of the bed-joint.

RULE XVII.—To find the proper thickness for an abutment with a rectangular horizontal base from the following data:—

- H, the horizontal component, and V, the vertical component, of the thrust to be resisted;
- x', the vertical height of the line of action of that thrust above the backward edge of the base of the abutment.
- b, the breadth of the abutment;
- h, its height;
- w, the heaviness of its material;
- n, the proportion which its bulk bears to that of the circumscribed rectangle; so that if t be its thickness at the base, $n\,w\,b\,h\,t$ is its weight;
- q, the ratio which the deviation of the centre of pressure from the centre of figure of the base is to bear to the thickness at the base. (See Rule XIV.)
- r, the ratio which the horizontal deviation of the centre of gravity of the abutment from the centre of figure of its base is to bear to the thickness at the base;

$$\text{make } \frac{H\,x'}{n\,(q \pm r)\,w\,h\,b} = A; \ \frac{(q + \frac{1}{2})\,V}{2\,n\,(q \pm r)\,w\,h\,b} = B;$$

using $q + r$ if q and r represent deviations in contrary directions, and $q - r$ if they represent deviations in the same direction; then the required thickness is

$$t = \sqrt{(A + B^2)} - B.$$

If the thrust to be resisted is wholly horizontal, $t = \sqrt{A}$ simply. In a vertical solid rectangular abutment $n = 1$ and $r = 0$.

RULE XVIII.—To find the direction of the resultant pressure at any bed-joint; let $W = n\,w\,b\,h\,t$ represent the weight of material in the abutment above that joint; then $\frac{H}{W + V}$ is the *tangent* of the angle made by that resultant with the vertical. In order that

the abutment may possess *stability of friction* (that is, be safe against giving way by the sliding of one course of masonry upon another), the normal to each bed-joint ought not to make a greater angle with the direction of the resultant pressure at that joint than the angle of repose of fresh masonry; that is, from about 25° to 36°. Should horizontal bed-joints prove too oblique to the pressure, sloping bed-joints may be substituted for them.

REMARK.—In an abutment which has to resist a thrust concentrated near one point, the risk of overturning is greatest at the base; but the risk of giving way by sliding is greatest at the bed-joint next below the place of application of the thrust; and it is to the latter joint, therefore, that Rule XVIII. is to be applied.

RULE XIX.—To find the proper thickness for a *vertical rectangular retaining wall*, of a height equal to that of the bank which presses it.

In each case let w' be the heaviness of the earth, ϕ its angle of repose, and let $\frac{p'}{p}$ be the ratio of the pressure exerted edgewise by the layers of earth to their vertical pressure, as found in Rule IV. of this section. Also, let h be the height of the wall, w, its heaviness, and q, ratio of the intended deviation of the centre of pressure from the centre of the base to the required thickness t.

Case I.—Bank in horizontal layers; $\frac{p'}{p} = \frac{1 - \sin \phi}{1 + \sin \phi}$;

$$\frac{t}{h} = \sqrt{\left(\frac{w' p'}{6\, q\, w\, p}\right)}.$$

$$\text{Let } q = \frac{3}{8}; \text{ then } \frac{t}{h} = \frac{2}{3} \cdot \sqrt{\left(\frac{w' p'}{w\, p}\right)}.$$

(*For a reservoir-wall*, make $w' = 62{\cdot}4$ lbs. per cubic foot; and $\frac{p'}{p} = 1$).

Case II.—Bank in layers of indefinite extent, at the natural slope ϕ; $\frac{p'}{p} = 1$.

$$\text{Make } \frac{w' \cos^2 \phi}{6\, q\, w} = a; \ \frac{(q + \frac{1}{2})\, w' \cos \phi \sin \phi}{4\, q\, w} = b; \text{ then}$$

$$\frac{t}{h} = \sqrt{a + b^2} - b.$$

Case III.—Bank in layers of indefinite extent, sloping at any angle θ less than ϕ. Find $\frac{p'}{p}$ by Rule IV. Then make

$$\frac{w'}{6\,q\,w} \cdot \frac{p'}{p} \cdot \cos^2 \theta = a; \; \frac{w'}{4\,q\,w}\left(q + \tfrac{1}{2}\right)\frac{p'}{p} \cos\theta \sin\theta = b; \text{ and}$$

$$\frac{t}{h} = \sqrt{a + b^2} - b.$$

Case IV.—*Surcharged Wall.*—Bank rises from wall at natural slope up to height c above top of wall, or $c + h$ above base; and at that height has a horizontal upper surface. Let the thickness, calculated as in Case I., be t; the thickness, calculated as in Case II., t'; and the required thickness, t''. Then

$$t'' = \frac{h\,t + 2\,c\,t'}{h + 2\,c}.$$

The *strength* of a retaining wall at its base should be tested by Rule XV. of this section, and the *stability of friction* by Rule XVIII.; and if the latter is found to be insufficient with horizontal beds, the beds may be sloped back; and then the back of the wall should be formed into steps, with the rise perpendicular to the beds.

Rule XX.—Having designed a vertical rectangular retaining wall, to *modify its figure* without diminishing its stability of position.

The face of the wall may be either battered, stepped, or panelled, so long as the centre of gravity of the part taken away does not fall behind a vertical line through the limiting position of the centre of pressure of the base. When the face has a straight or curved batter, the beds of the masonry or brickwork may be laid perpendicular to the battered face.

The masonry at the back of the wall may be diminished by steps, provided its place is filled with material of equal weight.

Rule XXI.—For retaining walls of uniform thickness which *lean* or *overhang* backwards, let r be the ratio which the backward deviation of the centre of gravity from that of an upright wall is to bear to the thickness; then put $q + r$ instead of q in the *denominators* of the expressions in Rule XIX., and they will become applicable, without material error, to the present case. The beds ought to be built perpendicular to the face.

Rule XXII.—Given, the dimensions of a wall with counterforts; to find the thickness of a plain wall of equal stability. Let t be the thickness and b the breadth between a pair of counterforts; c, the breadth of a counterfort, and T, the thickness of wall and counterfort together. Then the thickness of the plain wall of equal stability is nearly =

$$\sqrt{\left(\frac{b\,t^2 + c\,T^2}{b + c}\right)}.$$

8. **Stone and Brick Arches.**—Rule XXIII.—To find the *least proper thickness for the arch-ring* of a proposed arch; find the longest radius of curvature of the arch; then take a mean proportional between (that is, the square root of the product of) that radius and a constant whose values are as follows:—

	Foot.
For an arch above ground, standing solitary between its abutments,	0·12
For an arch forming one of a series of arches, with piers between them,	0·17
For an underground archway in hard material (such as rock or conglomerate),	0·12
For an underground archway in gravel or firm earth,	0·27
For an underground archway in wet clay or quicksand,	0·48

Rule XXIII a.—To find the level up to which the *backing* of the arch should be built before the centre is struck; take a mean proportional between the radius of curvature of the *intrados* (or inner profile) of the arch at its crown, and the thickness of the arch-ring; then lay off the length so calculated vertically downwards from the crown of the *outer surface* of the arch-ring.

Rule XXIV.—For a *rough approximation to the horizontal thrust* of an arch, take the weight of the vertical load that is supported between the crown of the arch and that point in the arch-ring where its inclination to the horizon is 45°.

Rule XXV.—To find a nearer approximation to the horizontal thrust of an arch, and also to determine whether a proposed arch will have sufficient stability.

Assume that the load is supported by a linear *rib* coinciding with the centre line of the arch-ring, and treat that rib by the method of Article 10 of the preceding section, page 178, so as to find its maximum horizontal thrust; this will be nearly equal to the horizontal thrust of the proposed arch. As to stability, the following cases may be distinguished:—

Case I.—If the supposed rib is either equilibrated under the vertical load alone, or requires horizontal pressure from without alone to give it equilibrium, the proposed arch will be stable throughout.

Case II.—If the supposed rib requires horizontal pressure from without up to a certain *point of rupture* only, and above that point requires horizontal tension to give it equilibrium, the actual arch is stable up to the point of rupture, but above that point it may be stable or unstable; and its stability must be further tested as follows:—

In fig. 83 let B C A represent one-half of a symmetrical arch; K L D E, an abutment, and C, the *joint of rupture*, drawn perpendicular to the assumed rib at the point of rupture. At A, the crown of the arch, suppose a vertical joint.

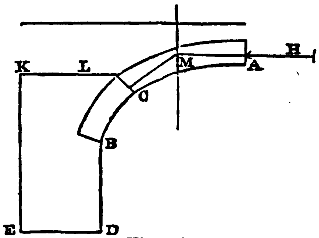


Fig. 83.

Find the centre of gravity of the load between the joint of rupture, C, and the crown, A; and draw through that centre of gravity a vertical line.

Then, if it be possible, from one point, such as M, in that vertical line, to draw a pair of lines, one parallel to a tangent to the assumed rib at the point of rupture, and the other horizontal, so that the former of those lines shall cut the joint of rupture, and the latter the supposed vertical joint at the crown, in a pair of points which are both within the middle third of the thickness of the arch-ring, the stability of the arch will be secure.

Should it be impossible to make the pair of points fall within the middle third of the arch-ring, its thickness must be increased.

RULE XXVI.—To adapt *Transformed Catenarian* curves to the figure of an arch of masonry. (See Article 7 of the preceding section, page 174.) For the *intrados* (or inner profile) of the arch, and the *extrados* (or outer profile) of the arch with its solid backing, take two transformed catenarian curves with the same directrix and parameter. For the extrados of the whole load (being usually the profile of the platform or roadway), take either the horizontal directrix itself, or a third and flatter transformed catenary with the same directrix and parameter. To find approximately the *horizontal thrust;* multiply the square of the parameter by the mean load per square foot area of spandril (allowing for the voids, if any, between the spandril walls); and then multiply the product by the ratio in which the depth from the platform to the crown of the intrados is greater than the depth from the directrix to the middle of the depth of the keystone.

RULE XXVII.—To adapt the figure of the *hydrostatic rib* to an arch of masonry. (See Article 9 of the preceding section, page 177). For the intrados take the figure of the hydrostatic rib, and make the arch-stones of an uniform thickness, determined from the radius of curvature at the crown by Rule XXIII. of this Article. The thrust will be nearly the same as in a supposed linear rib coinciding with the intrados, and under the same load.

RULE XXVIII.—To find the resultant horizontal thrust against a pier that stands between two equal arches, when one is loaded

with a travelling load in addition to its own weight, and the other with its own weight only; multiply the travelling load per unit of span by the radius of curvature of the centre line of the arch-ring at its crown.

RULE XXIX.—To represent approximately the amount and distribution of the load upon any part of the *centre* (or temporary framing) which supports an arch in progress of construction.

Case I. *Circular Arch.*—In fig. 84 let O A be the radius of the intrados, and A B a circular quadrant of which the intrados forms the whole or part. Conceive that the *half* of the radius A O represents the weight of the arch-ring per foot of intrados.

Let C be the point up to which the arch-ring has been built; and let it be required to find the amount and distribution of the load on the part C D of the centre.

From C draw C E || A O; bisect C E in F, from which draw F H || O B; draw D G || A O; then will D G represent the normal pressure on each lineal foot of the outer surface of the centre at the point D; and the shaded area, C D G F, will represent the vertical component of the load on the centre between C and D, both in amount and in distribution.

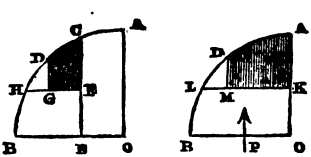


Fig. 84. Fig. 85.

The point H is that below which the arch-stones cease to press on the rib, when the arch has been built up to the point C.

The case in which the rib is completely loaded, the arch being finished all but the keystone, is represented by fig. 85. Bisect the vertical radius A O in K, and conceive A K to represent the weight per foot of intrados; draw K L || O B; L will be a point below which the stones do not press on the rib (supposing the arch to extend so far). Let D be any point in the intrados; draw D M || A O; then D M represents the normal pressure on the centre per foot of intrados at D, and the shaded area M D A K represents the vertical component of the load on the centre between A and D.

Case II. *Non-circular Arch.*—Find the two points at which the intrados is inclined 60° to the horizon; conceive a circular arc drawn through them and through the crown of the intrados, and proceed as in Case I.

PART VI.

TABLES AND RULES RELATING TO THE STRENGTH OF MATERIALS.

Section I.—Tables.

Table I a.—Tenacity of Wrought Iron and Steel.

Description of Material.	Tenacity in lbs. per Square Inch. Lengthwise.		Crosswise.		Ultimate Extension.	
Malleable Iron.						
Wire—very strong, charcoal, ...	114,000	Mo.				
Wire—average,..........	86,000	T.				
Wire—weak, ...	71,000	Mo.				
Yorkshire (Lowmoor),...	64,200	F.	52,490	F.		
„ from	66,390	N.			0·20	
to	60,075				0·26	
Yorkshire (Lowmoor) and Staffordshire rivet iron,...........	59,740	F.			0·2 to 0·25	
Charcoal bar,..............	63,620	F.			0·2	
Staffordshire bar,...from	62,231	N.			·302	
to	56,715				·186	
Yorkshire bridge iron,...	49,930	F.	43,940	F.	·04;	·029
Staffordshire bridge iron,	47,600	F.	44,385		·04;	·036
Lanarkshire bar,... from	64,795	N.			·158	
to	51,327				·238	
Lancashire bar,......from	60,110	N.			·169	
to	53,775				·216	
Swedish bar,.........from	48,933	N.			·264	
to	41,251				·278	
Russian bar,.........from	59,096	N.			·153	
to	49,564				·133	
Bushelled iron from turnings,............	55,878	N.			·166	
Hammered scrap,........	53,420	N.			·248	
Angle-iron from various districts, from	61,260	N.				
to	50,056					

TABLE—*continued.*

Description of Material.	Tenacity in lbs. per Square Inch. Lengthwise.		Crosswise.		Ultimate Extension.
Straps from various districts, ... from	55,937	N.			·108
to	41,386				·048
Bessemer's iron, cast ingot, ...	41,242	W.			
Bessemer's iron, hammered or rolled, ...	72,643	W.			
Bessemer's iron, boiler plate, ...	68,319	W.			
Yorkshire plates, ...from	58,487	N.	55,033	N.	·109; ·059
to	52,000		46,221		·170; ·113
Staffordshire plates, from	56,996	N.	51,251	N.	·04; ·034
to	46,404		44,764		·13; ·059
Staffordshire plates, best-best, charcoal,	45,010	F.	41,420	F.	·05; ·045
Staffordshire plates, best-best, from	59,820	F.	54,820	F.	·05; ·038
to	49,945	F.	46,470	F.	·067; ·04
Staffordshire plates, best,	61,280	F.	53,820	F.	·077; ·045
Staffordshire plates, common, ...	50,820	F.	52,825	F.	·05; ·043
Lancashire plates, ...	48,865	F.	45,015	F.	·043; ·028
Lanarkshire plates, from	53,849	N.	48,848	N.	·033; ·014
to	43,433		39,544		·093; ·046
Durham plates, ...	51,245	N.	46,712		·089; ·064

Effects of Reheating and Rolling.

Puddled bar, ...	43,904	C.
The same iron five times piled, reheated and rolled, ...	61,824	C.
The same iron eleven times piled, reheated and rolled, ...	43,904	C.

Strength of Large Forgings.

Bars cut out of large forgings, from	47,582	N.	44,578	·231; ·168
to	43,759		36,824	·205; ·064
Bars cut out of large forgings, ...	33,600	M.		

TABLE—*continued.*

Description of Material.		Tenacity in lbs. per Square Inch. Lengthwise.		Crosswise.		Ultimate Extension.
STEEL AND STEELY IRON.						
Cast steel bars, rolled and forged,	from	132,909	N.			·052
	to	92,015				·153
Cast steel bars, rolled and forged,..........		130,000	R.			
Blistered steel bars, rolled and forged...,		104,298	N.			·097
Shear steel bars, rolled and forged,..........		118,468	N.			·135
Bessemer's steel bars, rolled and forged,...		111,460	N.			·055
Bessemer's steel bars, cast ingots,..........		63,024	W.			
Bessemer's steel bars, hammered or rolled,		152,912	W.			
Spring steel bars, hammered or rolled,....		72,529	N.			·180
Homogeneous metal bars, rolled,.........		90,647	N.			·137
Homogeneous metal bars, rolled,.........		93,000	F.			
Homogeneous metal bars, forged,.........		89,724	N.			·119
Puddled steel bars, rolled and forged,..........	from	71,484	N.			·191
	to	62,768				·091
Puddled steel bars, rolled and forged,...		90,000	F.			
Puddled steel bars, rolled and forged,...		94,752	M.			
Mushet's gun-metal,.....		103,400	F.			0·034
Cast steel plates,....	from	96,289	N.	97,308	N.	·057; ·096
	to	75,594		69,082		·198; ·196
Cast steel plates,...	hard,	102,900	F.			·031
	soft,	85,400				·031
Homogeneous metal plates, first quality,		96,280	N.	97,150	N.	·086; ·144
Homogeneous metal plates, second quality,		72,408		73,580		·059; ·032
Puddled steel plates,............	from	102,593	N.	85,365	N.	·028; ·013
	to	71,532		67,686		·082; ·057
Puddled steel plates,....		93,600	F.			0·125

TABLE—*continued.*

Description of Material.	Tenacity in lbs. per Square Inch. Lengthwise.	Crosswise.	Ultimate Extension.
Coleford Gun-metal.			
Weakest, ...	108,970	F.	·190
Strongest, ...	160,540	F.	·030
Mean of ten sorts, ...	137,340	F.	·072

In the preceding table the following abbreviations are used for the names of authorities:—

C., Clay; F., Fairbairn; H., Hodgkinson; M., Mallet; Mo., Morin; N.,* Napier & Sons; R., Rennie; T., Telford; W., Wilmot.

The column headed "Ultimate Extension" gives the ratio of the elongation of the piece, at the instant of breaking, to its original length. It furnishes an index (but a somewhat vague one) to the ductility of the metal, and its consequent safety as a material for resisting shocks.

When two numbers separated by a semicolon appear in the column of ultimate extension (thus ·082; ·057), the first denotes the ultimate extension lengthwise, and the second crosswise.

TABLE I B.—RESILIENCE OF IRON AND STEEL.

Metal under Tension.	Ultimate Tenacity.	Working Tenacity.	Modulus of Elasticity.	Modulus of Resilience.
Cast iron—Weak, ...	13,400	4,467	14,000,000	1·425
" Average, ...	16,500	5,500	17,000,000	1·78
" Strong, ...	29,000	9,667	22,900,000	4·08
Bar iron—Good average, ..	60,000	20,000	29,000,000	13·79
Plate iron—Good average,	50,000	16,667	24,000,000?	11·57?
Iron wire—Good average,	90,000	30,000	25,300,000	35·57
Steel—Soft, ...	90,000	30,000	29,000,000	31·03
" Hard, ...	132,000	44,000	42,000,000	46·10

In the above Table of Resilience the working tenacity is for a "dead" or steady load. The modulus of resilience is calculated by dividing the square of that working tenacity by the modulus of elasticity.

* The experiments whose extreme results are marked N. were conducted for Messrs. R. Napier & Sons by Mr. Kirkaldy. For details, see *Transactions of the Institution of Engineers in Scotland*, 1858-59; also Kirkaldy *On the Strength of Iron and Steel.*

I.

Table of the Resistance of Materials to Stretching and Tearing by a Direct Pull, *in pounds avoirdupois per square inch.*

Materials.	Tenacity, or Resistance to Tearing.	Modulus of Elasticity, or Resistance to Stretching.
Stones, Natural and Artificial:		
Brick, Cement,	280 to 300	
Glass,	9,400	8,000,000
Slate,	9,600 to 12,800	13,000,000 to 16,000,000
Mortar, ordinary,	50	
Metals:		
Brass, cast,	18,000	9,170,000
„ wire,	49,000	14,230,000
Bronze or Gun Metal (Copper 8, Tin 1),	36,000	9,900,000
Copper, cast,	19,000	
„ sheet,	30,000	
„ bolts,	36,000	
„ wire,	60,000	17,000,000
Iron, cast, various qualities,	13,400 to 29,000	14,000,000 to 22,900,000
„ average,	16,500	17,000,000
Iron, wrought, plates,	51,000	
„ joints, double rivetted,	35,700	
„ „ single rivetted,	28,600	
„ bars and bolts,	60,000 to 70,000	29,000,000
„ hoop, best-best,	64,000	
„ wire,	70,000 to 100,000	25,300,000
„ wire-ropes,	90,000	15,000,000
Lead, sheet,	3,300	720,000
Steel bars,	100,000 to 130,000	29,000,000 to 42,000,000
Steel plates, average,	80,000	
Tin, cast,	4,600	
Zinc,	7,000 to 8,000	

Materials.	Tenacity, or Resistance to Tearing.	Modulus of Elasticity, or Resistance to Stretching.
Timber and other Organic Fibre:		
Acacia, false. See "Locust."		
Ash (*Fraxinus excelsior*),	17,000	1,600,000
Bamboo (*Bambusa arundinacea*),	6,300	
Beech (*Fagus sylvatica*),	11,500	1,350,000
Birch (*Betula alba*),	15,000	1,645,000
Box (*Buxus sempervirens*),	20,000	
Cedar of Lebanon (*Cedrus Libani*),	11,400	486,000
Chestnut (*Castanea Vesca*),	10,000 to 13,000	1,140,000
Elm (*Ulmus campestris*),	14,000	700,000 to 1,340,000
Fir: Red Pine (*Pinus sylvestris*),	12,000 to 14,000	1,460,000 to 1,900,000
„ Spruce (*Abies excelsa*),	12,400	1,400,000 to 1,800,000
„ Larch (*Larix Europœa*),	9,000 to 10,000	900,000 to 1,360,000
Hawthorn (*Cratægus Oxyacantha*),	10,500	
Hazel (*Corylus Avellana*),	18,000	
Hempen Cables,	5,600	
Holly (*Ilex Aquifolium*),	16,000	
Hornbeam (*Carpinus Betulus*),	20,000	
Laburnum (*Cytisus Laburnum*),	10,500	
Lancewood (*Guatteria virgata*),	23,400	
Lignum-Vitæ (*Guaiacum officinale*),	11,800	
Locust (*Robinia Pseudo-Acacia*),	16,000	
Mahogany (*Swietenia Mahagoni*),	8,000 to 21,800	1,255,000
Maple (*Acer campestris*),	10,600	
Oak, European (*Quercus sessiliflora* and *Quercus pedunculata*),	10,000 to 19,800	1,200,000 to 1,750,000
„ American Red (*Quercus rubra*),	10,250	2,150,000
Saul (*Shorea robusta*),	10,000	2,420,000
Sycamore (*Acer Pseudo-Platanus*),	13,000	1,040,000
Teak, Indian (*Tectona grandis*),	15,000	2,400,000
„ African, (?)	21,000	2,300,000
Whalebone,	7,700	
Yew (*Taxus baccata*),	8,000	

II.

Table of the Resistance of Materials to Shearing and Distortion, *in pounds avoirdupois per square inch.*

Materials.	Resistance to Shearing.	Transverse Elasticity, or Resistance to Distortion.
Metals:		
Brass, wire-drawn,		5,330,000
Copper,		6,200,000
Iron, cast,	27,700	2,850,000
" wrought,	50,000	8,500,000 to 9,500,000
Timber:		
Fir: Red Pine,	500 to 800	62,000 to 116,000
" Spruce,	600	......
" Larch,	970 to 1,700	......
Oak,	2,300	82,000
Ash and Elm,	1,400	76,000

III.

Table of the Resistance of Materials to Crushing by a Direct Thrust, *in pounds avoirdupois per square inch.*

Materials.	Resistance to Crushing.
Stones, Natural and Artificial:	
Brick, weak red,	550 to 800
" strong red,	1,100
" fire,	1,700
Chalk,	330
Granite,	5,500 to 11,000
Limestone, marble,	5,500
" granular,	4,000 to 4,500
Sandstone, strong,	5,500
" ordinary,	3,300 to 4,400
" weak,	2,200
Rubble masonry, about four-tenths of cut stone.	
Metals:	
Brass, cast,	10,300
Iron, cast, various qualities,	82,000 to 145,000
" " average,	112,000
" wrought,	about 36,000 to 40,000

Materials.	Resistance to Crushing.
Timber,* Dry, crushed along the grain:	
Ash,	9,000
Beech,	9,360
Birch,	6,400
Blue-Gum (*Eucalyptus Globulus*),	8,800
Box,	10,300
Bullet-tree (*Achras Sideroxylon*),	14,000
Cabacalli,	9,900
Cedar of Lebanon,	5,860
Ebony, West Indian (*Brya Ebenus*),	19,000
Elm,	10,300
Fir: Red Pine,	5,375 to 6,200
„ American Yellow Pine (*Pinus variabilis*),	5,400
„ Larch,	5,570
Hornbeam,	7,300
Lignum-Vitæ,	9,900
Mahogany,	8,200
Mora (*Mora excelsa*),	9,900
Oak, British,	10,000
„ Dantzic,	7,700
„ American Red,	6,000
Teak, Indian,	12,000
Water-Gum (*Tristania nerifolia*),	11,000

IV.

Table of the Resistance of Materials to Breaking Across, *in pounds avoirdupois per square inch.*

Materials.	Resistance to Breaking, or Modulus of Rupture.†
Stones:	
Sandstone,	1,100 to 2,360
Slate,	5,000

* The resistances stated are for *dry* timber. Green timber is much weaker, having sometimes only half the strength of dry timber against crushing.

† The modulus of rupture is eighteen times the load which is required to break a bar of one inch square, supported at two points one foot apart, and loaded in the middle between the points of support.

MATERIALS.	Resistance to Breaking, or Modulus of Rupture.
METALS:	
Iron, cast, open-work beams, average,	17,000
„ „ solid rectangular bars, var. qualities,	33,000 to 43,500
„ „ „ „ average,	40,000
„ wrought, plate beams,	42,000
TIMBER:	
Ash,	12,000 to 14,000
Beech,	9,000 to 12,000
Birch,	11,700
Blue-Gum,	16,000 to 20,000
Bullet-tree,	15,900 to 22,000
Cabacalli,	15,000 to 16,000
Cedar of Lebanon,	7,400
Chestnut,	10,660
Cowrie (*Dammara australis*),	11,000
Ebony, West Indian,	27,000
Elm,	6,000 to 9,700
Fir: Red Pine,	7,100 to 9,540
„ Spruce,	9,900 to 12,300
„ Larch,	5,000 to 10,000
Greenheart (*Nectandra Rodiæi*),	16,500 to 27,500
Lancewood,	17,350
Lignum-Vitæ,	12,000
Locust,	11,200
Mahogany, Honduras,	11,500
„ Spanish,	7,600
Mora,	22,000
Oak, British and Russian,	10,000 to 13,600
„ Dantzic,	8,700
„ American Red,	10,600
Poon,	13,300
Saul,	16,300 to 20,700
Sycamore,	9,600
Teak, Indian,	12,000 to 19,000
„ African,	14,980
Tonka (*Dipteryx odorata*),	22,000
Water-Gum,	17,460
Willow (*Salix*, various species),	6,600

V.—Supplementary Tables for Wrought Iron and Steel.

Mean results of experiments by W. H. Barlow, Esq., F.R.S.:—

	Tenacity. Lbs. on the Square Inch.	Proof Strength, Transversely Loaded. Lbs. on the Square Inch.	Modulus of Elasticity under Transverse Load. Lbs. on the Square Inch.
Puddled steel, specimen I.,...	95,233	—	—
" specimen II.,...	116,336	62,500	22,964,000
" " cast in ingots, ...	101,753	—	—
Puddled steel, specimen III.,	—	60,000	20,544,000
" specimen IV.,	—	63,750	24,802,000
" specimen V.,..	—	52,500	22,846,400
Homogeneous metal,.........	100,994	57,500	23,833,600
Steely iron,.....................	69,456	52,500	22,846,400

Weight of a cubic foot of puddled steel, 485·5 lbs.; of steely iron, 483·6 lbs. (See the *Engineer* of 3d January, 1862.)

Strength of Cold-rolled Iron.—The following results were obtained in some experiments by Mr. Fairbairn on the tenacity of iron. (See *Manchester Transactions*, 10th December, 1861.)

	Tenacity. Lbs. per Square Inch.	Ultimate Extension.
Black bar,.........................	58,627	·200
Same bar iron, turned,............	60,747	·220
Same bar iron, cold-rolled,.......	88,229	·079
Cold-rolled plate, ...	114,912	

Mean results of experiments by M. Tresca on bars cut out of cast steel boiler plates.

	Tenacity. Lbs. on the Square Inch.	Limit of Elasticity. Lbs. on the Square Inch.	Modulus of Elasticity.—Lbs. on the Square Inch.
Hard steel, untempered,...	74,300	36,000	29,500,000
" tempered,......	103,000 ?	71,900 ?	27,300,000
Soft steel, untempered,...	81,700	34,100	24,500,000
" tempered,......	121,700	105,800	28,300,000

The column headed "limit of elasticity" gives the tension up to which the elongation was sensibly proportional to the load. The results marked (?) are doubtful, because of discrepancies amongst the experiments of which they are the means.

VI.—Supplementary Table for Cast Iron.

Kinds of Iron.		Direct Tenacity.	Resistance to Direct Crushing.	Modulus of Rupture of Square Bars.	Modulus of Elasticity.
No. 1. Cold blast,......	from	12,694	56,455	36,693	14,000,000
	to	17,466	80,561	39,771	15,380,000
No. 1. Hot blast, ...	from	13,434	72,193	29,889	11,539,000
	to	16,125	88,741	35,316	15,510,000
No. 2. Cold blast,......	from	13,348	68,532	33,453	12,586,000
	to	18,855	102,408	39,609	17,036,000
No. 2. Hot blast, ...	from	13,505	82,734	28,917	12,259,000
	to	17,807	102,030	38,394	16,301,000
No. 3. Cold blast,......	from	14,200	76,900	35,881	14,281,000
	to	15,508	115,400	47,061	22,908,000
No. 3. Hot blast, ...	from	15,278	101,831	35,640	15,852,000
	to	23,468	104,881	43,497	22,733,000
No. 4. Smelted by coke without sulphur,.........		—	—	41,715	—
Toughened cast iron,	from	23,461	129,876	—	—
	to	25,764	119,457	—	—
No. 3. Hot blast after first melting, ...		—	98,560	39,690	—
No. 3. Hot blast after twelfth melting,.........		—	163,744	56,060	—
No. 3. Hot blast after eighteenth melting, ...		—	197,120	25,350	—
Malleable cast iron,............		48,000?	—	—	—

It is to be understood that the numbers in one line of the preceding table do not necessarily belong to the *same specimen* of iron, each number being an *extreme* result for the kind of iron specified in the first column.

VII.—Resistance of Timber to Twisting.

	Modulus of Rupture by Wrenching. f Lbs. on the Square Inch.	Modulus of Transverse Elasticity. C Lbs. on the Square Inch.
Red Pine of Prussia,............	1,540	116,300
„ of Norway,...........	950	61,800
Elm,...............................	1,390	76,000
Oak (of Normandy),.............	2,350	82,400
Ash,.................................	1,460	76,000

VIII.

Supplementary Table of Properties of Timber grown in Ceylon; selected and computed from a Table of the Properties of ninety-six kinds of Timber by Modliar Adrian Mendis.

Timber.	Modulus of Elasticity in lbs. on the Square Inch.	Modulus of Rupture in lbs. on the Square Inch.	Weight of a Cubic Foot in lbs.
Aludel (*Artocarpus pubescens*),...	1,850,000	12,800	51
Burute (*Chloroxylon Swietenia*),	2,700,000	18,800	55
Caha Milile (*Vitex altissima ?*),...	2,000,000	13,900	56
Caluvere. See "Ebony."			
Cos (*Artocarpus integrifolia*),......	1,810,000	11,000	42
Ebony or Caluvere (*Diospyros Ebenus*),.........................	1,360,000	13,000	71
Gal or Hal Mendora (*Vateria sp. — ?*).........................	1,530,000	13,300	57
Ilal Milile (*Berrya Ammonilla*),	970,000	15,200	48
Ironwood. See "Naw."			
Jack. See "Cos."			
Mee (*Bassia longifolia*),...........	1,880,000	13,000	61
Meean Milile (*Vitex altissima*),...	2,040,000	14,200	56
Naw (*Mesua Nagaha*), ...	2,580,000	17,900	72
Palmira. See "Tal."			
Paloo (*Mimusops hexandra*),......	2,430,000	18,900	68
Satinwood. See "Burute."			
Sooriya (*Thespesia populea*),.......	2,610,000	12,700	42
Tal (*Borassus flabelliformis*),......	2,810,000	14,700	65
Teak (*Tectona grandis*),...........	2,800,000	14,600	55

Additional Data from the Experiments of Captain Fowke, R.E., Captain Mayne, R.E., and Modliar Mendis.

Teak from Johore (Malay Peninsula),		19,400	
Teak from Cochin-China,..........	1,990,000	12,100	44
Teak from Moulmein,..............	1,900,000	11,520	42
Iron-bark (*Eucalyptus*—?) from Australia,......................	964,000	24,400	64
Iron-bark, rough-leaved,..........	1,157,000	22,500	64
Jarrah, or "Australian Mahogany" (*Eucalyptus*—?).........	1,157,000	20,238	59
Stringy-bark (*Eucalyptus gigantea*) from Australia, ...	1,709,000	13,000	54

IX.—Supplementary Table for Stone, Lime, and Cement.*

	Crushing Stress in lbs. on the Square Inch.
Grauwacke from Penmaenmaur,	16,893
Basalt, Whinstone,	11,970
Granite (Mount Sorrel),	12,861
„ (Argyllshire),	10,917
Syenite (Mount Sorrel),	11,820
Sandstone (Strong Yorkshire, mean of 9 experiments),	9,824
„ (weak specimens, locality not stated),	3,000 to 3,500
Limestone, compact (strong),	8,528
„ magnesian (strong),	7,098
„ „ (weak),	3,050

The above are from experiments by Mr. Fairbairn.

Mr. Fairbairn's experiments further show that the resistance of strong sandstone to crushing in a direction parallel to the layers, is only *six-sevenths* of the resistance to crushing in a direction perpendicular to the layers.

The hardest stones alone give way to crushing at once, without previous warning. All others begin to crack or split under a load less than that which finally crushes them, in a proportion which ranges from a fraction little less than unity in the harder stones, down to about *one-half* in the softest.

A Year and a Half after Mixture.	Crushing Force in lbs. on the Square Inch.
Mortar of Lime and River-Sand,	440
„ „ „ beaten,	600
Mortar of Lime and Pit-Sand,	580
„ „ „ beaten,	800
Hydraulic Mortar, of lime and pounded tiles,	680
„ „ „ beaten,	930
Beton, or concrete, of mortar and broken flints,	420

Sixteen Years after Mixture, the increase of strength is in the following proportions:—

For common mortar,	1-8th.
For hydraulic mortar,	1-4th.

Six Months after Mixture.	Lbs. on the Sq. In.
Adhesion of common mortar to compact limestone,	15
Adhesion of common mortar to brick,	33

* See page 305.

One Year after Mixture.	Tenacity in lbs. on the Square Inch.
Good hydraulic lime,	170
Ordinary hydraulic lime, from	140
Ordinary hydraulic lime, to	100
Rich lime,	40
Good hydraulic mortar,	140
Ordinary hydraulic mortar,	85
Good common mortar,	50
Bad common mortar,	20

Cement from chalk lime and blue clay, a few days after mixture, ... 125
Portland cement (from compact limestone and clay) 30 to 50 days after mixture, ... 1,200 to 1,550

X.—Miscellaneous Supplementary Table.

Material.	Dimensions.	Tearing Load, lbs.	Length of 1 lb. weight, in feet.	Tenacity in feet of the Material.
Cast steel bar,	1 in. × 1 in.	130,000	0·297	38,610
Charcoal iron wire,	area 1 sq. in.	100,000	0·3	30,000
Iron wire rope,	girth 1·27 in.	4,480	6·0	26,880
Iron bar, strong,	1 in. × 1 in.	60,000	0·3	18,000
Boiler plate, strong,	area 1 sq. in.	50,000	0·3	15,000
Teak wood,	1 in. × 1 in.	15,000	3·6	54,000
Deal,	1 in. × 1 in.	12,000	4·0	48,000
Hempen rope, hawser-laid,	girth 1 in.	1,050	26·0	27,300
Hempen rope, cable-laid,	girth 10 in.	67,200	0·279	18,750
Silken thread,	area 0·000115 sq. in.	6	19,950	119,700
Flaxen thread,	unknown.	6	15,833	95,000

Modulus of elasticity of silken thread;

3,000,000 feet of itself = 1,300,000 lbs. on the square inch.

Modulus of resilience of silken thread;

473 foot-lbs. for a cord weighing 2 lbs.; or
205 foot-lbs. for a cord 2 feet long × 1 square inch area.

The tenacity of silk-worm gut, in lineal feet of itself, is about the same with that of silken thread.

Royal Navy Canvas.

	Mean of Nos. 1, 2, 3, 4, 5, and 6.	Mean of Nos. 7 and 8.
Tenacity of warp in lineal feet of canvas,	21,552	27,200
Tenacity of weft in lineal feet of canvas,	30,788	32,000
Mean tenacity of the flaxen yarn in lineal feet of itself, being the sum of the tenacities of the warp and weft,.........	52,340	59,200

(The above are from the *Trans. of the Institution of Engineers in Scotland* for 1865-6, on the authority of Professor Rankine, Mr. Peter Carmichael, and Mr. John P. Smith.)

Aluminium bronze contains from 5 to 10 per cent. of aluminium, and from 95 to 90 per cent. of copper.

Its mechanical properties are as follows, according to Mr. John Anderson, of the Woolwich Gun Factory:—

Specific gravity, 7·68; heaviness, 480 lbs. per cubic foot.
Tenacity,..........................73,000 lbs. per square inch.
Resistance to Crushing,........132,000 lbs. per square inch.

Cast steel in small blocks; resistance to crushing, in lbs. on the square inch, according to Mr. Fairbairn,.. 269,000

Section II.—Rules.

1. Factors of Safety and Moduli of Strength:—

	Dead Load.	Live Load.
Factors of safety for perfect materials and workmanship,..................................	2	4
For good ordinary materials and workmanship:—		
Metals,..............................	3	6
Timber,..............................	4 to 5	8 to 10
Masonry,.............................	4	8

A *dead load* on a structure is one that is put on by imperceptible degrees, and that remains steady; such as the weight of the structure itself.

A *live load* is one that is put on suddenly, or accompanied with vibration; such as a swift train travelling over a railway bridge, or a force exerted in a moving machine.

Rule I.—Given, the proportions of live and dead load on a structure; to find the factor of safety for the mixed load; multiply the factor of safety for a dead load by a number proportional to

the dead part of the load, and the factor of safety for a live load by the number proportional to the live part of the load; add together the products, and divide by the sum of the multipliers.

EXAMPLE.—In an iron bridge, suppose dead load : live load : : 5 : 4; then $(3 \times 5) + (6 \times 4) = 39$; and $39 \div (5 + 4) = 4\frac{1}{3}$, factor of safety for mixed load.

RULE II.—Given, the *breaking load* of a piece of material; to find the *proof load;* divide by the factor of safety for a dead load.

RULE III.—Given, the intended *working load* on a piece of material; to find the least proper *breaking load;* multiply by the proper factor of safety as found by Rule I.

RULE IV.—To find the *working modulus* or co-efficient of strength of a given piece of material; divide the modulus or co-efficient of *ultimate strength* by the proper factor of safety. (The co-efficients in the tables of the preceding section relate, with a few exceptions, to ultimate strength, or breaking load.)

2. **Uniform Tension.**—RULE V.—To find *the intensity of the stress* on a bar bearing a tensile load; divide the load by the sectional area of the bar.

RULE VI.—To find the *breaking load,* or the *working load,* of a bar subjected to tension; multiply the sectional area of the bar by the modulus of ultimate or working tenacity, as the case may be (having due regard in the latter case to the proper factor of safety).

RULE VII.—To find the *sectional area* of a bar to bear a given load; divide the load by the proper modulus. (See Rule IV.)

RULE VIII.—To find the *proportionate extension* of a stretched bar; divide the intensity of the tensile stress by the "*modulus of elasticity.*" (See Tables.)

To find the *elongation;* multiply the length of the bar by the proportionate extension.

N.B.—This Rule holds only when the load is not beyond the proof strength of the material. In applying it to a *live load,* that load must be doubled, so as to reduce it to the *equivalent dead load.*

RULE IX.—To find the *resilience* of a bar under tension; multiply the proof load by half the corresponding elongation: or otherwise; multiply the *modulus of resilience* by half the volume of the bar.

The five preceding Rules are applicable when the resultant of the stretching load traverses the centre of each cross-section of the bar.

3. **Uniformly Varying Tension.**—When the resultant of the stretching load does not traverse the centre of the cross-section of the bar, the intensity of the stress will sensibly vary at an uniform

rate; and will be least at that edge of the section *from* which the resultant deviates, and greatest at that edge *towards* which the resultant deviates. The *mean intensity* will be the same with that given by the Rules of the preceding Article. To find the ratio in which the greatest intensity exceeds the mean, proceed as follows:—

RULE X.—Find the *centre of magnitude* of the cross-section as in the Rules of pages 81, 82, 83, and 85. Then find its *centre of percussion* relatively to the edge from which the resultant load deviates. (See pages 155, 156, 157.) Divide the deviation of the resultant of the load from the centre of magnitude by the deviation of that centre of percussion from the centre of magnitude. Divide the distance of the centre of magnitude from the edge towards which the resultant load deviates by the distance of the same centre from the opposite edge. (In symmetrical sections this second quotient is = 1.) Multiply together the two quotients, and to the product add 1. (In symmetrical sections add 1 to the first quotient.) The sum will be the ratio in which the greatest intensity of the stress is greater than the mean intensity.

4. **Resistance of Thin Shells to Bursting.**—Let r denote the radius of a thin hollow cylinder, such as the shell of a high pressure boiler; t, the thickness of the shell; f, the tenacity of the material, in pounds on the square inch; p, the intensity of the pressure, in pounds on the square inch, required to burst the shell. This ought to be taken at *six times* the effective working pressure—*effective pressure* meaning the excess of the pressure from within above the pressure from without, which last is usually the atmospheric pressure of 14·7 lbs. on the square inch, or thereabouts.

RULE XI.—To find the bursting pressure of a given *thin cylindrical shell;* make

$$p = \frac{f\,t}{r}.$$

RULE XII.—To find the proper proportion of thickness to radius for a given ultimate tenacity and bursting pressure;

$$\frac{t}{r} = \frac{p}{f}.$$

Value of f for well-made wrought-iron boilers, with single-rivetted joints, properly crossed; about 34,000 lbs. on the square inch (Fairbairn).

RULE XIII.—To find the bursting pressure of a *thin spherical shell;* take double the bursting pressure of a thin cylindrical shell of the same radius, thickness, and material.

RULE XIV.—To find the least proper thickness for a thin spherical shell of a given material and radius, for a given bursting

pressure; take half the corresponding thickness for a cylindrical shell.

N.B.—When a cylindrical boiler has hemispherical ends, it is advisable to make them as thick as the cylindrical barrel, notwithstanding that they are thereby made twice as strong.

RULE XV.—Suppose a shell of the figure of a segment of a sphere to have a *circular flange* round its base, through which it is bolted to a flange upon a cylindrical shell, or upon another spherical shell. Let r denote the radius of the sphere, in inches; r', the radius of the circular base of the segmental shell, in inches; p, the bursting pressure, in lbs. on the square inch; then the number and dimensions of the bolts by which the flange is held should be such, that the load required to tear them asunder all at once shall be

$$3{\cdot}1416\, r'^2 p;$$

and the flange itself should require, in order to crush it, the following thrust in the direction of a tangent to it:—

$$\frac{1}{2} p\, r' \cdot \sqrt{r^2 - r'^2}.$$

If the segment is a complete hemisphere, $r' = r$, and the last expression becomes $= 0$.

5. **Resistance of Thick Shells to Bursting.**—Let R represent the external and r the internal radius of a thick hollow cylinder, such as a hydraulic press, the tenacity of whose material is f, and whose bursting pressure is p.

RULE XVI.—To find the bursting pressure of a given thick hollow cylinder; make

$$p = f \cdot \frac{\mathrm{R}^2 - r^2}{\mathrm{R}^2 + r^2}.$$

RULE XVII.—To find the proper proportion of outside to inside radius for a given tenacity and bursting pressure; make

$$\frac{\mathrm{R}}{r} = \sqrt{\left(\frac{f + p}{f - p}\right)}.$$

The corresponding formulæ for a *thick hollow sphere* are

RULE XVIII.— $$p = f \cdot \frac{2\,\mathrm{R}^3 - 2\,r^3}{\mathrm{R}^3 + 2\,r^3}.$$

RULE XIX.— $$\frac{\mathrm{R}}{r} = \sqrt[3]{\left(\frac{2f + 2p}{2f - p}\right)}.$$

6. **Resistance to Shearing.**—In rivets, keys, pins, bolts, treenails, and other fastenings exposed to shearing stress, the *greatest intensity*

of the stress is liable to become greater than the mean intensity, through unequal distribution. The strength of fastenings, allowing for that inequality of stress, is to be made equal to that of the main pieces which they connect together.

RULE XX.—To find the strength of an *easy-fitting fastening* against shearing; multiply the sectional area by the modulus of strength; then take $\frac{2}{3}$ of the product if the fastening is rectangular in section, or $\frac{3}{4}$ if it is circular or elliptical in section.

For a *perfectly tight-fitting fastening* the strength is the whole product just mentioned. Many actual fastenings are intermediate between easy and perfectly tight fastenings.

RULE XXI.—Ordinary dimensions of *rivets*:—

Diameter for plates less than half an inch thick, about double the thickness of the plate.

For plates of half an inch thick and upwards, about once and a-half the thickness of the plate.

Length before clenching, measuring from the head = sum of the thickness of the plates to be connected + $2\frac{1}{2} \times$ diameter of the rivet.

RULE XXI. A.—*Rivetted Joints.*—Make the joint sectional area of the rivets equal to the area of plate left after making the rivet holes; or in symbols,—

Let t denote the thickness of the plate iron;
d, the diameter of a rivet;
n, the number of rows of rivets transverse to the pull;
c, the *pitch* from centre to centre of the rivets in one row; then

$$c = d + \frac{\cdot 7854\, n\, d^2}{t}.$$

Each plate is weakened by the rivet holes in the ratio

$$\frac{c - d}{c} = \frac{\cdot 7854\, n\, d}{t + \cdot 7854\, n\, d};$$

In "single-rivetted" joints, $n = 1$; in "double-rivetted" joints, $n = 2$; in "chain-rivetted" joints, n may have any value greater than 1. A single-rivetted joint is weakened by unequal distribution of the tension in the ratio of 4 : 5.

Suppose that in a chain-rivetted joint the pitch, c, is fixed; then

$$n = \frac{(c - d)\, t}{\cdot 7854\, d^2}.$$

7. **Resistance to Compression and Direct Crushing.**—Resistance to *longitudinal compression*, when the proof stress is not exceeded, is sensibly equal to the resistance to stretching, and is expressed by the same modulus. When that limit is exceeded, it becomes irregular. (See Rule VIII., page 206.)

The present Article has reference to direct and simple crushing only, and is limited to those cases in which the pillars, blocks, struts, or rods along which the thrust acts are not so long in proportion to their diameter as to have a sensible tendency to give way by bending sideways. Those cases comprehend—

Stone and brick pillars and blocks of ordinary proportions;

Pillars, rods, and struts of cast iron, in which the length is not more than five times the diameter, approximately;

Pillars, rods, and struts of wrought iron, in which the length is not more than ten times the diameter, approximately;

Pillars, rods, and struts of dry timber, in which the length is not more than about twenty times the diameter.

In such cases the Rules of this Section, from V. to VII., and also Rule X. (pages 206, 207), are approximately applicable, substituting *thrust* for *tension*, and using the proper modulus of resistance to direct crushing instead of the tenacity.

Blocks whose lengths are less than about once-and-a-half their diameter offer greater resistance to crushing than that given by the Rules; but in what proportion is uncertain.

8. **Strength of Long Struts and Pillars.**—Long struts and pillars give way by bending sideways and breaking across. Let P be the breaking load of such a pillar; S, its sectional area; l, its length; r, the *least radius of gyration* of its cross-section (see page 154); f and c, two co-efficients depending on the material; then

RULE XXII.—For a strut or pillar fixed in direction at both ends,

$$\frac{P}{S} = \frac{f}{1 + \frac{l^2}{c r^2}}.$$

RULE XXIII.—For a strut or pillar jointed at both ends;

$$\frac{P}{S} = \frac{f}{1 + \frac{4 l^2}{c r^2}}.$$

RULE XXIV.—For a strut or pillar jointed at one end and fixed at the other;

$$\frac{P}{S} = \frac{f}{1 + \frac{16 l^2}{9 c r^2}}.$$

Values of the Constants.

	f	c
	Lbs. on the Square Inch.	
Malleable iron,......................	36,000	36,000
Cast iron,..........................	80,000	6,400
Dry timber,........................	7,200	3,000

Table of Values of r^2 for Different Forms of Cross-Section.

Solid rectangle; least dimension $= h$;....................	$h^2 \div 12.$
Thin square cell; side $= h$;....	$h^2 \div 6.$
Thin rectangular cell; breadth, b; depth, h;......	$\frac{h^2}{12} \cdot \frac{h + 3b}{h + b}$
Solid cylinder; diameter $= h$;	$h^2 \div 16.$
Thin hollow cylinder; diameter $= h$;.................	$h^2 \div 8.$
Angle iron of equal ribs; breadth of each $= b$;......	$b^2 \div 24.$
Angle iron of unequal ribs; greater, b, less, h;..........	$b^2 h^2 \div 12 (b^2 + h^2).$
Cross of equal arms;...........	$h^2 \div 24.$
H-iron; breadth of flanges, b; their joint area, A; area of web, B;..................	$\frac{b^2}{12} \cdot \frac{A}{A + B}.$
Channel iron; depth of flanges $+ \frac{1}{2}$ thickness of web, h; area of web, B; of flanges, A;..................	$h^2 \cdot \left\{ \frac{A}{12 (A + B)} + \frac{A B}{4 (A + B)^2} \right\}.$
Barlow rail; cross-section composed of two quadrants of radius R, measured to middle of thickness, connected by a table of sectional area $=$ joint area of quadrants $\times$ ·273;	$R^2 \div 7$ nearly.
Pair of Barlow rails as above, rivetted base to base;......	$·393 R^2.$
Circular segment of radius R and length $2 R \theta$;.......	$\left\{ \frac{1}{2} + \frac{\cos\theta \sin\theta}{2\theta} - \frac{\sin^2\theta}{\theta^2} \right\} R^2$

9. **Resistance of Tubes to Collapsing.**—Rule XXV.—Collapsing pressure in lbs. on the square inch =

$$\frac{9{,}672{,}000 \text{ thickness}^2}{\text{length} \times \text{diameter}};$$

all the dimensions being in the same units of measure.

When tubes are stiffened by rings, the length in the rule is to be measured from ring to ring.

10. **Action of a Transverse Load on a Beam.**—If the load consists of several parts, find the *resultant load* by the Rules of Part V., page 164, and Part IV., page 153. Then find the *supporting forces* by the proper rule (XIX.) in page 163.

RULE XXVI.—To find the *shearing actions* exerted in a series of intervals of the length of the beam:—

Case I.—If the loaded part of the beam projects outward from its point of support, and the load is applied at detached points, the shearing action in the outermost interval is equal to the load at the outermost point.

To the shearing action in any interval add the load applied at the inner end of that interval; the sum will be the downward shearing action in the next interval inwards.

For a distributed load, in symbols; let dx be an interval of the length; w, the load per unit of length; F, the shearing action at the distance x inwards from the outermost loaded point; then

$$F = \int_o^x w\,dx.$$

Case II.—If the loaded part of the beam lies between its points of support, and the load is applied at detached points; the upward shearing action in the interval next one of the points of support is equal to the supporting force at that point.

From the shearing action in any interval subtract the load applied at the point next beyond that interval; the remainder will be the shearing action in the interval next beyond.

For a distributed load, in symbols; let P_o be the supporting pressure at the end where the calculations commence, and F the shearing action at the distance x from that end; then

$$F = P_o - \int_o^x w\,dx.$$

REMARK.—In calculating the series of shearing actions in Case II., a point is reached where the shearing action changes its direction, as shown by its algebraical sign changing from positive to negative. This is the point where the *load divides* (as in page 171). At the further end of the span the shearing action is equal in amount to the supporting force at that end, but of contrary algebraical sign. Let l be the span; P_l, the supporting force at its further end; and F_l, the shearing action close to that end; then

$$F_l = P^o - \int_o^l w\,dx = -P_l;$$

and this formula serves as a check on the accuracy of the calculations by the preceding formula.

Rule XXVII.—To find the *bending moments* exerted at a series of points in the length of the beam. Multiply the length of each interval by the shearing action exerted in that interval; add together the products corresponding to the intervals which lie between one end of the beam and the point where the bending moment is required; the sum will be the required bending moment.

In symbols, let M be the bending moment at the distance x from one end of the beam; then

$$M = \int_o^x F\, d\, x.$$

Remark.—The accuracy of the calculation of the bending moments at a series of points may be checked by trying whether at the further end of the span the bending moment vanishes; that is

$$M_l = \int_o^l F\, d\, x = 0.$$

Rule XXVIII.—To find the *greatest bending moment;* take the bending moment at the point where the load divides; that is, where $F = 0$.

For tables of the comparative values of different *units of bending moment*, see pages 104, 110, 113.

11. **Explanation of the Table of Examples.**—W, total load; l, length of beam fixed at one end, or span of beam supported at both ends; F, shearing action, and M, bending moment, at distance x' from one end; x'_1, distance from one end at which shearing action is greatest; k, ratio of greatest shearing action to total load W; x'_0, distance from same end at which $F = 0$ and $M =$ a maximum; m, ratio of maximum bending moment to $W\,l$. That is to say, let F_1 = greatest shearing action, and M_0 = greatest bending moment; then $F_1 = k\,W$; $M_0 = m\,W\,l$.

To transform the expressions in the following table, Cases IV. to VII., which are suited for co-ordinates measured from one point of support of a beam supported at both ends, into expressions suited for co-ordinates measured from the middle of the beam, let c be the *half-span*, and substitute $2\,c$ for l, $c - x$ for x', and $c + x$ for $l - x'$, throughout the whole of that part of the table.

12. **Travelling Load on a Beam.**—A beam of the span l is supported at the two ends; a permanent load of the uniform intensity of w lbs. per lineal foot is distributed over it. An additional load, such as the weight of a railway train, of w' lbs. per lineal foot, gradually rolls on to the beam from one end, covering it at last from end to end, and then rolls off at the other end. (For the continuation of this Article see page 216.)

TABLE OF EXAMPLES.

CASES.	F	x'_1	k	M	x'_0	m
A. BEAMS FIXED AT ONE END.						
I. Loaded at extreme end with W,...............	$-W$	anywhere	-1	$-Wx'$	l	-1
II. Uniform load of intensity $w = W \div l$.	$-wx'$	l	-1	$-\frac{wx'^2}{2}$	l	$-\frac{1}{2}$
III. Uniform load of intensity w, and additional load W' at extreme end,.........	$-W'-wx'$	l	-1	$-W'x'-\frac{wx'^2}{2}$	l	$-\frac{W'+\frac{wl}{2}}{W'+wl}$
B. BEAMS SUPPORTED AT BOTH ENDS.						
IV. Single load W, in the middle;						
half of beam next origin,	$\frac{W}{2}$	o to $\frac{l}{2}$	$\frac{1}{2}$	$\frac{Wx'}{2}$	$\frac{l}{2}$	$\frac{1}{4}$
farther half, ...	$-\frac{W}{2}$	$\frac{l}{2}$ to l.	$-\frac{1}{2}$	$\frac{W(l-x')}{2}$		

TABLE OF EXAMPLES—*continued.*

CASES.	F	x'_1	k	M	x'_0	m
V. Single load W, applied at x'';						
between x'' and origin;	$\frac{l-x''}{l}\mathrm{W}$	anywhere	$\frac{l-x''}{l}$	$\frac{x'(l-x'')}{l}\mathrm{W}$	x''	$\frac{x''(l-x'')}{l^2}$
beyond x'';	$-\frac{x''\mathrm{W}}{l}$	anywhere	$-\frac{x''}{l}$	$\frac{(l-x')x''}{l}\mathrm{W}$		
VI. Uniform load of intensity $w = \mathrm{W} \div l$,	$w\left(\frac{l}{2}-x'\right)$	0 and l	$\pm\frac{1}{2}$	$\frac{w\,x'(l-x')}{2}$	$\frac{l}{2}$	$\frac{1}{8}$
VII. Partial load of uniform intensity $w = \mathrm{W} \div x''$ from 0 to x''; remainder unloaded;						
between x'' and origin;	$w\left(x''-\frac{x''^2}{2l}-x'\right)$	0	$1-\frac{x''}{2l}$	$w\left\{\left(x''-\frac{x''^2}{2l}\right)x' - \frac{x'^2}{2}\right\}$	$x''-\frac{x''^2}{2l}$	$\frac{x''}{2l}\left(1-\frac{x''}{2l}\right)^2$
beyond x''............	$-\frac{w\,x''^2}{2l}$	x'' to l	$-\frac{x''}{2l}$	$\frac{w\,x''^2}{2l}(l-x')$		

RULE XXIX.—The *Greatest Shearing Action* at a given cross-section occurs when the longer of the two segments into which it divides the beam is loaded with the travelling load as well as with the permanent load, and the shorter loaded with the permanent load only. Let F' denote that action, and x' the distance of the section in question from the nearer end of the beam; then

$$F' = w\left(\frac{l}{2} - x'\right) + \frac{w'(l - x')^2}{2l}.$$

Let x be the distance of the cross-section in question from the *middle* of the beam, and c the half-span; then

$$F' = wx + \frac{w'(c + x)^2}{4c}.$$

The *Greatest Bending Moment* at a given cross-section occurs when the whole span is loaded with the travelling load, and is therefore given by Case VI. of the table; viz.,

$$M = \frac{(w + w')\,x'(l - x')}{2} = \frac{(w + w')(c^2 - x^2)}{2}.$$

REMARK.—If the travelling load is liable to rush *suddenly* on to the bridge, like a swift railway train, its actual weight should be *doubled* in taking the value of w', in order to reduce it to the equivalent steady load; and when this has been done, the factor of safety employed in further calculations may be that suited for a dead load.

13. **The Moment of Resistance** of a Beam at a given cross-section ought to be at least equal to the greatest bending moment.

RULE XXX.—In a *skeleton beam*, consisting of stringers and braces only (see fig. 72, page 169), to find the moment of resistance at a given joint; multiply the sectional area of the stringer opposite that joint by the greatest safe intensity of stress along it (tensile or compressive as the case may be) and by the perpendicular distance of the centre line of the stringer from the joint; the product will be the required moment of resistance.

RULE XXXI.—In a *thin-webbed beam* with parallel flanges along the edges of the web (in other words, of a thin-webbed I-shaped section) the flange which becomes convex by the bending of the beam is stretched, and that which becomes concave compressed. Multiply the sectional area of each flange by the greatest safe stress along it (tension or thrust according as the flange is stretched or compressed); then multiply the *lesser* of the two products by the perpendicular distance between the centre lines of the flanges; the final product will be the required moment of resistance, *approximately*. In this method the moment of resistance of the web is neglected.

N.B. For the best economy of material, the two products first mentioned should be equal to each other. The cross-section of the beam is then said to be of *equal strength.*

RULE XXXII.—In a *solid beam,* to find the moment of resistance at a given cross-section:—

Step 1.—Find the *neutral axis* of the cross-section by taking its centre of magnitude (see pages 81 to 84), and drawing through that point a straight line perpendicular to the plane in which the bending of the beam takes place.

Step 2.—Find the *geometrical moment of inertia* of the cross-section relatively to its neutral axis, by dividing that section into narrow strips parallel to the neutral axis, multiplying the area of each strip by the square of its distance from the neutral axis, and adding the products together. (In Rules I., II., and III. of page 154, put "cross-section" for "body," and "area" for "mass," and those rules become applicable to the present purpose.) In symbols, let y be the distance of any strip from the neutral axis; z, its length parallel to that axis; $d\,y$, its breadth; and I, the geometrical moment of inertia of the section; then $\mathrm{I} = \int y^2\,z\,d\,y$ ($= n'\,b\,h^3$, where b is the breadth, h the depth, and n' a factor depending on the form of section). Also, let S be the sectional area, and r the radius of gyration of the section relatively to its neutral axis (see page 211); then $\mathrm{I} = r^2\,\mathrm{S}$.

Step 3.—Divide the greatest safe tensile stress on the material by the greatest distance of the stretched particles of the cross-section from the neutral axis, and the greatest safe compressive stress by the greatest distance of the compressed particles from the neutral axis; multiply the lesser of those quotients by the moment of inertia of the cross-section; the product will be the required moment of resistance.

In symbols, let y_a and y_b be the greatest distances of compressed and stretched particles from the neutral axis; f_a and f_b, the greatest safe thrust and tension on those particles respectively; let $\frac{f_1}{y_1}$ stand for the lesser of the two quotients, $\frac{f_a}{y_a}, \frac{f_b}{y_b}$; then the moment of resistance is

$$\mathrm{M} = \frac{f_1\,\mathrm{I}}{y_1} = n\,f_1\,b\,h^2;$$

where n is a factor depending on the form of cross-section. Another expression for the moment of resistance is as follows:—

$$\mathrm{M} = \frac{f_1\,\mathrm{I}}{y_1} = q\,f_1\,h\,\mathrm{S};$$

in which S is the area of the cross-section, and q a suitable numerical factor.

For the best economy of material, the two quotients ought to be equal; that is to say,

$$\frac{f_1}{y_1} = \frac{f_a}{y_a} = \frac{f_b}{y_b} = \frac{f_a + f_b}{h}.$$

This gives a *cross-section of equal strength.*

Examples of the Numerical Factors.

Form of Cross-Sections.	$n' = \frac{I}{b h^2}$	$m' = \frac{y_1}{h}$	$n = \frac{M_0}{f b h^2}$
I. Rectangle $b h$, (including square)	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{1}{6}$
II. Ellipse— Vertical axis h, Horizontal axis b, (including circle)	$\frac{\pi}{64} = \frac{1}{20 \cdot 4}$ $= 0 \cdot 0491$	$\frac{1}{2}$	$\frac{\pi}{32} = \frac{1}{10 \cdot 2}$ $= 0 \cdot 0982$
III. Hollow rectangle, $b h - b' h'$; also I-formed section, where b' is the sum of the breadths of the lateral hollows,	$\frac{1}{12}\left(1 - \frac{b' h'^3}{b h^3}\right)$	$\frac{1}{2}$	$\frac{1}{6}\left(1 - \frac{b' h'^3}{b h^3}\right)$
IV. Hollow square, $h^2 - h'^2$,	$\frac{1}{12}\left(1 - \frac{h'^4}{h^4}\right)$	$\frac{1}{2}$	$\frac{1}{6}\left(1 - \frac{h'^4}{h^4}\right)$
V. Hollow ellipse,	$\frac{1}{20 \cdot 4}\left(1 - \frac{b' h'^3}{b h^3}\right)$	$\frac{1}{2}$	$\frac{1}{10 \cdot 2}\left(1 - \frac{b' h'^3}{b h^3}\right)$
VI. Hollow circle,	$\frac{1}{20 \cdot 4}\left(1 - \frac{h'^4}{h^4}\right)$	$\frac{1}{2}$	$\frac{1}{10 \cdot 2}\left(1 - \frac{h'^4}{h^4}\right)$
VII. Isosceles triangle; base b, height h; y_1 measured from summit,	$\frac{1}{36}$	$\frac{2}{3}$	$\frac{1}{24}$

FORM OF CROSS-SECTION.

	q.
I. Rectangle, ...	$\frac{1}{6}$.
II. Ellipse and circle, ...	$\frac{1}{8}$.
III. Hollow rectangle, $S = b\,h - b'\,h'$; also I-shaped section, b' being the sum of the depths of the lateral hollows, ...	$\dfrac{1 - \frac{b'\,h'^3}{b\,h^3}}{6\left(1 - \frac{b'\,h'}{b\,h}\right)}$.
IV. Hollow square, $S = h^2 - h'^2$, ...	$\frac{1}{6}\left(1 + \frac{h'^2}{h^2}\right)$.
V. Do., very thin (approx.),	$\frac{1}{3}$.
VI. Hollow ellipse, ...	$\frac{1}{8}\left(1 - \frac{b'\,h'^3}{b\,h^3}\right) \div \left(1 - \frac{b'\,h'}{b\,h}\right)$.
VII. Hollow circle, ...	$\frac{1}{8}\left(1 + \frac{h'^2}{h^2}\right)$.
VIII. Do., very thin (approx.),	$\frac{1}{4}$.
IX. T-shaped section; flange A, web C; $S = A + C$ (approx.),	$\dfrac{C\,(C + 4\,A)}{6\,(C + A)\,(C + 2\,A)}$.
X. I-shaped section; flanges A, B; web C; $S = A + B + C$; the beam supposed to give way at the flange A (approx.), ...	$\dfrac{C\,(C + 4\,A + 4\,B) + 12\,A\,B}{6\,(C + 2\,B)\,(A + B + C)}$.
X. A. Do., do., the beam supposed to give way at the flange B (approx.), ...	$\dfrac{C\,(C + 4\,A + 4\,B) + 12\,A\,B}{6\,(C + 2\,A)\,(A + B + C)}$.
XI. I-shaped section; with equal flanges; $A = B$; $S = C + 2\,A$ (approx.), ...	$\frac{1}{6}\left(1 + \frac{4\,A}{C + 2\,A}\right)$.

14. **Cross-Sections of Equal Strength** have already been mentioned. The following rules are applicable where the beam is I-shaped, consisting of a vertical web, rectangular or nearly so in section, with flanges of small depth compared with the depth of the web, running along its upper and lower edges.

Let f_a be the greatest safe thrust; f_b, the greatest safe tension; y_a and y_b, the distance from the neutral axis to the centres of the compressed and stretched flanges respectively; $h = y_a + y_b$, the depth between the centres of the flanges; A and B, the sectional areas of the compressed and stretched flanges respectively; C, the sectional area of the web measured from centre to centre of the flanges.

RULE XXXIII.—f_a greater than f_b (as in cast iron). Given, A, C; to find B;

$$B = \frac{f_a}{f_b} A + \frac{f_a - f_b}{2 f_b} C.$$

REMARK.—The moment of resistance is

$$M = h\left\{f_a A + (2 f_a - f_b)\frac{C}{6}\right\} = h\left\{f_b B - (f_a - 2 f_b)\frac{C}{6}\right\}.$$

In practice, $h f_b$ B is often used as an approximation to this moment.

RULE XXXIII A.—f_a less than f_b (as in wrought iron). Given, B, C; to find A;

$$A = \frac{f_b}{f_a} \cdot B + \frac{f_b - f_a}{2 f_a} C.$$

REMARK.—The moment of resistance is

$$M = h\left\{f_b B + (2 f_b - f_a)\frac{C}{6}\right\} = h\left\{f_a A + (2 f_a - f_b)\frac{C}{6}\right\}.$$

In designing I-shaped beams, fix C by considerations of practical convenience, and then find A and B so as to give the required moment of resistance.

15. **Longitudinal Sections of Equal Strength.**—RULE XXXIV.—To give a beam a longitudinal section of equal strength, make $b h^2$, or h S, at different points of the length of the beam, vary proportionally to M; taking care near the points of support to leave enough of material to resist the shearing action.

To effect this with the greatest economy of material, let the depth, h, be uniform, and make the breadth, b, or the sectional area, S, vary proportionally to M.

To effect the same thing, and give the beam the greatest possible flexibility, either let b be constant, and make h vary proportionally to $\sqrt{M}$; or let S be constant, and make h vary proportionally to M.

16. **Allowance for Weight of Beam.**—RULE XXXV.—Let W′ be the external working load, dead, live, or mixed, on a beam; s', its proper factor of safety; and let s be the factor of safety for a dead

load. Having fixed the depth beforehand, calculate a *provisional breadth*, or a *provisional sectional area*, suited to bear safely the external load alone; and thence compute a *provisional weight* for the beam,—say B′. Then increase the breadth, or the sectional area, in the following ratio:—

$$\frac{s' \mathrm{W}'}{s' \mathrm{W}' - s \mathrm{B}'};$$

and the beam will safely bear its own weight in addition to the given external load.

RULE XXXVI.—Given, the span l, weight B, and external working load W′ of an actual beam of a given sort; to find the *limiting span*, L, of a beam of the same sort, and with the same proportion ($h \div l$) of depth to span, which will just bear its own weight safely and no more.

$$\mathrm{L} = l \cdot \frac{s' \mathrm{W}' + s \mathrm{B}}{s \mathrm{B}}.$$

RULE XXXVII.—Given, for a certain sort of beam, with a given proportion, $h \div l$, of depth to span, the span l, and the *limiting span*, L, of similar beams; to estimate the probable proportion of weight of beam to external load;

$$\frac{\mathrm{B}}{\mathrm{W}'} = \frac{s'}{s} \cdot \frac{l}{\mathrm{L} - l}.$$

17. **Deflection of Beams.**—RULE XXXVIII.—To find the *curvature* (that is the reciprocal of the radius of curvature) of an originally straight beam at a given cross-section.

Case I.—The bending moment given. Divide the bending moment by the moment of inertia of the given cross-section (see Article 13 of this section, page 217), and by the modulus of elasticity of the material. In symbols, let r be the radius of curvature; then

$$\frac{1}{r} = \frac{\mathrm{M}}{\mathrm{E\,I}}.$$

Case II.—The cross-section under its proof stress. Divide the proof stress (f_1) by the distance of the most severely strained particles from the neutral axis, and by the modulus of elasticity; the quotient will be the *proof curvature*;

$$\frac{1}{r} = \frac{f_1}{\mathrm{E}\, y_1}.$$

In *cross-sections of equal strength* the proof curvature is

$$\frac{1}{r} = \frac{f_a + f_b}{E\,h}.$$

RULE XXXIX.—To find the *slope* of the beam (originally level) at a given point. Divide the length of the beam into small intervals ($d\,x$); multiply the length of each interval by the curvature at its centre $\left(\text{giving the product } \frac{d\,x}{r}\right)$; add together the products for the intervals from a point where the beam continues horizontal to the point where the slope is required; the sum $\left(i = \int \frac{d\,x}{r}\right)$ will be the required slope.

RULE XL.—To find the deflection. Multiply the length of each small interval by its slope (obtaining the product $i\,d\,x$); add together those products for the intervals extending between the highest and lowest points of the beam, the sum ($v = \int i\,d\,x$) will be the required deflection.

The preceding is the general method. The following are special rules:—

Let c be the *half-span* of a beam supported at both ends, or the *length* of a beam fixed at one end; h, the extreme depth, and b, the extreme breadth of the beam; W, any given load; f_1, the proof stress; or f_a, the proof thrust, and f_b, the proof tension, in cross-sections of equal strength; $m'\,h$, the distance of the most severely strained layer from the neutral axis; $n'\,b\,h^3$, the moment of inertia of the greatest cross-section; m'', n'', m''', n''', numerical multipliers.

RULE XLI.—Steepest slope under proof load;

$$i_1 = \frac{m''\,f_1\,c}{E\,m'\,h}; \ \left(\text{or } \frac{m''\,(f_a + f_b)\,c}{E\,h}\right).$$

RULE XLII.—Proof deflection;

$$v_1 = \frac{n''\,f_1\,c^2}{E\,m'\,h}; \ \left(\text{or } \frac{n''\,(f_a + f_b)\,c^2}{E\,h}\right).$$

RULE XLIII.—Steepest slope under a given load, W;

$$i_1 = \frac{m'''\,W\,c^2}{E\,n'\,b\,h^3}.$$

RULE XLIV.—Deflection under a given load, W;

$$v_1 = \frac{n'''\,W\,c^3}{E\,n'\,b\,h^3}.$$

Case.	Proof Load. Factors for Slope. m''	Proof Load. Factors for Deflection. n''	Given Load. Factors for Slope. m'''	Given Load. Factors for Deflection. n'''
A. Uniform Cross-Section.				
I. Constant Moment of Flexure, ...	1	$\frac{1}{2}$		
II. Fixed at one end, loaded at other, ...	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
III. Fixed at one end, uniformly loaded, ...	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$
IV. Supported at both ends, loaded in middle, ...	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$
V. Supported at both ends, uniformly loaded, ...	$\frac{2}{3}$	$\frac{5}{12}$	$\frac{1}{6}$	$\frac{5}{48}$
B. Uniform Strength and Uniform Depth.				
(The curvature of these is uniform).				
VI. Fixed at one end, loaded at other, ...	1	$\frac{1}{2}$	1	$\frac{1}{2}$
VII. Fixed at one end, uniformly loaded, ...	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
VIII. Supported at both ends, loaded in middle, ...	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
IX. Supported at both ends, uniformly loaded, ...	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
C. Uniform Strength and Uniform Breadth.				
X. Fixed at one end, loaded at other, ...	2	$\frac{2}{3}$	2	$\frac{2}{3}$
XI. Fixed at one end, uniformly loaded, ...	infinite	1	infinite	$\frac{1}{2}$
XII. Supported at both ends, loaded in middle, ...	2	$\frac{2}{3}$	1	$\frac{1}{3}$
XIII. Supported at both ends, uniformly loaded, ...	1·5708	0·5708	0·3927	0·1427

Rule XLV.—Given, the half-span, c, and the *intended proof deflection*, v_1, of a proposed beam; to find the proper value of the *greatest depth*, h_0; make

$$h_0 = \frac{n'' f_1 c^2}{E m' v_1};$$

(taking n'' from the preceding table, and making $m' h_0$ as before, denote the distance from the layer in which the stress is f_1 to the neutral axis.)

If the cross-section is to be of equal strength, make

$$h_0 = \frac{n'' (f_a + f_b) c^2}{E v_1}.$$

Rule XLVI.—To deduce the greatest stress in a given layer of a beam from the deflection found by experiment.

Let h be the depth of the beam at the section of greatest stress, and y the distance from the neutral axis of that section to that layer of the beam at which the greatest stress is required:—

c, the half-span of a beam supported at both ends, or the length of the loaded part of a beam supported at one end;
n'', the factor for proof deflection, already explained;
E, the modulus of elasticity of the material;
v, the observed deflection;

then the intensity of the required stress is

$$p = \frac{E y v}{n'' c^2}.$$

Rule XLVII.—To find the *resilience* of a beam loaded at one point; multiply half the proof load by the proof deflection.

18. **Continuous Girders.**—In the following rules the girder is supposed to be of uniform cross-section, and to be continuous over two or more piers. The half-span of one bay is denoted by c; the fixed load per unit of span by w; the travelling load per unit of span, if brought on slowly, by w'; if the travelling load comes on suddenly, w' must be understood to stand for the equivalent dead load; that is *twice* the actual travelling load per unit of span. The moment of resistance of the uniform cross-section is to be adapted to the most severe bending moment.

Rule XLVIII.—To find the bending moment at mid-span (M_0), and the reverse bending moment over each pier ($-M_1$), when every span is loaded with the travelling load;

$$M_0 = \frac{(w + w') c^2}{6}; \ -M_1 = \frac{(w + w') c^2}{3}.$$

RULE XLIX.—To find the said bending moment when the span under consideration is loaded with the travelling load and the adjoining spans with the weight of the bridge only;

$$M_0 = \frac{(w + 2\,w')\,c^2}{6}; \; - M_1 = \frac{(2\,w + w')\,c^2}{6}.$$

Every continuous girder bridge has two *end bays* at which the continuity stops; and these must be of less span than the intermediate bays.

RULE L.—The proper span of an end bay should be not less than $c\sqrt{\dfrac{2\,w + w'}{3\,w}}$ (or it will be too light); and not greater than

$c\left(1 + \sqrt{\dfrac{w + 2\,w'}{3\,(w + w')}}\right)$ (or it will be too weak).

To calculate the *proof deflection* of continuous girders, use Rule XLIV., page 223, with the following values of the multiplier n'';

	n''
Every span fully loaded,..................................	$\frac{1}{8}$
One span fully loaded; the adjoining spans loaded with the weight of the bridge alone; the lesser of the two following factors,......................	$\frac{w + 3\,w'}{4\,w + 8\,w'}$; $\frac{w + 3\,w'}{8\,w + 4\,w'}$

19. **Arched Ribs.**—In the following rule the rib, of iron or timber, is supposed to have its centre line of the form of a parabola, of the half-span, c, and rise, k. The sectional area of the rib at its crown is denoted by A, and at other points that area is supposed to vary as the secant of the inclination of the rib to the horizon. The depth of the rib, h, is supposed uniform. The moment of resistance of the rib to cross-breaking is supposed to be denoted by $f_1\,q\,h\,A$; q being the multiplier of which values are given in page 219. The uniform fixed load per unit of span is denoted by w; and the travelling load per unit of span, if gradually put on, by w'; if suddenly put on, w' denotes *twice* the actual travelling load per unit of span. The rib is supposed to be jointed at the crown and at the springing.

RULE LI.—When the rib is fully loaded, to find the horizontal thrust (H), and the intensity of the stress (p),

$$H = \frac{(w + w')\,c^2}{2\,k}; \; p = \frac{H}{A}.$$

RULE LII.—When one-half of the span only is loaded with the travelling load, the horizontal thrust is,

$$H' = \left(w + \frac{w'}{2}\right) \frac{c^2}{2k};$$

Also, let $\frac{w' c^2}{16} = M'$; then the *greatest* intensity of stress is

$$\frac{1}{A} \left(H' + \frac{M'}{q h}\right).$$

REMARK.—That greatest stress is compressive; and is exerted near the middle of the length of the inner edge of the unloaded half of the rib, and of the outer edge of the loaded half.

RULE LIII.—Given, the greatest safe stress, f_a; to find the proper area, A, for the rib at its crown; calculate the two following quantities: H as in Rule LI.; and $H' + \frac{M'}{q h}$, as in Rule LII.; divide the *greater* of them by f_a; the quotient will be the required area.

20. **Stiffening Girder.**—RULE LIV.—To find the proper *moment of resistance* for a stiffening girder for a suspension bridge; calculate M′ as in Rule LII. The greatest *shearing action* in that girder is $\frac{w' c}{4}$.

The stiffening girder is liable to be bent upwards and downwards alternately; and therefore it should be made alike above and below.

21. **Resistance to Twisting.**—Let h be the external diameter of a shaft; h', the internal diameter (if it is hollow); f', a modulus of stress.

RULE LV.—Moment of resistance of

a solid cylindrical shaft,.......$0{\cdot}196\, f' h^3$;

a hollow cylindrical shaft,....$0{\cdot}196\, f' \cdot \frac{h^4 - h'^4}{h}$;

a solid square shaft,.............$0{\cdot}28\, f' h^3$.

RULE LVI.—To find the thickness of a shaft which shall have a given moment of resistance to twisting, M.

$$\text{solid cylindrical shaft, } h = \sqrt[3]{\left(\frac{M}{0{\cdot}196 f'}\right)};$$

$$\text{hollow cylindrical shaft, } h' = n h;\ h = \sqrt[3]{\left(\frac{M}{0{\cdot}196\,(1 - n^4) f'}\right)}.$$

$$\text{solid square shaft, } h = \sqrt[3]{\left(\frac{M}{0{\cdot}28 f'}\right)}.$$

	Stress in Lbs. on the Square Inch. Breaking.	Working.
Cast iron,	27,700	4,000 to 4,500
Wrought iron,	50,000	8,000 to 9,000

Rule LVII.—When bending and twisting actions are combined on one shaft, let M be the bending moment, and T the twisting moment; then make the shaft of the diameter suited to resist the following *twisting moment*:—

$$M + \sqrt{(M^2 + T^2)}.$$

Rule LVIII.—The *angle of torsion* of a bar, whether cylindrical or square, when under the *proof* stress f', is $\frac{2 f' l}{C h}$; in which l is the length, and h the thickness of the bar, and C the modulus of transverse elasticity.

22. **Buckled Plates.**—Rule LIX.—To calculate the load uniformly distributed over a buckled plate, which will crush it; the plate being square, and fastened all round the edges. Multiply the depth to which the plate is buckled by the square of the thickness, both in inches and by 165; the product will be the crushing load in tons, nearly. *Central* load which crushes a buckled plate, about $\frac{1}{3}$ of uniformly distributed load.

23. **Suspension Bridges.**—As to the horizontal tension, see page 173. As to stiffening girders, see page 226.

Rule LX.—Given, the working horizontal tension, H, the half span, x, and the depression, y, of the chain or cable; to calculate the weight of a half-span of it. (Factor of safety, 6.)

For the strongest wire cables, make $C = \frac{H x}{4,500 \text{ feet}}$;

For cable iron chains, make $C = \frac{H x}{3,000 \text{ feet}}$.

Then for a chain or cable of *uniform cross-section*, the weight of a half-span is

$$C' = C\left(1 + \frac{8 y^2}{3 x^2}\right);$$

and for a chain or cable of *uniform strength* (the area varying as the tension) the weight of a half-span is

$$C'' = C\left(1 + \frac{4 y^2}{3 x^2}\right).$$

For eyes and fastenings of links, add one-eighth to net weight.

PART VII.

MACHINES IN GENERAL.

SECTION I.—RULES RELATING TO THE COMPARISON OF MOTIONS.

1. **Motion of a Point.**—As to measures of speed of advance, or linear velocity, and of speed of turning, or angular velocity, see page 102. In the following rules, when not otherwise specified, linear velocity is supposed to be expressed in *fect per second,* and angular velocity in *circular measure per second.* Linear velocities and angular velocities are represented by lines, and compounded and resolved, like forces and couples. (See pages 158 to 163.) If there be three bodies, 1, 2, and 3, and 3 has a given motion relatively to 2, and 2 a given motion relatively to 1, the *resultant* of those two motions is the motion of 3 relatively to 1.

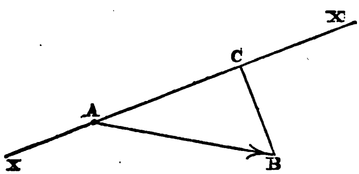


Fig. 86.

RULE I. (See fig. 86.)—Given, the velocity and direction, A B, of the motion of a point, A; to find the *component* of that velocity along a given line, X A X; from B, let fall B C perpendicular to X X; A C will be the required component. In symbols;

$$A\,C = A\,B \cdot \cos C\,A\,B.$$

RULE II.—A point moves in a curve of a given radius (r) with a given linear velocity (v); to find the *angular velocity of revolution,* divide the linear velocity by the radius. In symbols;

$$a = \frac{v}{r}.$$

RULE III.—In the same case, to find the *rate of deviation;* divide the square of the linear velocity by the radius; or otherwise, multiply the square of the angular velocity by the radius. In symbols;

$$\text{rate of deviation} = \frac{v^2}{r} = a^2\,r.$$

2. **Translation of a Rigid Body** is that kind of motion in which all points in the body move with equal velocities and in parallel directions along equal and similar paths, straight or curved.

RULE IV.—During translation the *relative motion* of two points in a rigid body is = 0. Their *comparative motion* at any instant consists in equality of speed and identity of direction.

3. **Rotation of a Rigid Body.**—RULE V.—Given, an axis of rotation in a rigid body, and the angular velocity of rotation; to find the direction and velocity of the motion of any point in the body. Let fall a perpendicular from the point on the axis; the required direction will be perpendicular to that perpendicular and to the axis; and the required velocity will be the product of the angular velocity into the length of that perpendicular.

RULE VI.—Given, the linear velocity of a point in a rigid body rotating about an axis; to find the angular velocity; divide the linear velocity by the perpendicular distance of the point from the axis.

RULE VII.—Given, an axis of rotation, and two points not in that axis; to find the *comparative motion* of those two points. The ratio of their velocities, or *velocity-ratio*, is equal to the ratio of their perpendicular distances from the axis.

RULE VIII.—A rigid body moves parallel to a given plane, and the directions of motion of two points in it are given; to find its axis of rotation, if any.

If the two points are not in one plane parallel to the given plane of motion, take their projections on such a plane (A, B, in figs. 87, 88, 89); the motions of those projections will be identical with

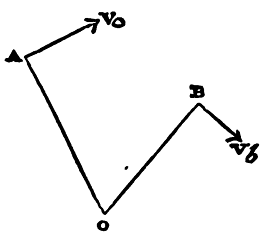


Fig. 87.

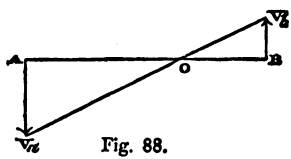


Fig. 88.

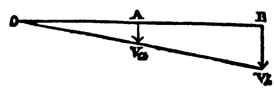


Fig. 89.

those of the original points. In each figure the arrows represent the given directions of motion of the points.

Case I.—Directions not parallel (fig. 87). Perpendicular to the given directions, draw A O, B O, cutting each other in O; the required axis will traverse O, and be perpendicular to the plane of motion.

Case II.—Directions parallel to each other, and not perpendicular to line of connection, A B. In this case the motion is one of translation, and there is no axis.

Case III.—Directions perpendicular to A B. (See figs. 88, 89.) In this case the problem is indeterminate unless the velocity-ratio of A and B is given. Then draw A V_a, B V_b, in the directions of motion of A and B, and bearing to each other the given ratio; draw the straight line V_a V_b, cutting A B (produced if necessary) in O; this will give the position of the required axis.

Remark.—The axis found by Rule VIII. may be either *permanent* or *instantaneous*.

Rule IX. (See fig. 90.)—In a body rotating with a given speed about a given axis, O, to find the component, in a given direction, B A, perpendicular to that axis, of the velocity of a point, A. On A B let fall the perpendicular O B, and multiply its length by the angular velocity.

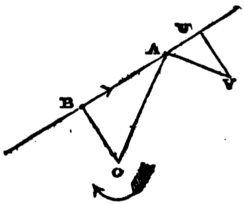


Fig. 90.

4. **Motion of Rigidly-Connected Points.**—A pair of points, A and B (fig. 91), are so connected that their distance from each other, A B, is invariable.

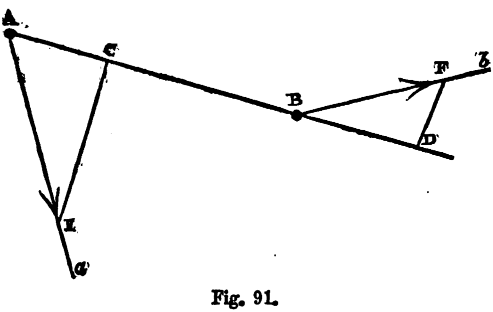


Fig. 91.

Rule X.—Given, the directions, A *a* and B *b*, of the motions of a pair of rigidly-connected points at a given instant; required, their velocity-ratio. Draw the straight *line of connection*, A B, and produce it if necessary. Then lay off in it any convenient equal distances, A C = B D. Through C and D draw perpendiculars to the line of connection, cutting A *a* and B *b* in E and F. Then, velocity of A : velocity of B : : A E : B F.

5. **Points in Sliding Contact.**—In fig. 92 let A B and C D represent a pair of smooth surfaces moving in sliding contact, and let

T mark the position of the pair of particles which at a given instant touch each other.

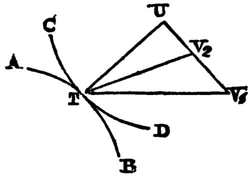


Fig. 92.

RULE XI.—Given, the directions T V_1 and T V_2 of the motions of the contiguous particles; to find the ratio of their velocities. At the point of contact draw T U of any convenient length normal to the two surfaces at that point. Through U draw U V_1 V_2 parallel to the common tangent plane of those surfaces, and cutting the directions of motion of the contiguous particles in V_1 and V_2. Then velocity of particle 1 : velocity of particle 2 : : T V_1 : T V_2.

SECTION II.—RULES RELATING TO MECHANISM.

1. **Rolling Contact.**—The conditions of rolling contact between two pieces in a machine (such as two smooth wheels, or a smooth wheel and a sliding bar) are as follows:—If the two pieces turn about axes, the two axes and the straight line of contact of the two pieces must be in the same plane, and must either be parallel or intersect in one point. If one piece turns on an axis, and the other slides, the axis and the line of contact must be parallel to each other, in one plane perpendicular to the direction of sliding.

RULE I.—Two pieces (smooth wheels) are to turn in rolling contact with each other about a pair of parallel axes, with a given ratio of angular velocities; say that of $a : b$. To find the position of the line of contact of the *pitch-surfaces;* let c be the *line of centres;* that is, the perpendicular distance between the axes; then the distances of each point of contact are,—

From the axis about which the angular velocity is as a; $\dfrac{b\,c}{a+b}$;

From the axis about which the angular velocity is as b; $\dfrac{a\,c}{a+b}$.

In other words, *the radii are inversely as the angular velocities.*

RULE II.—A rotating piece (such as a smooth wheel) and a sliding piece move in rolling contact. Given, the angular velocity of the rolling piece; to find the linear velocity of the sliding piece; multiply the angular velocity of the rolling piece by the perpendicular distance from its axis to the line of contact of the pitch-surfaces.

RULE III.—Given, the ratio of the angular velocities of two *conical* or *smooth bevel* wheels about their axes (which meet in one point); to find the line of contact of the pitch-surfaces of those

wheels. In fig. 93 let O A, O C be the two axes, intersecting in O. Lay off on those axes, O a, O b, respectively proportional to the angular velocities of the wheels which are to turn about them. Complete the parallelogram O b c a; the diagonal O c (produced as far as required) will be the line of contact of the two pitch-surfaces; and those surfaces will be cones made by sweeping that line round the two axes respectively.

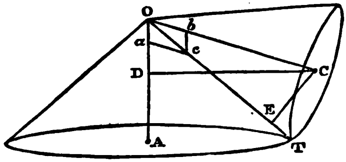


Fig. 93.

2. **Skew-Bevel Wheels.**—The pitch-surfaces of skew-bevel wheels are hyperboloïds, generated by the revolution of the line of contact about each of the axes, to which it is neither parallel nor intersecting.

Rule IV.—The directions and positions of the axes being given, and the required angular velocity-ratio, $a : b$, it is required to find the *obliquities* of the line of contact to the two axes, and its least perpendicular distances from those axes.

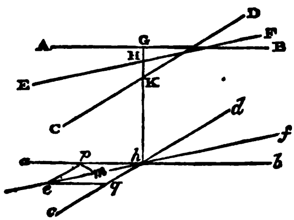


Fig. 94.

In fig. 94 let A B, C D be the two axes, and G K their common perpendicular.

On any plane normal to the common perpendicular draw $a\ b \parallel$ A B, $c\ d \parallel$ C D, in which take lengths in the following proportions:—

$$a : b :: \overline{h\,p} : \overline{h\,q};$$

complete the parallelogram $h\ p\ e\ q$, and draw its diagonal, $e\ h\ f$; the line of contact, E H F, will be parallel to that diagonal.

From p let fall $p\ m$ perpendicular to $h\ e$. Then divide the common perpendicular, G K, in the ratio given by the proportional equation,

$$\overline{h\,e} : \overline{e\,m} : \overline{m\,h} :: \overline{\mathrm{G\,K}} : \overline{\mathrm{G\,H}} : \overline{\mathrm{K\,H}};$$

and the two segments thus found will be the least distances of the line of contact from the axes.

The first pitch-surface is generated by the rotation of the line E H F about the axis A B, with the radius vector $\overline{\mathrm{G\,H}}$; the second, by the rotation of the same line about the axis C D, with the radius vector $\overline{\mathrm{H\,K}}$.

3. **Teeth of Wheels.**—RULE V.—To find the *least thickness* suitable for the teeth of a wheel. Divide the pressure to be transmitted by 1,500 lbs., and extract the square root of the quotient for the thickness on the pitch-circle in inches.

RULE VI.—To find the *least pitch* suited for the teeth of a wheel; multiply the least thickness on the pitch-line by $2\frac{1}{5}$.

RULE VII.—To find the *least breadth* suited for the teeth of a wheel; divide the pressure to be transmitted, in lbs., by 160, and by the pitch in inches; the quotient will be the required breadth in inches.

RULE VIII.—To find the proper circumference for a wheel; multiply the pitch by the intended number of teeth.

RULE IX.—To set out *involute teeth.* In fig. 95 let C_1, C_2 be the centres of two circular wheels whose pitch circles are B_1, B_2. Through the pitch-point, I, draw the intended *line of connection*, $P_1 P_2$, making the angle $C I P = \theta$ with the line of centres. This angle is usually about 75°. From C_1, C_2, draw

$$\overline{C_1 P_1} = \overline{I C_1} \cdot \sin \theta, \ \overline{C_2 P_2} = \overline{I C_2} \cdot \sin \theta,$$

perpendicular to $P_1 P_2$, with which two perpendiculars as radii, describe circles (called *base circles*), D_1, D_2. The proportions of the triangles, $C_1 I P_1$, $C_2 I P_2$, are in practice nearly as follows:—

$$65 : 63 : 16 :: I C : C P : I P.$$

Make a circular mould of the figure of one of the base circles, D; wrap a cord round the edge of it; make fast one end of the cord, and tie a pencil or tracing-point to the other end; on unwrapping the cord, the point will trace the figure of a tooth for the wheel to which the base circle belongs.

All involute teeth of the same pitch work smoothly together.

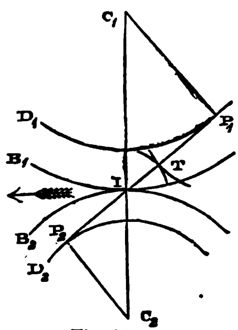


Fig. 95.

To mark the *path of contact* of the teeth; lay off a distance equal to the pitch $\times \sin \theta \left(\text{say} = \frac{63}{65} \text{ pitch}\right)$, along $P_1 P_2$ in either direction from I. The distance of the tip of a tooth of either wheel from the centre of that wheel is equal to the distance from that centre to the further end of the path of contact.

The teeth of a *rack*, to work correctly with wheels having involute teeth, should have plane surfaces perpendicular to the line of connection, and consequently making, with the direction of motion of the rack, angles equal to the before-mentioned angle θ.

The *smallest possible number* of involute teeth in a pinion is the

whole number next above $2\pi \tan \theta$. When $\tan \theta = \frac{63}{16}$ the number is 25.

RULE X.—To set out *epicycloidal teeth*. Make two moulds of the figure of the pitch-circle of the wheel, one convex, the other concave. Make a circular disc called the *describing circle*, with a tracing-point in its circumference; the usual size of the describing circle is such that its circumference is *six times the pitch*, and its radius therefore = pitch × 0·955. To trace the *flanks* of the teeth, roll the describing circle *inside* the concave mould; to trace their *faces*, roll it *outside* the convex mould.

In fig. 96 let B B be the pitch-circle; C I C′, part of a radius of the wheel; R, the describing circle when inside the pitch-circle; R′, the describing circle when outside the pitch-circle. On the circumferences of the describing circles lay off I D = I D′ = the pitch; D will be the inner end of the flank of a tooth, and D′ the outer end of the face of a tooth.

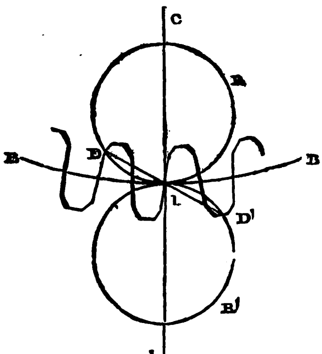


Fig. 96.

All wheels having epicycloidal teeth set out with the same pitch and the same describing circle work accurately together.

The smallest practicable pinion having epicycloidal teeth is that the circumference of whose pitch-circle is twice that of the describing circle. According to usual proportions, it has twelve teeth. Their flanks are radial straight lines.

RULE XI.—To set out *approximate epicycloidal teeth*; let p denote the pitch, n the number of teeth in the wheel.

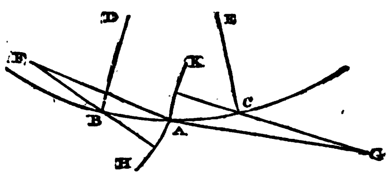


Fig. 97

In fig. 97 let B C be the part of the pitch-circle, A the point where a tooth is to cross it. Set off A B = A C $= \frac{p}{2}$. Draw radii of the pitch-circle, D B, E C. Draw F B, C G, making angles of $75\frac{1}{2}°$ with those radii, in which take

$$\overline{BF} = \frac{p}{2} \cdot \frac{n}{n+12}; \ \overline{CG} = \frac{p}{2} \cdot \frac{n}{n-12}$$

Round F, with the radius F A, draw the circular arc A H; this will be the face of the tooth. Round G, with the radius G A, draw the circular arc G K; this will be the flank of the tooth. (See Willis *On Mechanism.*)

4. **Screws.**—RULE XII.—To find the *advance* of a screw corresponding to a given number of turns; multiply that number by the *pitch* (measured parallel to the axis, between corresponding points on two successive turns of the thread).

RULE XIII.—Given, the pitch of a screw; to find the *obliquity* of the thread to the axis at a given distance from the axis; multiply that distance by 6·2832 (so as to find the corresponding *circumference*), and divide by the pitch; the quotient will be the tangent of the required obliquity.

RULE XIV.—To find the *normal pitch* of a screw (measured perpendicularly to the thread) at a given distance, r, from the axis; let p be the pitch; then

$$\text{Normal pitch} = \frac{2 \pi r p}{\sqrt{(4 \pi^2 r^2 + p^2)}}.$$

RULE XV.—To make two screws of given numbers of threads and given cylindrical pitch-surfaces *gear together;* make the normal pitches of the screws proportional to their numbers of threads, and the angle between their axes equal to the sum of the obliquities of their threads, if both are right-handed or both left-handed; or equal to the difference of those obliquities if one screw is right-handed and the other left-handed.

N.B.—The angular velocities of two gearing screws are inversely as their numbers of threads.

5. **Pulleys and Bands** (whether belts, cords, or chains).—RULE XVI.—To find the *ratio of the speed of turning* of two pulleys connected by a band. Measure the *effective radii* of the pulleys from the axis of each to the centre line of the band; then the speeds of turning will be inversely as the radii.

RULE XVII.—To design a pair of *tapering speed-cones,* so that the belt may fit equally tight in all positions.

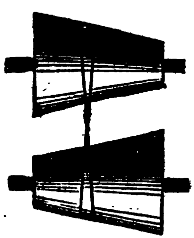


Fig. 98.

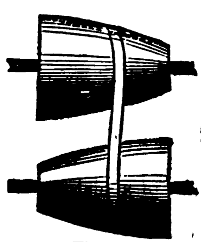


Fig. 99.

Case I.—Belt crossed (fig. 98). Use a pair of equal and similar cones tapering opposite ways.

Case II.—Belt uncrossed (fig. 99.) Use a pair of equal and similar conoids tapering opposite ways, and *bulging* in the middle according to the following formula:—Let c denote the distance between the axes of the conoids; r_1, the radius at the larger end of each; r_2, the radius at the smaller end; then the *radius in the middle*, r_0, is found as follows:—

$$r_0 = \frac{r_1 + r_2}{2} + \frac{(r_1 - r_2)^2}{6 \cdot 28\, c}.$$

6. **Linkwork.**—When two pins are connected together by a *link* or *connecting-rod*, to find their velocity-ratio at any instant, use Rule X. of the preceding Section (see page 230), taking the centres of the pins as a pair of *rigidly-connected points*.

When the points thus connected move in one plane, use Rule VIII. of the preceding Section to find the *instantaneous axis* of the link; the velocities of the connected points will be proportional to their perpendicular distances from that axis. Should the triangle formed by the connected points and their instantaneous centre be inconveniently large, proceed as follows:—

Rule XVIII.—Draw any triangle having one side parallel to the line of connection or centre-line of the link, and the other two sides respectively *perpendicular* to the directions of motion of the connected points; the last two sides will be proportional to the velocities of those points.

Example.—*Crank and Piston-Rod.*—In fig. 100 let R T_1 be a piston-rod; T_1, its head; C T_2, a crank; T_2, the crank-pin; T_1 T_2, the connecting-rod. Through T_1 draw T_1 K perpendicular to R T_1, and produce C T_2; the intersection, K, of those straight lines will be the instantaneous centre of the connecting-rod; and if v_1 and v_2 be the velocities of T_1 and T_2 respectively, $v_1 : v_2 :: \text{K } T_1 : \text{K } T_2$:—or otherwise; through C draw C A perpendicular to R T_1, and cutting the line of connection, T_1 T_2 (produced if necessary) in A. Then $v_1 : v_2 :: \text{C A} : \text{C } T_2$.

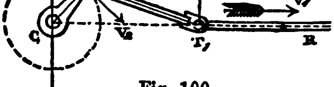


Fig. 100.

7. **Parallel Motions.**—Rule XIX.—Given (in fig. 101), the line of motion, G D, of a piston-rod, the middle position of its head, B, and the centre, A, of a lever which, in its middle position, A D, is perpendicular to D G; to find the radius of the lever, so that the

link connecting it with B shall deviate equally to the two sides of G D during the motion; also, the length of the link.

Make D E = $\frac{1}{4}$ stroke; join A E; and perpendicular to it, draw E F cutting A D produced in F; A F will be the required radius. Join F B; this will be the link.

Rule XX.—Given, the data and results of Rule XIX.; also the point, G, where the middle position of a second lever connected with the same link cuts G D: to find the second lever, so that the two extreme positions of B shall lie in the same straight line, G B D, with the middle position.

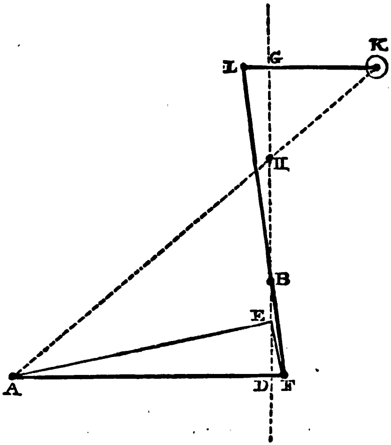


Fig. 101.

Through G draw a straight line, L G K, perpendicular to G D; produce F B till it cuts that line in L; this point will be one end of the required second lever at mid-stroke, and F L will be the entire link. Then in D G lay off D H = G B; join A H, and produce it till it cuts L K G in K; this will be the centre for the second lever.

When the two extreme positions and the middle position of B lie in the straight line G D, the whole of its positions are near enough to that line for practical purposes.

Rule XXI.—Given (in fig. 102), the *main centre*, A, the middle position of the *main lever*, A F, the piston-rod-head, B, and its length of stroke; the radius, A F, of the lever, and the *main link*, F B, having been found by Rule XIX. Let the figure represent those parts at mid-stroke; and let it be required to construct a parallel motion consisting of a parallelogram, C E D F (in which C E = F D is called the *parallel bar*,

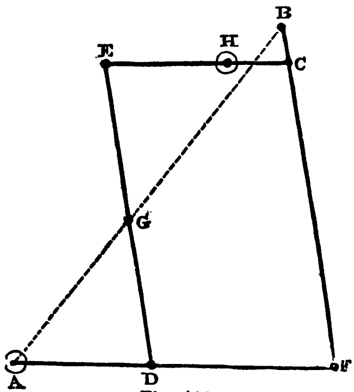


Fig. 102.

and D E = F C the *back link*), and a *radius lever*, or *bridle*, H E, jointed to the angle E of the parallelogram.

Draw the straight line A B, cutting the back link D E in G; then by Rule XX. find the lever H E, such that the middle and extreme positions of G shall lie in one straight line.

(The point G shows where a pump-rod may, if convenient, be jointed to the back link).

8. **Blocks and Tackle.**—RULE XXII.—The ratio of the velocity of the *fall* of a tackle to the velocity of the *moving block* is equal to the number of plies of rope by which the fixed and moving blocks are connected with each other.

9. **Pistons.**—The *area* of a piston is to be measured on a plane perpendicular to its direction of motion. The *stroke* of a piston moving in a straight line may be measured along the line of motion of any point in the piston; when it moves in a circle the stroke is to be measured on the line described by the centre of the area.

RULE XXIII.—To find the *volume swept* by a piston per stroke; multiply the stroke by the area.

RULE XXIV.—Two pistons have an invariable volume of fluid between them; to find the ratio of their velocities; take the reciprocal of the ratio of their areas.

SECTION III.—RULES RELATING TO WORK AT UNIFORM AND PERIODICAL SPEED.

1. **General Principles.**—In a machine moving at an uniform speed the driving and resisting forces are balanced. If the speed is varied, but in such a manner that the variations are periodic, the *mean* driving and resisting forces during one period, or complete revolution, are balanced. The energy exerted is equal to the whole work performed; in the former case, at all times; in the latter, during any whole number of periods or revolutions. As to units of work, see page 103.

2. **Computation of Work Done.**—To compute the quantity of work done:—

RULE I.—When a weight is lifted to a given height:—multiply the weight by the height.

RULE II.—When a body shifts through a given distance against a given force:—

Case I. If the force is directly opposed to the motion (being a *direct resistance*), multiply the force by the distance moved;

Case II. If the force is obliquely opposed to the motion; either resolve the force into a *resistance* directly opposed to the motion, and a *lateral force* perpendicular to the motion (see page 160, Rule VIII.), and multiply the resistance by the distance moved; *or otherwise:*—resolve the motion into a *direct component* opposed

to the entire force, and a *transverse component* at right angles to it, and multiply the entire force by the direct component of the motion. (In symbols, let F be the force, s the distance moved, θ the angle of obliquity; then work done $= F s \cos \theta$).

RULE III.—When a rotating body turns through a given angle against a resisting couple of a given moment (see pp. 104, 161):—

Multiply that moment by the extent of turning in circular measure. (See page 102.)

RULE IV.—When a piston moves against a pressure of a given intensity (see p. 103):—

Multiply that intensity by the *volume swept* by the piston. (See page 238, Rule XXIII.)

REMARK.—The unit of volume and unit of intensity should be adapted to each other, so that the product of their numbers may express units of work. For example:—

Unit of Intensity.	Unit of Volume.	Unit of Work.
Lbs. on the square foot.	Cubic foot.	Foot-pound.
Lbs. on the square inch.	Prism 1 ft. × 1 in. × 1 in.	do.
Lbs. on the circular inch.	Cylinder 1 ft. long × 1 in. diam.	do.
Kilo. on the square mètre.	Cubic mètre.	Kilogrammètre.

3. **Computation of Energy, Power, and Efficiency.**—(I.) When a given weight descends through a given height, or (II.) a given force drives a body shifting through a given distance, or (III.) a rotating body is driven by a couple of a given moment, or (IV.) a piston is driven by a pressure of a given intensity, the rules are the same as in the preceding Article; except that for *resistance* is to be put *effort*, or *driving force*, and for *work done*, *energy exerted*.

For *stored* or *potential energy*, use the same rules, substituting possible for actual motions.

RULE V.—To find the energy which must be exerted to make a machine perform a given motion at an uniform or periodical speed against given resistances. Find, by the rules of the preceding article, the quantities of work done during the given motion against the resisting forces, and add them together; the sum will be the *total work done*, to which the *energy to be exerted* will be equal.

As to *Power*, see page 104.

RULE VI.—To find the *Efficiency* of a machine; distinguish the resistances, and the work done against them, into *useful* and *wasteful;* then divide the *useful work* by the *total work;* the quotient will be the efficiency.

RULE VII.—To find the efficiency of a *train of machines;* multiply together the efficiencies of the elementary machines of which the train consists.

4. **Computation of Driving Force.**—Suppose a machine to be driven against given resistances by an *effort* or *driving force* applied at, and in the direction of motion of, the *driving point;* and that it is required to find the effort which will maintain an uniform speed.

RULE VIII.—Find the *energy* to be exerted, by Rule V., and divide it by the space moved through by the driving point;—*or otherwise:*

RULE VIII. A.—Find, by the principles of mechanism (see Section I. of this part, pages 231 to 238), the ratios of the velocities of the several *working points*, where resistances are overcome, to the velocity of the driving point. Multiply each direct resistance by the velocity-ratio belonging to its point of application, and add together the products; the sum will be the required effort.

REMARKS.—This is called "*reducing the resistances to the driving point.*" Rule VIII. A. may be applied to a machine capable of motion, though not actually moving; it is then called the "*principle of virtual velocities.*" When only one resistance is overcome, the effort and resistance are to each other inversely as the velocities of their points of application.

5. **Friction in Machines.**—RULE IX.—To calculate the resistance of friction to the sliding of two surfaces (when the pressure is not so great as to grind the surfaces, or force out the unguent), multiply the *amount* of the load, or direct pressure between the surfaces, by the *co-efficient* of friction.

Explanation of the Table.—ϕ, angle of repose; $f = \tan \phi$, co-efficient of friction; $1 : f = \cot \phi$, reciprocal of that co-efficient.

SURFACES.	ϕ	f	$1 : f$
Wood on wood, dry,	14° to 26½°	·25 to ·5	4 to 2
,, ,, soaped,	11½° to 2°	·2 to ·04	5 to 25
Metals on oak, dry,	26½° to 31°	·5 to ·6	2 to 1·67
,, ,, wet,	13½° to 14½°	·24 to ·26	4·17 to 3·85
,, ,, soapy,	11½	·2	5
Metals on elm, dry,	11½° to 14°	·2 to ·25	5 to 4
Hemp on oak, dry,	28°	·53	1·89
,, ,, wet,	18½°	·33	3
Leather on oak,	15° to 19½°	·27 to ·38	3·7 to 2·86
Leather on metals, dry,	29½°	·56	1·79
,, ,, wet,	20°	·36	2·78
,, ,, greasy,	13°	·23	4·35
,, ,, oily,	8½°	·15	6·67
Metals on metals, dry,	8½° to 11½°	·15 to ·2	6·67 to 5
,, ,, wet and clean,	16½°	·3	3·33
,, ,, damp and slimy,	8°	·14	7·14
Smooth surfaces, occasionally greased,	4° to 4½°	·07 to ·08	14·3 to 12·5
,, ,, continually greased,	3°	·05	20
,, ,, best results,	1¾° to 2°	·03 to ·036	33·3 to 27·6
Bronze on lignum vitæ, constantly wet,	3° ?	·05 ?	20 ?

In order that the load may neither grind the surfaces nor force out the unguent of the bearings of machinery, the pressure is to be limited by the following rules; in which, by *area of bearing* is meant the product of the length and diameter of a cylindrical bearing; although the real area on which pressure acts is much smaller.

RULE X.—Add 20 to the velocity of sliding in feet per minute, and divide 44,800 by the sum; the quotient will be the greatest proper intensity of pressure in lbs. on the square inch, with the further limitation that the intensity is in no case to exceed 1,200 lbs. on the square inch.

RULE XI.—To calculate the *moment of friction* of an axle; multiply the resultant load by the radius of the axle, and by the sine of the angle of repose (which is sensibly equal to the co-efficient of friction).

6. **Pulley and Strap.**—Let T_1 be the tension at the tighter side of the strap, and T_0 the tension at the slacker side, so that $T_1 - T_0$ is the force to be exerted between the strap and pulley; also let c be the *arc of contact* between the strap and pulley, in fractions of a circumference, and f the co-efficient of friction.

RULE XII.—Given, c, f, and the force $T_1 - T_0$; to find the tensions, greatest, least, and mean. Let N be the number corresponding to the common logarithm $2{\cdot}73\, f\, c$; then

$$T_0 = \frac{T_1 - T_0}{N - 1};\ T_1 = \frac{N}{N - 1}(T_1 - T_0);$$

$$\frac{T_1 + T_0}{2} = \frac{N + 1}{2\,(N - 1)} \cdot (T_1 - T_0).$$

REMARK.—Whether the calculation relates to driving belts or to strap-brakes, the co-efficient, f, should be estimated on the supposition of the surfaces being oily; say 0·15 for leather on metal, and 0·08 for metal on metal.

7. **Balancing of Machinery.**—In a machine every piece which turns on an axis should, as far as possible, have its re-actions balanced.

RULE XIII.—In order that there may be no tendency to shift the axis, arrange the weights that turn together about it so that their common centre of gravity shall be in the axis. (This constitutes a "*standing balance.*")

RULE XIV.—In order that there may be no tendency to turn the axis into varying directions; multiply each of the masses that turn together about the axis by its *arm* or perpendicular distance from the axis. Regard the products as representing forces, each pulling the axis towards the mass to which that product belongs,

and arrange the masses so that the moments of those forces shall balance each other.

RULES XIII. and XIV. are thus expressed algebraically. At a fixed point in the axis of rotation, let three planes fixed relatively to the rotating masses cut each other at right angles; two intersecting each other in the axis, and the third perpendicular to it. Let m be any one of the masses which rotate with one angular velocity about the axis, and x, y, z, its distances from the first, second, and third planes respectively. Then for a standing balance, make

$$\Sigma \cdot m\, x = 0;\ \Sigma \cdot m\, y = 0;$$

and for a running balance, make also

$$\Sigma \cdot m\, z\, x = 0;\ \Sigma \cdot m\, z\, y = 0.$$

8. **Work of Variable Force.**—RULE XV.—To find the work done against a varying resistance, or the energy exerted by a varying effort. Construct a *diagram* in which intervals of the length, or base-line, shall represent distances, and breadths or ordinates shall represent forces acting through those distances. The area of the diagram (measured by the Rules of pages 64, 65, 66, 67) will represent the work done, or the energy exerted. The common trapezoidal Rule, D, page 67, is usually accurate enough for this purpose.

REMARK.—If intervals of the length be taken to represent volumes swept through by a piston, and breadths to represent intensities of pressure (as in page 239), the area of the diagram will still represent work done or energy exerted.

RULE XVI.—To find the mean value of the varying force; divide the area of the diagram by its length, so as to find its *mean breadth;* this will represent the required mean force.

9. **Resistance on Lines of Land-Carriage.**—RULE XVII.—To find the resistance of a load drawn on a line of conveyance by land; to the *co-efficient of resistance on a level* (f) add the sine of the inclination (i) if ascending (or subtract that sine if the inclination is descending); multiply the load by the sum (or difference).

In symbols, let W be the load, R the resistance; then

$$\mathrm{R} = (f \pm i)\,\mathrm{W}.$$

VALUES OF THE CO-EFFICIENT OF RESISTANCE ON A LEVEL.

I. *Roads.*—Let v be the velocity in feet per second; r, the radius of the wheels of the carriage in *inches;* then

$$f = \frac{a + b\,(v - 3{\cdot}28)}{r} \text{ (Morin).}$$

		a.	b.
For good broken stone roads,	from	·4	·025
	to	·55	
For pavements,..................	from	·27	·068
	to	·39	·03

Values of f, from experiments by Sir John Macneill,—

Sandy and gravelly ground, ·14; gravel road, ·07;

Broken stone road, from ·03 to ·02; pavement, ·015.

II. *Railways.**—Let V be the speed in miles an hour; then

$$f = \text{from } \cdot 0027 \text{ to } \cdot 004 \left(1 + \frac{V^2}{1440}\right).$$

On *curves*, add to the above value of f,

For carriages with parallel axles, $\dfrac{3 \cdot 3}{\text{radius in feet}}$;

For carriages with moveable axles, $\dfrac{1 \cdot 36}{\text{radius in feet}}$.

RULE XVIII.—To calculate the *probable adhesion* of a locomotive engine; multiply the weight which rests on the driving wheels by the co-efficient of adhesion $\left(= \text{about } \frac{1}{7}\right)$. In symbols, let E be the weight of the engine, q the fraction resting on driving wheels; then

$$\text{Adhesion} = \text{about } \frac{q\,E}{7}.$$

ORDINARY VALUES OF q AND $\frac{q}{7}$.

	No. of Driving Wheels.		q.	$\frac{q}{7}$.
Passenger engines,............	2	from	·33	·048
		to	·5	·071
Goods engines,................	4	from	·67	·095
		to	·75	·107
Do do.	all		1·00	·143

* Proportion of gross to net load in railway trains; goods, from $1\frac{1}{8}$ to $1\frac{3}{4}$; minerals, from $1\frac{3}{4}$ to 2; passengers, about 3. Passengers without luggage weigh on an average about 15 or 16 to the ton; with luggage, about 10 to the ton.

Ordinary Weights of Locomotive Engines.*

Weights of Engines with separate Tenders,—

(The Tender weighs from 10 to 15 tons.)	Tons.
Narrow gauge passenger locomotives, six-wheeled, with one pair of driving wheels,	19 to 23
Do. do. do. unusually heavy,	24 to 27
Broad gauge passenger locomotive, eight-wheeled, with one pair of driving wheels 8 feet in diameter,..........................	35
Goods locomotive, from four to six wheels, coupled,..	27 to 32

Weights of Tank Engines, carrying Fuel and Water,—

	Tons.
For light traffic on branch lines, ...	12 to 20
For heavy traffic on steep inclined planes, with from six to twelve wheels,...........	40 to 60

Rule XIX.—To calculate the *greatest tractive force* (P) of a locomotive engine ascending a given gradient. Multiply the weight of the engine (E) by the sine of the inclination (i), and subtract the product from the adhesion. In symbols,—

$$P = \left(\frac{q}{7} - i\right) E.$$

In order that an engine may be able to draw a given load, P must be not less than R, (Rule XVI.) That is to say, on the *ruling gradient,* let E be the weight of the heaviest engine, T that of the heaviest load drawn behind the engine; then

$$\left(\frac{q}{7} - i\right) E = \left(f + i\right) T.$$

Hence the following rules:—

Rule XX.—Given, q, i, f; then $\dfrac{E}{T} = \dfrac{f + i}{\frac{q}{7} - i}$

Rule XXI.—Given, E, q, T, f; then $i = \dfrac{\frac{q\,E}{7} - f\,T}{E + T}$.

* Proper weight of rails, in lbs. to the yard = 15 × greatest load on a driving wheel in tons.

Weight of a chair; common = 1 foot of rail; joint = from 1¼ to 1½ foot of rail.

RULE XXII.—To find the total work done by a locomotive engine in a given time; multiply the resistance of engine and train as carriages by the distance run, for the *net* work; then multiply by about $1\frac{1}{3}$, to allow for resistance of mechanism of engine. In symbols, let x be the distance run; then

$$\text{Total work} = 1\tfrac{1}{3}\, x\, (f \pm i)\, (\mathrm{E} + \mathrm{T}).$$

SECTION IV.—RULES RELATING TO VARYING SPEED.

1. **General Principles.**—An unbalanced force applied to a body produces change of momentum equal in amount to and coincident in direction with the impulse exerted by the force. *Impulse* is the product of the force in absolute units (see page 104) into the time during which it acts in seconds. *Momentum* is the product of the mass of a body into its velocity in units of distance per second. The unit of mass is the mass of an unit of weight—such as a pound avoirdupois, or a kilogramme. A body receiving an impulse *re-acts* against the body giving the impulse, with an equal and opposite impulse.

2. **Acceleration and Retardation.**—RULE I.—To find what *impulse* is required to produce a given change in the velocity of a given mass; multiply the weight of the mass by the change in its velocity, in units of distance per second.

(If the change consists in acceleration, the impulse must be forward; if in retardation, backward.)

RULE II.—To find what *energy* must be exerted upon or taken away from a given mass to produce a given increase or diminution of its velocity; find the impulse required; divide it by the number of absolute units of force in the weight of an unit of mass, and multiply the quotient by the mean velocity during the change;—*or otherwise:* multiply the weight of the mass by the change in the value of the *half-square* of its velocity, and divide by the number of absolute units of force in the weight of an unit of mass.

REMARK.—Absolute units of force in the weight of an unit of mass; in British Measures (velocities being in feet per second), 32·2 nearly; in French Measures (velocities being in metres per second), 9·809 nearly. (See page 104.) This constant is denoted by g,* and sometimes called "*gravity*."

* More exact formula for g,

$$g = g_1\,(1 - 0{\cdot}00284 \cos 2\lambda)\left(1 - \frac{2h}{\mathrm{R}}\right).$$

in which $g_1 = 32{\cdot}1695$ in British Measures, or $9{\cdot}8051$ in French Measures;

RULE III.—To calculate the *actual energy* of a moving mass; multiply its weight by the half-square of its velocity, and divide by g.

RULE IV.—To calculate what unbalanced effort, or unbalanced resistance, as the case may be, is required to produce a given increase or diminution of a body's speed, in a given time, or in a given distance.

Case I.—If the *time* is given; multiply the weight of the mass by its change of velocity; divide by g, and by the time in seconds.

Case II.—If the *distance* is given; multiply the weight of the mass by the change in the half-square of its velocity, and divide by g, and by the distance.

RULE V.—To find the *re-action* of an accelerated or retarded body; find, by Rule IV., the force required to produce the change of velocity; the re-action will be equal and opposite.

REMARK.—The momentum, energy, and re-action of a body of any figure undergoing *translation* are the same as if its whole mass were concentrated at its centre of gravity.

3. **Deviated Motion and Centrifugal Force.**—To make a body move in a curve, some other body must guide it by exerting on it a *deviating force* directed towards the centre of curvature. The revolving body re-acts on the guiding body with an equal and opposite *centrifugal force.*

RULE VI.—To find the deviating and centrifugal force of a given mass revolving with a given velocity in a circle of a given radius. Multiply the weight of the mass by the square of its linear velocity, and *divide* by the radius;—*or otherwise:* multiply the mass by the square of its angular velocity of revolution (see page 228), and *multiply* by the radius:—the result will be the value of the deviating and centrifugal forces in absolute units, which may be converted into units of weight by dividing by g.

REMARK.—The *resultant centrifugal force* of a rigid body of any shape is the same in amount and direction (though not the same in distribution) as if the whole mass were collected at its centre of gravity.

RULE VII.—To find the *height of a revolving pendulum* which makes a given number of revolutions per second; divide $\frac{g}{4\pi^2}$ by the square of the number of revolutions per second. (Approximate values of $\frac{g}{4\pi^2}$, being the height of the pendulum, which makes

λ, latitude of the place; observing that when 2 λ becomes obtuse, the term containing it is to be added instead of being subtracted; h, height above the level of the sea; and R, the earth's radius = 20,900,000 feet, or 6,370,000 metres, nearly.

one revolution per second; 0·815 foot = 9·78 inches = 0·248 metre nearly.)

N.B.—The *height* of a revolving pendulum is measured vertically, from the level of its centre of gravity to the level of the point where the line of suspension cuts the axis of revolution.

4. **Rotating Bodies—Fly-Wheels.**—As to the *moment of inertia* of a body turning about an axis, see pages 154 to 156.

RULE VIII.—To find the *angular momentum* of a rotating body; multiply its moment of inertia by its angular velocity in circular measure. (See page 102.)

RULE IX.—To find the *actual energy* of a rotating body; multiply either its angular momentum by half its angular velocity, or its moment of inertia by the half-square of its angular velocity; divide the product by g.

RULE X.—To find the moment of the couple required in order to produce a given change in the angular velocity of a rotating body, in the course of a given time, or of a given angular motion, as the case may be.

Case I.—If the time is given; divide the change of angular momentum by g, and by the time in seconds.

Case II.—If the angular motion is given; divide the change of actual energy by the angular motion in circular measure.

RULE XI.—Given, the alternate excess and deficiency (Δ E) of energy exerted as compared with work performed in a machine; to find the moment of inertia of a *fly-wheel*, such that the fluctuation of speed (or difference between the greatest and least speed) shall not exceed a given fraction of the mean speed $\left(\text{say } \frac{1}{m}\right)$. Let a be the mean angular velocity of the fly-wheel, I its required moment of inertia; then

$$\mathrm{I} = \frac{m\, g\, \Delta\, \mathrm{E}}{a^2}.$$

Ordinary values of m, from 30 to 60 nearly; of $m\ g$, in British Measures, from about 1,000 to 2,000.

TABLE of values of the ratio of the alternate excess and deficiency of energy, Δ E, to the whole work per revolution, $\int \mathrm{P}\, d\, s$, in steam-engines of various kinds (Morin).

NON-EXPANSIVE ENGINES.

$\dfrac{\text{Length of connecting rod}}{\text{Length of crank}}$	=	8	6	5	4
$\Delta\,\mathrm{E} \div \int \mathrm{P}\, d\, s$	=	·105	·118	·125	·132

Expansive Condensing Engines.

Connecting rod = crank × 5.

Fraction of stroke at which steam is cut off	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
$\Delta E \div \int P\,ds =$	·163	·173	·178	·184	·189	·191

Expansive Non-Condensing Engines.

Steam cut off at	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\Delta E \div \int P\,ds =$	·160	·186	·209	·232

For double cylinder expansive engines, the value of the ratio $\Delta E \div \int P\,ds$ may be taken as equal to that for single cylinder non-expansive engines.

For *tools working at intervals*, such as punching, slotting, and plate-cutting machines, coining presses, &c., ΔE is nearly equal to the whole work performed at each operation.

5. **Falling Bodies.**—The following rules apply to a body falling without sensible resistance from the air:—

Rule XII.—To find the velocity acquired at the end of a given time; multiply the time by g. (See page 245.)

Rule XIII.—To find the height of fall in a given time; multiply the square of the time by $\frac{1}{2} g$.

Rule XIV.—To find the height of fall corresponding (or "due") to a given velocity; divide the half-square of the velocity by g.

Rule XV.—To find the velocity due to a given height; multiply the height by $2g$, and extract the square root (or, in British Measures, multiply the square root of the height in feet by 8·025 for the velocity in feet per second; or, in French Measures, multiply the square root of the height in metres by 4·429 for the velocity in metres per second).

Table of Heights due to Velocities.

Explanation of Symbols.

v = Velocity in feet per second.
h = Height in feet = $v^2 \div 64{\cdot}4$.

This table is exact for latitude 54°¾, and near enough to exactness for practical purposes in all parts of the earth's surface.

v	h	v	h	v	h
1	·01553	27	11·320	54	45·280
2	·06211	28	12·174	56	48·695
3	·13975	29	13·059	58	52·235
4	·24845	30	13·975	60	55·901
5	·38820	31	14·922	62	59·688
6	·55901	32	15·901	64	63·602
7	·76087	32·2	16·100	64·4	64·400
8	·99379	33	16·910	66	67·640
9	1·2578	34	17·950	68	71·800
10	1·5528	35	19·022	70	76·087
11	1·8789	36	20·124	72	80·496
12	2·2360	37	21·257	74	85·029
13	2·6242	38	22·422	76	89·688
14	3·0435	39	23·618	78	94·472
15	3·4938	40	24·845	80	99·379
16	3·9752	41	26·102	82	104·41
17	4·4876	42	27·391	84	109·56
18	5·0311	43	28·711	86	114·84
19	5·6056	44	30·062	88	120·25
20	6·2112	45	31·444	90	125·78
21	6·8478	46	32·857	92	131·43
22	7·5155	47	34·301	94	137·20
23	8·2143	48	35·776	96	143·10
24	8·9441	49	37·283	98	149·13
25	9·7050	50	38·820	100	155·28
26	10·497	52	41·987		

6. **Reduced Inertia.**—RULE XVI.—To reduce the inertia or mass of a machine to the driving point. Multiply the weight of each moving portion of the machine by the square of the ratio of its velocity to the velocity of the driving point; and add together the products; the sum will be the weight of the mass which, if concentrated at the driving point, would require the same force to produce a given change in its speed, in the course of a given time or of a given motion, that is required by the actual machine.

SECTION V.—STRENGTH OF MACHINERY.

1. **Shafts.**—See pages 226, 227 for the relations between greatest twisting moment, greatest working stress, and diameter. As to the twisting moment for which provision is to be made, regard must be had not merely to the *mean moment* transmitted by the shaft, but to the *greatest moment.*

RULE XVII.—Given, the horse-power of the prime mover that drives a shaft, and the number of revolutions per minute; to find the *mean twisting moment:* multiply the horse-power by 5250, and divide by the turns per minute; the quotient will be the mean twisting moment in foot-lbs.; which, multiplied by 12, will give inch-lbs.

RULE XVIII.—In a shaft driven by steam-power, given, the mean twisting moment; to find the *greatest twisting moment;*

If the shaft is driven by a single engine, multiply by	1·6
If by a pair of engines, with cranks at right angles, multiply by	1·1
If by three engines, with cranks at angles of $\frac{1}{3}$ revolution, multiply by................................	1·05

2. **Rods.**—*Piston-rods* are to be treated as struts fixed at one end and jointed at the other. (See page 210, Rule XXIV.) *Connecting-rods* are to be treated as struts jointed at both ends. (See page 209, Rule XXIII.)

3. **Arms and Teeth of Wheels.**—RULE XIX.—To find the greatest bending moment on an *arm of a wheel;* divide the greatest twisting moment on the shaft by twice the number of arms.

RULE XX.—To find the greatest pressure exerted on a *tooth of a wheel;* divide the greatest twisting moment on the shaft by the perpendicular distance from the axis of the shaft to the line of action of the teeth.

As to the *thickness of teeth,* see page 233.

SECTION VI.—MUSCULAR POWER.

1. **General Principles.**—Let P be the effort exerted by an animal in performing work, V the velocity of the point at which the effort is applied, and T the time for which the effort P is exerted at the velocity V during a day's work; so that P V T is equal, or proportional, to the work done per day. Let P_1, V_1, T_1, be the values of P, V, and T, corresponding to the greatest day's work of the animal, $P_1 V_1 T_1$. Then for values of P, V, and T, not greatly deviating from P_1, V_1, and T_1, we have

$$\frac{P}{P_1} + \frac{V}{V_1} + \frac{T}{T_1} = 3;$$

so that when any five of those quantities are given, the sixth may be found.

Animals.	Approximate Values of				
	P_1	V_1		T_1	
	Lbs.	Ft. per sec.	Miles per hour.	Seconds.	Hours.
Good average draught horse,	120	3·6	2½ nearly.	28,800	8
High-bred horse,	64	7·2	5 „	28,800	8
Ox, ...	120	2·4	1·6 „	28,800	8
Mule, ...	60	3·6	2½ „	28,800	8
Ass, ...	30	3·6	2½ „	28,800	8

2. **Tables of Performance of Horses.**—Explanation of Table I.:—P, effort in lbs.; V, velocity, feet per second; T, hours' work per day; P V, work per second, in foot-lbs.; 3,600 P V T, work per day, in foot-lbs.

I.—Work of a Horse against a known Resistance.

Kind of Exertion.	P	V	T	P V	3,600 P V T
1. Cantering and trotting, drawing a light railway carriage (thoroughbred), ...	min. 22½ mean 30½ max. 50	14⅔	4	447½	6,444,000
2. Horse drawing cart or boat, walking (draught horse), ...	120	3·6	8	432	12,441,600
3. Horse driving a gin or mill, walking, ...	100	3·0	8	300	8,640,000
4. Ditto, trotting, ...	66	6·5	4½	429	6,949,800

Explanation of Table II.:—L, net load drawn or carried horizontally, in lbs.; V, velocity, feet per second; T, hours' work per day; L V, lbs. conveyed horizontally one foot per second; 3,600 L V T, lbs. conveyed horizontally one foot per day.

II.—Performance of a Horse in Transporting Loads Horizontally.

Kind of Exertion.	L	V	T	L V	3,600 L V T
5. Walking with cart, always loaded, ...	1,500	3·6	10	5,400	194,400,000
6. Trotting ditto, ...	750	7·2	4½	5,400	87,480,000
7. Walking with cart, going loaded, returning empty; V = ⅓ of mean velocity, ...	1,500	2·0	10	3,000	108,000,000
8. Carrying burden, walking, ...	270	3·6	10	972	34,992,000
9. Ditto, trotting, ...	180	7·2	7	1,296	32,659,200

3. **Tables of Work of Men.**—Explanation of Table I.:—P, effort, lbs.; V, velocity, feet per second; T, hours' work per day; P V, work, foot-lbs. per second; 3,600 P V T, work, foot-lbs. per day.

I.—Work of a Man against Known Resistances.

Kind of Exertion.	P	V	T	P V	3,600 P V T
1. Raising his own weight up stair or ladder, ...	143	0·5	8	72·5	2,088,000
2. Hauling up weights with rope, and lowering the rope unloaded, ...	40	0·75	6	30	648,000
3. Lifting weights by hand,.....	44	0·55	6	24·2	522,720
4. Carrying weights up stairs, and returning unloaded,...	143	0·13	6	18·5	399,600
5. Shovelling up earth to a height of 5 ft. 3 in.,.........	6	1·3	10	7·8	280,800
6. Wheeling earth in barrow up slope of 1 in 12, ½ horiz. veloc. 0·9 ft. per sec., and returning unloaded,.........	132	0·075	10	9·9	356,400
7. Pushing or pulling horizontally (capstan or oar),......	26·5	2·0	8	53	1,526,400
8. Turning a crank or winch,...	12·5	5·0	?	62·5	...
	18·0	2·5	8	45	1,296,000
	20·0	14·4	2 mins.	288	...
9. Working pump,.................	13·2	2·5	10	33	1,188,000
10. Hammering,......................	15	?	8 ?	?	480,000

Explanation of Table II.:—L, load conveyed horizontally, lbs.; V, velocity, feet per second; T, hours' work per day; L V, lbs. conveyed horizontally one foot in a second; 3,600 L V T, lbs. conveyed horizontally one foot in a day.

II.—Performance of a Man in Transporting Loads Horizontally.

Kind of Exertion.	L	V	T	L V	3,600 L V T
11. Walking unloaded, transport of own weight, ...	140	5	10	700	25,200,000
12. Wheeling load in 2-wheeled barrow: returning unloaded,	224	$1\frac{2}{3}$	10	373	13,428,000
13. Ditto in 1-wh. barrow, ditto,	132	$1\frac{2}{3}$	10	220	7,920,000
14. Travelling with burden, ...	90	$2\frac{1}{2}$	7	225	5,670,000
15. Carrying burden, returning unloaded, ...	140	$1\frac{2}{3}$	6	223	5,032,800
16. Carrying burden for 30 seconds only,....................	252	0	...	0	...
	126	11·7	...	1474·2	...
	0	23·1	...	0	...

III.—Day's Work of a Man required for various Operations. (Day = 10 Hours.)

Shovelling earth, one cubic yard, thrown not more than 5 feet vertically up; if dry,........	from	·05	to ·0625
Ditto, wet mud,..	„	·06	to ·08
Excavating earth with the pick, one cubic yard,..	„	·025	to ·2
Wheeling one cubic yard of earth in barrows from 100 to 120 feet horizontally; if up a slope at the same time, deduct 6 feet from horizontal distance for each foot of total rise,..	„	·05	to ·0625
Spreading and ramming earth in layers from 9 to 18 inches deep, one cubic yard,..............	„	·06	to ·07
Dressing slopes of cuttings, one square yard,....	about	·008	
Soiling slopes, 6 inches thick, one square yard,	„	·008	
Making clay puddle, one cubic yard,............	„	·3	
Spreading do., do., ...	„	·3	
Quarrying rock of moderate hardness with wedges,..................................average	„	·4	
Quarrying rock of moderate hardness by blasting,*..average	„	·45	
Jumping holes in rock, 100 cylindrical inches, granite,..from		1·0	to ·5
Do. do. do., limestone,	„	·2	to ·15
Driving mines in rock; dimensions from $3\frac{1}{2}$ feet × $3\frac{1}{2}$ feet to $3\frac{1}{2}$ feet × 5 feet; one foot forward,..	„	2·0	to 5·0
Quarrying rock in tunnels, one cubic yard,......	„	·75	to 3·0
Making one thousand bricks,...... men's time,		1·125	
Making one thousand bricks,...... boys' time,		0·75	
Mixing mortar by hand, one cubic yard,.........		·75	
Mixing concrete, wheeling and laying, one cubic yard,..		·3	
Loading barrows with stone, one cubic yard,...		·06	
Wheeling one cubic yard of stone 100 feet horizontally; if on an ascent, allow 6 feet of distance for each foot of rise,................		·045	
Unloading barrows of stone, one cubic yard,....		·03	

* Weight of rock loosened ÷ weight of powder exploded = in small blasts from 7,000 to 14,000; average 10,000: in great blasts from 4,500 to 13,000; average between 6,000 and 7,000. One lb. of blasting powder fills about 30 cubic inches = 38 cylindrical inches. If gun-cotton be used instead of powder, allow one-sixth of the weight and one-half of the space.

Stone Masonry, one cubic yard.		Breaking Stone.	Cutting Stone.	Building.	Labourers' Work.
Dry stone,		·64	—	1·00	·50
Coursed rubble,		·64	—	·90	·90
Block-in-course,		·90	1·5	·90	·90
Do. arching,		·90	2·25	·90	·90
Ashlar (soft sandstone),	from	1·80	2·50	1·00	1·00
	to	2·50	6·00	2·00	2·00

Breaking and stone cutting for harder stones;
hard sandstone = soft sandstone × 2.
hard limestone, marble, granite = soft sandstone × from 3 to 4.

Facing ashlar (soft sandstone), per square foot—
stroked, ·05; droved, ·07; polished, ·1.

$$\text{Curved facing} = \text{flat} \times \left(1 + \frac{2\frac{1}{2}}{\text{radius in feet}}\right).$$

Taking down old masonry, one cubic yard, from ·5 to ·6.

	Bricklayer.	Labourer.	Erecting scaffolding.
Brickwork, ordinary, one cubic yard,	·6	·6	·2
" arching and other curved work,	·9	·9	various; depending on centering.

" in tunnels, about double of similar brickwork above ground.

	Bricklayer.	Labourer.
Laying and jointing drain pipes, one lineal foot, *per inch diameter*,	·0025	·0025

Sinking cylinders for foundations under water with compressed air*; per cubic yard of earth removed, ·67

Sawing timber, one square foot;

Pine and fir,	from	·0045 to ·005
Ash, elm, beech, mahogany,	"	·0065 to ·007
Oak,	"	·0075 to ·009
Teak,		·01

Shaping timber; pine-woods; one cubic foot,	from	·04 to ·135
Planing pine woods, per square foot,		·013
Boring hole $\frac{3}{4}$ diameter, one lineal foot, in pine-woods,		·02
Do. do., in hard leaf-woods,		·03

* Supply of air should be at the rate of 30 cubic feet per man per minute.

	Carpenter.	Labourer.
Erecting centres for arches; per 100 square feet area of soffit,................	from 1·55 to... 1·70	·75 ·80

	Men's time.	Boys' time.
Rivetting iron ships; from 100 to 140 rivets,..................................	3·0	from 1·0 to 2·0
Making plank roads; breadth planked, 8 feet; total breadth, 16 feet; 1 lineal foot,..		1·0

PART VIII.

HYDRAULICS.

SECTION I.—RULES RELATING TO THE FLOW OF WATER.

1. **Head of Water.**—RULE I.—To find the *head* of a particle of water; add together the *head of elevation*, or height of the particle above some fixed or "datum" level, and the *head of pressure*, or intensity of the pressure exerted by the particle expressed as the height of an equivalent column of water. (See pages 103, 115.)

In stating the pressure, it is usual *not* to include the atmospheric pressure; so that the absolute pressure exceeds the pressure stated in the common way by one atmosphere. When the absolute pressure is equal to the atmospheric pressure, the pressure stated in the common way is = 0; when the absolute pressure falls short of the atmospheric pressure, their difference is called *vacuum*.

The atmospheric pressure, at the level of the sea, varies from about 32 to 35 feet of water, and diminishes nearly at the rate of 1-100th part of itself for each 262 feet of elevation.

In the rest of this Section, heads in feet of water will be denoted by h.

2. **Volume and Velocity of Flow.**—RULE II.—To find the *volume of flow* of a stream; multiply the mean velocity by the sectional area.

RULE III.—To find the *mean velocity* of flow of a stream; divide the volume of flow by the sectional area.

RULE IV.—In a stream like a river channel the ratio of the mean velocity to the greatest velocity (which occurs at the middle of the stream) is nearly =

$$\frac{\text{greatest velocity} + 7\cdot71 \text{ feet per second}}{\text{greatest velocity} + 10\cdot28 \text{ feet per second}}.$$

The *least velocity*, being that of the particles in contact with the bed, is nearly as much less than the mean velocity as the greatest velocity is greater than the mean. In ordinary currents the least, mean, and greatest velocities are nearly as 3 : 4 : 5; in very slow currents, as 2 : 3 : 4.

In what follows, *volume of flow in cubic feet per second* will be denoted by Q; the *mean velocity* of a stream in *feet per second*

by v; and the *sectional area* in *square feet* by A; so that $Q = v A$.

3. **Relation between Head and Velocity.**—RULE V.—*Theoretical head, h, due to a given velocity, v;*

$$h = \frac{v^2}{2g} = \frac{v^2}{64 \cdot 4}. \quad \text{(See Table, page 249.)}$$

RULE VI.—*Theoretical velocity, v,* due to a given head, h;

$$v = 8 \cdot 025 \sqrt{h}.$$

RULE VII.—To find the *loss of head, h,* due to a given *gain of velocity* in a stream; let the *velocity of approach* (or original velocity, at the point where the greater head is) be the fraction, n, of the *velocity of discharge;* let v be the velocity of discharge; and let F be a *factor of resistance* (as to which, see next Article); then

$$h = (1 + F - n^2) \frac{v^2}{64 \cdot 4}.$$

RULE VIII.—To find the velocity of discharge due to a given loss of head;

$$v = 8 \cdot 025 \sqrt{\left(\frac{h}{1 + F - n^2}\right)}.$$

REMARK.—n is the ratio of the sectional area of the channel of discharge to that of the channel of approach. When those areas are equal, as in an *uniform channel* or an *uniform pipe,* $1 - n^2 = 0$; and then the formulæ become

$$h = \frac{F v^2}{64 \cdot 4}; \; v = 8 \cdot 025 \sqrt{\frac{h}{F}}.$$

4. **Factors of Resistance.**—Values of F in Rules VII. and VIII.
(1.) *Friction of an orifice in a thin plate*—

$$F = 0 \cdot 054.$$

(2.) *Friction of mouthpieces, or entrances from reservoirs into pipes.*—Straight cylindrical mouthpiece, perpendicular to side of reservoir—

$$F = 0 \cdot 505.$$

The same mouthpiece making the angle θ with a perpendicular to the side of the reservoir—

$$F = 0 \cdot 505 + 0 \cdot 303 \sin \theta + 0 \cdot 226 \sin^2 \theta.$$

For a mouthpiece of the form of the "contracted vein"—that is,

one somewhat bell-shaped—and so proportioned that if d be its diameter on leaving the reservoir, then at a distance $d \div 2$ from the side of the reservoir it contracts to the diameter $\cdot 7854\ d$,—the resistance is insensible, and F nearly $= 0$.

(3.) *Friction at sudden enlargements.*—Let A_1 be the sectional area of a channel, in which a sluice, or slide valve, or some such object, produces a sudden contraction to the smaller area a, followed by a sudden enlargement to the area A_2. Let v in the formulæ of Rules VII. and VIII. stand for the velocity in the second enlarged part of the channel, so that $Q = A_2 v$. Let

$$n = \frac{A_2}{a} \sqrt{\left(2{\cdot}618 - 1{\cdot}618 \frac{a^2}{A_1^2}\right)}.$$

Then

$$F = (n - 1)^2.$$

(4.) *Friction in pipes and conduits.*—Let A be the sectional area of a channel; b, its *border*—that is, the length of that part of its girth which is in contact with the water; l, the length of the channel, so that $l\,b$ is the frictional surface; and for brevity's sake let $A \div b = m$; then, for the friction between the water and the sides of the channel,

$$F = f \cdot \frac{l\,b}{A} = \frac{f\,l}{m};$$

Let d = diameter of pipe in feet; then

For iron pipes (not pitch-lined)*... $f = 0{\cdot}005\left(1 + \frac{l}{12d}\right)$;

For open conduits,............... $f = 0{\cdot}00741 + \frac{0{\cdot}000227}{v}$.

The quantity $m = A \div b$ is called the "*hydraulic mean depth*" of channel, and for cylindrical and square pipes running full is *one-fourth* of the diameter.

Rule IX.—To find the *declivity* (i) in an uniform channel of a given hydraulic mean depth (m);

$$i = \frac{h}{l} = \frac{f}{m} \cdot \frac{v^2}{2\,g}.$$

In an open channel this is an actual slope of the surface of the water. In a close pipe it may be a *virtual declivity*, due wholly or partly to diminution of pressure.

* In iron pipes lined with smooth pitch the co-efficient of friction is about one-sixth part less than in unlined pipes.

(5.) For *bends in circular pipes*, let d be the diameter of the pipe; ρ, the radius of curvature of its centre line at the bend; θ, the angle through which it is bent; π, two right angles; then

$$F = \frac{\theta}{\pi} \left\{ 0{\cdot}131 + 1{\cdot}847 \left(\frac{d}{2\,\rho} \right)^{\frac{7}{2}} \right\}.$$

(6.) For *bends in rectangular pipes*,

$$F = \frac{\theta}{\pi} \left\{ 0{\cdot}124 + 3{\cdot}104 \left(\frac{d}{2\,\rho} \right)^{\frac{7}{2}} \right\}.$$

(7.) For *knees*, or sharp turns in pipes, let θ be the angle made by the two portions of the pipe at the knee; then

$$F = 0{\cdot}946 \sin^2 \frac{\theta}{2} + 2{\cdot}05 \sin^4 \frac{\theta}{2}.$$

RULE X. *Summary of losses of head.*—When several successive causes of resistance occur in the course of one stream, the losses of head arising from them are to be added together; and this process may be extended to cases in which the velocity varies in different parts of the channel, in the following manner:—

Let the final velocity, at the cross-section where the loss of head is required, be denoted by v;

Let the ratios borne to that velocity by the velocities in other parts of the channel be known; $n_0\, v$ being the "velocity of approach," $n_1\, v$ the velocity in the first division of the channel, $n_2\, v$ in the second, and so on; and let F_1 be the sum of all the factors of resistance for the first division, F_2 for the second, and so on; then the loss of head will be

$$h = \frac{v^2}{64{\cdot}4} (1 - n_0^2 + F_1\, n_1^2 + F_2\, n_2^2 + \&c.)$$

5. **Contraction of Stream—Co-efficients of Discharge.**—RULE XI.—To find the *effective area* of an outlet; multiply the total area by a fraction called the *co-efficient of contraction.*

For uniform streams there is no contraction, and the co-efficient is 1.

REMARK.—Sometimes it is impossible to distinguish between the effect of friction in diminishing the velocity (expressed by $1 \div \sqrt{1 + F}$), and that of contraction in diminishing the area of the stream. In such cases the ratio in which the actual discharge is less than the product of the theoretical velocity and the total area of the orifice is called the *co-efficient of efflux* or of *discharge.*

The quantities given in the following statements and tables are some of them real co-efficients of contraction, and some co-efficients

of discharge. In hydraulic formulæ such co-efficients are usually denoted by the symbol c.

(1.) *Sharp-edged circular orifices in flat plates;* $c = \cdot 618$.

(2.) *Sharp-edged rectangular orifices in vertical flat plates.*—In this case the co-efficient is intended to be used in the following formula for the discharge in cubic feet per second, A being the area of the orifice in square feet; and h the head, measured from the *centre* of the orifice to the *level of still water*.

$$Q = 8{\cdot}025\, c\, A\, \sqrt{h}.$$

Co-efficients of Discharge for Rectangular Orifices.

Head ÷ Breadth.	Height of Orifice ÷ Breadth. 1	0·5	0·25	0·15	0·1	0·05
0·05	...	...	...	...	...	·709
0·10	...	...	...	...	·660	·698
0·15	...	...	...	·638	·660	·691
0·20	...	...	·612	·640	·659	·685
0·25	...	...	·617	·640	·659	·682
0·30	...	·590	·622	·640	·658	·678
0·40	...	·600	·626	·639	·657	·671
0·50	...	·605	·628	·638	·655	·667
0·60	·572	·609	·630	·637	·654	·664
0·75	·585	·611	·631	·635	·653	·660
1·00	·592	·613	·634	·634	·650	·655
1·50	·598	·616	·632	·632	·645	·650
2·00	·600	·617	·631	·631	·642	·647
2·50	·602	·617	·631	·630	·640	·643
3·50	·604	·616	·629	·629	·637	·638
4·00	·605	·615	·627	·627	·632	·627
6·00	·604	·613	·623	·623	·625	·621
8·00	·602	·611	·619	·619	·618	·616
10·00	·601	·607	·613	·613	·613	·613
15·00	·601	·603	·606	·607	·608	·609

(3.) *Sharp-edged rectangular notches in flat vertical weir boards.*—The area of the orifice is measured up to the *level of still water* in the pond behind the weir.

Let b = breadth of the notch;

B = total breadth of the weir; then

$$c = {\cdot}57 + \frac{b}{10\,B};$$

provided b is not less than B ÷ 4.

(4.) *Sharp-edged triangular or V-shaped notches in flat vertical weir boards* (from experiments by Professor James Thomson).—Area measured up to the level of still water.

$$\text{Breadth of notch} = \text{depth} \times 2;\ c = \cdot 595;$$
$$\text{Breadth of notch} = \text{depth} \times 4;\ c = \cdot 620.$$

(5.) *Partially-contracted sharp-edged orifice.*—(That is to say, an orifice towards part of the edge of which the water is guided in a direct course, owing to the border of the channel of approach partly coinciding with the edge of the orifice.)

Let c be the ordinary co-efficient;
n, the fraction of the edge of the orifice which coincides with the border of the channel;
c', the modified co-efficient; then

$$c' = c + \cdot 09\, n.$$

(6.) *Flat or round-topped weir*, area measured up to the level of still water—

$$c = \cdot 5 \text{ nearly.}$$

(7.) *Sluice in a rectangular channel*—

$$\text{vertical};\ c = 0\cdot 7;$$

Inclined backwards to the horizon at 60°; $c = 0\cdot 74$;
" " " at 45°; $c = 0\cdot 8$.

(8.) *Incomplete contraction.*—Let A be the area of a pipe partially closed by a partition, having in it an orifice of the total area a and effective area $c\,a$; then

$$c = \frac{\cdot 618}{\sqrt{\left(1 - \cdot 618 \frac{a^2}{A^2}\right)}}$$

6. **Discharge from Sluices and Notches.**—Let b be the breadth of the orifice; h_0, the depth of its upper edge, and h_1, that of its lower edge, below the level of still water in the pond; c, the co-efficient of contraction (see last Article); Q, the discharge in cubic feet per second.

RULE XII.—*Rectangular orifice*—

$$Q = 8\cdot 025\, c \times \frac{2}{3} b \left(h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}\right) = 5\cdot 35\, c\, b \left(h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}\right).$$

RULE XIII.—*Rectangular notch, with a still pond;* $h_0 = 0$; h_1 measured from the lower edge of the notch to the level of still water.

$$Q = 8\cdot 025\, c \times \frac{2}{3} b\, h_1^{\frac{3}{2}} = 5\cdot 35\, c\, b\, h_1^{\frac{3}{2}} = \left(3\cdot 05 + \cdot 535 \frac{b}{B}\right) b\, h_1^{\frac{3}{2}}.$$

TABLE OF VALUES OF c AND 5·35 c.

$\frac{b}{B}$,.....	1·0	0·9	0·8	0·7	0·6	0·5	0·4	0·3	0·25
c,......	·67	·66	·65	·64	·63	·62	·61	·60	·595
5·35 c,	3·58	3·53	3·48	3·42	3·37	3·32	3·26	3·21	3·18

The *cube of the square root of the head*, $h_1^{\frac{3}{2}}$, is easily computed as follows, by the aid of an ordinary table of squares and cubes: look in the column of squares for the nearest square to h_1; then opposite, in the column of cubes, will be an approximate value of $h_1^{\frac{3}{2}}$.

RULE XIV.—*Rectangular notch, with current approaching it.*—When still water cannot be found, to measure the head h_1 up to, let v_0 denote the velocity of the current at the point up to which the head is measured, or *velocity of approach*: compute the height due to that velocity as follows:—

$$h_0 = v_0^2 \div 64 \cdot 4;$$

then,

$$Q = 5 \cdot 35\ c\ b \left\{(h_1 + h_0)^{\frac{3}{2}} - h_0^{\frac{3}{2}}\right\}.$$

RULE XV.—*Triangular or V-shaped notch, with a still pond;* h_1 measured from the apex of the triangle to the level of still water.

Let a denote the ratio of the *half-breadth* of the notch at any given level to the height above the apex, so that, for example, at the level of still water, the whole breadth of the notch is $2\ a\ h_1$;

$$Q = 8 \cdot 025\ c \times \frac{8}{15}\, a\, h_1^{\frac{5}{2}} = 4 \cdot 28\ c\ a\ h_1^{\frac{5}{2}};$$

and adopting the values of c already given, we have,

$$\text{for } a = 1,\ Q = 2 \cdot 54\ h_1^{\frac{5}{2}};\ \text{for } a = 2,\ Q = 5 \cdot 3\ h_1^{\frac{5}{2}}.$$

For squares and fifth powers, see page 32.

RULE XVI.—*Drowned orifices* are those which are below the level of the water in the space into which the water flows as well as in that from which it flows. In such cases the difference of the levels of still water in those two spaces is the head to be used in computing the flow.

RULE XVII.—*Drowned rectangular notch.*—Let h_1 and h_2 be the heights of the still water above the lower edge of the notch at the up-stream and down-stream sides of the notch-board respectively;

$$Q = 5 \cdot 35\ c\ b\ \left(h_1 + \frac{h_2}{2}\right) \surd\,(h_1 - h_2).$$

RULE XVIII.—For *weirs with broad flat crests*, drowned or undrowned, the formulæ are the same as for rectangular notches, except that the co-efficient c is about $\cdot 5$.

RULE XIX.—*Computation of the dimensions of orifices.*—Most of the preceding formulæ can be used in an inverse form, in order to find the dimensions of orifices that are required to discharge given volumes of water per second.

For example, if RULE XII. is applicable, the breadth of the orifice is given as follows:—

$$b = Q \div 5{\cdot}35\, c\, (h_1^{\frac{3}{2}} - h_0^{\frac{3}{2}}).$$

If RULE XIII. is applicable, the depth of the bottom of the notch below still water is given by the equation,

$$h_1 = \{Q \div 5{\cdot}35\, c\, b\}^{\frac{2}{3}}.$$

If RULE XV. is applicable,

$$h_1 = \{Q \div 4{\cdot}28\, c\, a\}^{\frac{2}{5}}.$$

7. **Discharge of Water-Pipes.**—RULE XX.—To find the loss of head, h, in a length, l, of a pipe of the uniform diameter, d (all dimensions in feet);

$$h = \frac{4fl}{d} \cdot \frac{v^2}{64{\cdot}4} = \cdot 02\left(1 + \frac{l}{12d}\right)\frac{l}{d} \cdot \frac{v^2}{64{\cdot}4}.$$

RULE XXI.—*To compute the discharge of a given pipe;* the data being h, l, and d, all in feet.

For a rough approximation, we may take an average value for $4f$. The value commonly assumed is $\cdot 0258$. This gives for the approximate velocity

$$v = 8{\cdot}025\sqrt{\frac{hd}{\cdot 0258\, l}} = 50\sqrt{\frac{hd}{l}};$$

or, *a mean proportional between the diameter and the loss of head in* 2,500 *feet of length.* When greater precision is required, make

$$4f = \cdot 02\left(1 + \frac{l}{12d}\right); \quad v = 8{\cdot}025\sqrt{\frac{hd}{4fl}}.$$

Then the discharge is given by the formula,

$$Q = \cdot 7854\, v\, d^2.$$

RULE XXII.—*To find (in feet) the diameter* d *of a pipe, so that it shall deliver* Q *cubic feet of water per second, with a loss of head at the rate of* h *feet in each length of* l *feet.*

Assume, as a first approximation, $4f' = \cdot 0258$. This gives, as a first approximation to the diameter,

$$d' = 0\cdot 23\left(\frac{l\,Q^2}{h}\right)^{\frac{1}{5}};$$

Compute a second approximation,

$$4f'' = 0\cdot 02\left(1 + \frac{l}{12d'}\right);$$

if this is $= 4f'$, d' is the true diameter; if not, a corrected diameter is to be calculated as follows:—

$$d = d'.\left(\frac{f''}{f'}\right)^{\frac{1}{5}} = d'.\left(\frac{4}{5} + \frac{f''}{5f'}\right) \text{ nearly.}$$

In the preceding formulæ the pipe is supposed to be free from all curves and bends so sharp as to produce appreciable resistance. Should such obstructions occur in its course, they may be allowed for in the following manner:—Having first computed the diameter of the pipe as for a straight course, calculate the additional loss of head due to curves by the proper formula (Article 4, page 259); let h'' denote that additional loss of head; then make a further correction of the diameter of the pipe, by increasing it in the ratio of

$$1 + \frac{h''}{5\,h} : 1.$$

By a similar process an allowance may be made for the loss of head on first entering the pipe from the reservoir, viz.:—

$(1 + \text{F})\,v^2 \div 64\cdot 4$; F being the factor of friction of the mouthpiece.

The preceding rules are for clean iron pipes. To allow for incrustation, add *one inch* to the diameter of all pipes.

8. **Discharge and Dimensions of Channels.**—RULE XXIII.—To find the declivity, i, of the upper surface of the water in a channel of the hydraulic mean depth m;

$$i = \frac{h}{l} = \frac{f}{m} \cdot \frac{v^2}{64\cdot 4} = \left(\cdot 00741 + \frac{\cdot 000227}{v}\right) \cdot \frac{v^2}{64\cdot 4\,m}.$$

RULE XXIV.—*To compute the discharge of a given stream,* the data being i, m, and the sectional area A. Assume an *approximate*

value for the co-efficient of friction, such as $f' = \cdot 007565$; then the *first approximation* to the velocity is

$$v' = 8\cdot025\sqrt{\frac{i\,m}{\cdot007565}} = \sqrt{8512\,i\,m} = 92\cdot26\sqrt{i\,m};$$

or, *a mean proportional between the hydraulic mean depth and the fall in* 8,512 *feet*. A *first approximation to the discharge* is $Q' = v'\,A$.

These first approximations are in many cases sufficiently accurate. To obtain second approximations, compute a corrected value of f according to the expression in brackets in Rule XXIII; should it agree nearly or exactly with f', the first assumed value, it is unnecessary to proceed further; should it not so agree, correct the values of the velocity and discharge by multiplying each of them by the factor, $\frac{3}{2} - \frac{f}{\cdot01513}$.

RULE XXV.—*To determine the dimensions of an uniform channel which shall discharge* Q *cubic feet of water per second with the declivity* i. Assume a figure for the intended channel, so that the *proportions* of all its dimensions to each other, and to the hydraulic mean depth m, may be fixed. This will fix also the proportion $A \div m^2$ of the sectional area to the square of the hydraulic mean depth, which will be known although those areas are still unknown; let it be denoted by n.

Compute *first approximations* to the hydraulic mean depth and velocity as follows:—

$$m' = \left(\frac{Q^2}{8{,}512\,n^2\,i}\right)^{\frac{1}{5}};\ v' = \frac{Q}{n\,m'^2};$$

from these data, by means of Rule XXIII., compute an *approximate declivity*, i'. If this agrees exactly or very nearly with the given declivity, i, the first approximation to the hydraulic mean depth is sufficient; if not, a *corrected hydraulic mean depth* is to be found by the following formula:—

$$m = m'\left(\frac{4}{5} + \frac{i'}{5\,i}\right).$$

From the hydraulic mean depth all the dimensions of the channel are to be deduced, according to the figure assumed for it.

9. **Swell and Backwater Produced by a Weir.**—When a weir or dam is erected across a river, to calculate the height, h_1, in feet, at which the water in the pond, close behind the weir, will stand above its crest; Q being the discharge in cubic feet per second, and b the breadth of the weir in feet;

RULE XXVI.—*Weir not drowned,* with a flat or slightly rounded crest—

$$h_1 = \left(\frac{Q^2}{7\,b^2}\right)^{\frac{1}{3}}, \text{ nearly.}$$

RULE XXVII.—*Weir drowned.*—Let h_2 be the height of the water in front of the weir above its crest.

$$\textit{First approximation; } h'_1 = h_2 + \left(\frac{Q^2}{7\,b^2}\right)^{\frac{1}{3}}.$$

$$\textit{Second approximation; } h''_1 = h'_1 - h_2\left(1 - \frac{5}{4}\cdot\frac{h_2}{h'_1 - h_2}\right).$$

RULE XXVIII.—In a channel of uniform breadth and declivity—

Let i denote the rate of inclination of the *bottom* of the stream, which is also the rate of inclination of its surface before being altered by the weir.

Let δ_0 be the natural depth of the stream, before the erection of the weir.

Let δ_1 be the depth as altered, close behind the weir.

Let δ_2 be any other depth in the *backwater,* or altered part of the stream.

It is required to find x, the distance from the weir in a direction up the stream at which the altered depth δ_2 will be found.

Denote the ratio in which the depth is altered at any point by $\delta \div \delta_0 = r$; and let ϕ denote the following function of that ratio:—

$$\phi = \int \frac{d\,r}{r^3 - 1} = \frac{1}{6}\,\text{hyp. log.}\left\{1 + \frac{3\,r}{(r-1)^2}\right\}$$

$$+ \frac{1}{\sqrt{3}}\,\text{arc. tan.}\,\frac{2\,r+1}{\sqrt{3}} = \frac{1}{2\,r^2} + \frac{1}{5\,r^5} + \frac{1}{8\,r^8}, \text{ nearly.}$$

Compute the values, ϕ_1 and ϕ_2, of this function, corresponding to the ratios $r_1 = \delta_1 \div \delta_0$ and $r_2 = \delta_2 \div \delta_0$. Then

$$x = \frac{\delta_1 - \delta_2}{i} + \left(\frac{1}{i} - 264\right)\cdot(\phi_1 - \phi_2)\,\delta_0.$$

The following table gives some values of ϕ:—

r	ϕ	r	ϕ
1·0	∞	1·8	·166
1·1	·680	1·9	·147
1·2	·480	2·0	·132
1·3	·376	2·2	·107
1·4	·304	2·4	·089
1·5	·255	2·6	·076
1·6	·218	2·8	·065
1·7	·189	3·0	·056

10. **Time of Emptying a Reservoir.**—RULE XXIX.—Let Q be the rate of discharge at the outlet, supposing the reservoir kept constantly full; W, the whole volume of water in it. Then

	Time in Seconds =
For a vertical-sided reservoir of uniform depth,........	$\frac{2W}{Q}$
For a wedge-shaped reservoir (triangular vertical sections; maximum depth of the sections uniform),	$\frac{4W}{3Q}$
For a pyramidal reservoir (base at the surface, apex at the outlet),...	$\frac{6W}{5Q}$

RULE XXX.—To find the time required to *equalize the water-level* in two adjoining basins with vertical sides; calculate the time required to empty a vertical-sided reservoir containing a volume of water equal to the volume transferred, and of a depth equal to the greatest difference of water-level between the basins.

11. **Cascade from a Weir-Crest.**—RULE XXXI.—To find the horizontal distance to which the cascade of water from a weir-crest will shoot in the course of a given fall below that crest; take once-and-a-third of a mean proportional between that fall and the height from the weir-crest to still water in the pond.

12. **Rain-Fall.**

Inches Depth of Rain-fall.	Cubic feet on an acre.	Gallons on an acre.	Cubic feet on a square mile.	Gallons on a square mile	Inches Depth of Rain-fall.
1	3,630	22,635	2,323,200	14,486,314	1
2	7,260	45,270	4,646,400	28,972,627	2
3	10,890	67,905	6,969,600	43,458,941	3
4	14,520	90,539	9,292,800	57,945,254	4
5	18,150	113,174	11,616,000	72,431,568	5
6	21,780	135,809	13,939,200	86,917,882	6
7	25,410	158,444	16,262,400	101,404,195	7
8	29,040	181,079	18,585,600	115,890,509	8
9	32,670	203,714	20,908,800	130,376,822	9
10	36,300	226,349	23,232,000	144,863,136	10

For the conversion of cubic feet into gallons, and gallons into cubic feet, see page 109.

An *inch of rain per annum on an acre* is roughly equivalent to *ten cubic feet per day.*

An *inch of rain per annum on a square mile* is roughly equivalent to *forty thousand gallons per day.*

Annual depth of rain-fall in different countries and seasons ranges from 0 to 150 inches.

In Britain, different seasons and districts, 15 to 100 and upwards.

Ratio of available to total rain-fall on gathering-grounds; steep impervious rock, from 1·0 to 0·8; moorland and hilly pasture, from

·8 to ·6; cultivated land, from ·5 to ·4, and sometimes less; chalk, 0.

Greatest depths of rain in short periods: one hour, 1 inch; four hours, 2 inches; twenty-four hours, 5 inches.

13. **Stability of Bed of Stream.**—Greatest velocities *of the current close to the bed*, consistent with the stability of various materials:—

Soft clay,	0·25 foot per second.
Fine sand,	0·50 " "
Coarse sand, and gravel as large as peas,	0·70 " "
Gravel as large as French beans,	1·00 " "
Gravel 1 inch in diameter,	2·25 feet per second.
Pebbles 1½ inch diameter,	3·33 " "
Heavy shingle,	4·00 " "
Soft rock, brick, earthenware,	4·50 " "
Rock, various kinds,	6·00 " " and upwards.

14. **Strength of Water-Pipes.**—Rule XXXII.—To find the *least proper thickness* of metal for a cast-iron pipe of a given bore, to bear a given pressure from within.

First; divide the greatest pressure, *in feet of water* (see page 103) by 12,000, and multiply the bore or internal diameter of the pipe by the quotient: secondly; take a mean proportional between the internal diameter and *one forty-eighth of an inch*: the *greater* of those two quantities will be the required thickness.

Rule XXXIII.—To find the *greatest working pressure*, in feet of water, which a cast-iron pipe will safely bear; multiply the thickness by 12,000, and divide by the internal diameter.

The *bursting pressure* should be six times the working pressure.

As to the *weight* of pipes, *in lbs. to the foot*, see pages 149 and 153.

Rule XXXIV.—For the weight of one foot of a cast-iron pipe, in fractions of a ton; multiply the difference of the squares of the outside and inside diameters by ·00108.

A *faucet* on a 9 feet length of pipe adds between one-tenth and one-twentieth to the weight.

15. **Demand for Water in Towns.**

	Gallons per head per day.
Used for domestic purposes (liberal supply),	15
Washing streets, extinguishing fires, supplying fountains &c.,	3
Trade and manufactures,	7
Total usefully consumed,	25
Waste, under careful regulation,	2½
Total, under careful regulation,	27½
Additional waste, in some cases,	22½
Total in some cases,	50

Greatest hourly demand = from 2 to $2\frac{1}{3}$ × average hourly demand.

Demand as to head, 20 feet above house-tops (after deducting loss of head due to velocity and friction in pipes).

SECTION II.—RULES RELATING TO HYDRAULIC PRIME MOVERS.

1. **General Rules.**—RULE I.—To calculate the total or *gross power* of a fall of water. To the actual *head*, or depth of fall (from the surface of the head-race to the surface of the tail-race), add the height due to the velocity of the water in the head-race. (As to heights due to velocities, see pages 248, 249.) Multiply the sum (or *total head*) by the volume of the flow of water per second, and by the heaviness of water (62·4 lbs. to the cubic foot). The product will be the gross power in foot-lbs. per second. This divided by 550 gives the gross horse-power.

REMARK.—The dimensions of the head-race and tail-race are to be fixed by means of the principles of the preceding section, pages 264, 265.

RULE II.—To estimate the *net* or *effective power* of a fall of water; multiply the gross power by the probable efficiency of the kind of prime mover to be used. That efficiency is a fraction ranging,

for water-pressure engines, from 0·65 to 0·75;
for overshot and breast wheels, from 0·7 to 0·8;
for undershot wheels, from 0·4 to 0·6;
for a drowned wheel, $\frac{3}{4}$ of the efficiency of the same wheel not drowned;
for turbines, from 0·6 to 0·8.

RULE III.—The *velocity of greatest efficiency* for a water-wheel is as follows:—

Case I.—For wheels which act wholly by impulse, or partly by impulse and partly by weight, from 0·4 to 0·6 (or on an average one-half) of the velocity of the feed-water;

Case II.—For turbines acting by pressure, the velocity due to half the head (that is, 0·7 of the velocity due to the whole head).

In Cases I. and II. the surface-velocity is measured at the place where the wheel receives the water.

Case III.—For re-action wheels, the velocity measured at the *outlets* to be that due to the whole head.

REMARK.—If the whole head is used to impel the feed-water (as in wheels which act wholly by impulse), Case I. of Rule III. determines the best speed for the wheel. If the wheel acts partly by impulse and partly by weight, and its velocity is given, Case I. determines how much of the head is to be used in giving velocity to the feed-water—viz., the head due to from $2\frac{1}{2}$ to $1\frac{2}{3}$, or an average,

to *double* of the mean speed of the wheel. For relations between head and velocity, see page 249.

2. **Overshot and Breast Wheels.**—RULE IV.—*Diameter of overshot wheel* = fall — head required for velocity of feed. Velocity of feed = 2 × velocity of outer surface of wheel. Ordinary velocity of outer surface of wheel = 6 feet per second; velocity of feed-water, 12 feet per second; head for that velocity, about 2·25 feet.

A *breast wheel* may be made of any greater diameter.

RULE V.—To find the *clear breadth* (l) between the *crowns* (or flat rims of the wheel), called also the *length* of the buckets.

Let Q be the volume of water, in cubic feet per second; u, the surface velocity of the wheel, in feet per second; r, the outside radius of the wheel; b, the depth of shrouding (= from 1 to 1·75 foot); (all measurements in feet). The buckets are supposed to run two-thirds full. Then,

$$l = \frac{3\,Q}{2\,u\,b\left(1 - \frac{b}{2\,r}\right)}$$

RULE VI.—Other *dimensions of buckets*. Distance between their bottoms, measured on the *sole* (or inner circumference) = b. Opening between lip of bucket and front of the next bucket above —when the slope of the circumference of the wheel at the point where the water is fed to it is between 0° and 24°, $\frac{b}{5}$; for steeper slopes, $\frac{b}{2}$ × sin. slope.

RULE VII.—To find the best positions for the *guide-blades*, between which the water flows on to the wheel.

In fig. 103 let A B be a section of a bucket, B its lip. Draw the straight line B D H a tangent to the circumference of the wheel; and make $\overline{\mathrm{B\,D}} = u$, the surface velocity; and $\overline{\mathrm{B\,H}} = 2\,u$. Draw D L parallel to a tangent to the lip of the bucket; draw H C perpendicular to B H, cutting D L in C; join B C.

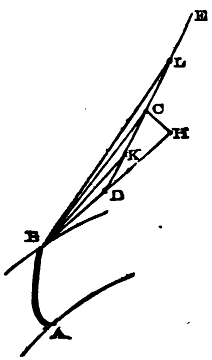


Fig. 103.

Then $\overline{\mathrm{B\,C}}$ represents the best velocity for the supply of water to the wheel; and the middle outlet between the series of guide-blades is to be placed at the depth below the topwater level in the penstock due to that velocity.

Also, ∠ H B C will be the proper angle for the guide-blades of the middle outlet to make with the tangents to the circumference of the wheel at the points where they meet

it, in order that the water may glide into the bucket without collision. The *co-efficient of contraction* for orifices between guide-blades is about $c = 0{\cdot}75$; consequently the total area of the outlets required for the flow Q, is given approximately by the formula, $A = \frac{2\,Q}{3\,u}$; and this is to be provided by having a sufficient number of outlets before and behind the middle outlet.

The positions of the guide-blades for these outlets are found as follows:—

Take the depth of the narrowest part of each outlet below the topwater level of the penstock; compute the velocity due to that depth; from B lay off distances, such as $\overline{BK}$, $\overline{BL}$, representing those velocities, so as to find a series of points, such as K, L, in the line D C L; then will $\angle$ H B K, $\angle$ H B L, be respectively the proper inclinations to tangents to the wheel, for the guide-blades of outlets where the velocities are $\overline{BK}$, $\overline{BL}$; and so on for other guide-blades.

The formula gives a total area of outlet rather greater than is absolutely necessary; but this is the best side to err on, as any excess of outlet can be closed by the regulator.

Besides computing the area of the outlets between the guide-blades, the height of the topwater above the regulator, necessary to give the required flow Q, treating the regulator as an overfall with the co-efficient of contraction 0·7, should be computed by the formula $h' = \left(\frac{Q}{3{\cdot}75\,l}\right)^{\frac{2}{3}}$; and the depth of the upper edge of the lowest guide-blade below the topwater level should be made not less than the height so found.

3. **Undershot Wheels (Poncelets').**—Rule VIII.—(*Usual dimensions of wheel and sluice.*) Diameter = fall × 2, nearly. (The *fall* is measured from the topwater of the penstock to the centre of its outlet.) Depth of shrouding $= \frac{1}{2}$ fall. Greatest depth of opening of sluice $= \frac{1}{8}$ fall. To calculate breadth (b) of opening of sluice; let Q be the volume of water, in cubic feet per second; h, the fall in feet; then $b = \frac{5\,Q}{4\,h^{\frac{1}{2}}}$.

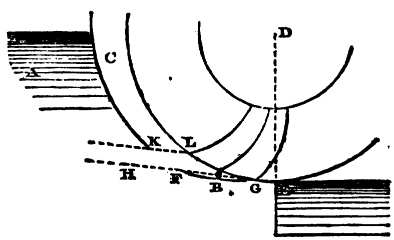


Fig. 104.

Rule IX.—To design the *wheel-race.* In fig. 104 draw H F G a tangent to the wheel, with a declivity of one in ten.

At the height $\frac{h}{10}$ above H F G, draw K L to represent the upper surface of the stream, meeting the circumference of the wheel at the point L. Then make the section of the bottom of the wheel-race from G to F an arc of a circle, equal to G L, and of the same radius; that is, the outside radius of the wheel.

From G to E the wheel-race is formed so as to clear the wheel by about 0·4 inch.

RULE X.—To design the *floats*:—

In fig. 105 draw B C to represent the direction and velocity of the stream of feed-water A, and B N a tangent to the circumference of the wheel at the centre of that stream; and from C let fall C N perpendicular to B N. Make B D $= \frac{6}{10}$ of B N, and join C D. This line will be parallel to a tangent to the lip E of the float. The rest of the float may be made of the figure of a circular arc, touching a radius of the wheel at its inner edge. From two to three floats in the length of the arc L G (fig. 103) are in general a sufficient number.

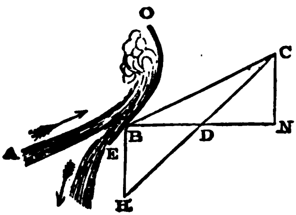


Fig. 105.

The *efficiency* of this wheel is about ·6 when not drowned, and ·48 when drowned.

4. **Undershot Wheel in an Open Current.**—Wheels of this class have their floats usually plane and radial, and fixed at distances apart equal to their depth.

RULE XI.—The following is the useful work per second of such a wheel; v being the velocity of the current; u, that of the centre of a float; A, the area of a float, in square feet; and D, the weight of a cubic foot of water:—

$$\mathrm{R}\,u = 0{\cdot}8\,\frac{\mathrm{D\,A}\,v\,(v-u)\,u}{g}.$$

The velocity of the centres of the floats for the greatest efficiency is half the velocity of the current; and the efficiency at that speed is 0·4.

5. **Turbines.**—RULE XII.—For the *velocity of the feed-water;* in impulse turbines take the velocity produced by the whole head; in pressure turbines, the velocity produced by half the head.

RULE XIII.—To find the proper *obliquity of the guide-blades* to the receiving surface of the wheel; divide the volume of feed-water per second by the area of the receiving surface of the wheel

(diminished by $\frac{1}{10}$ for contraction), and by the velocity of feed; the quotient will be the sine of the required angle.

RULE XIV.—To find the proper *obliquity of the floats* to the receiving surface of the wheel; in *impulse turbines* proceed as in Rule X., page 272; in *pressure turbines* make the receiving ends of the floats perpendicular to the receiving surface of the wheel.

RULE XV.—(In this rule the discharging surface of the wheel is supposed to be, as it ought, equal to the receiving surface.) To find the obliquity of the floats to the *discharging* surface of the wheel. In impulse turbines take the tangent of the obliquity of the receiving ends of the floats; in pressure turbines take the tangent of the obliquity of the guide-blades. Multiply the tangent so found by the radius of the receiving surface of the wheel, and divide the product by the radius of the discharging surface. The quotient will be the tangent of the obliquity of the discharging ends of the floats.

6. **Re-action Wheels.**—RULE XVI.—To find the proper total area of orifices for a re-action wheel; divide the volume of water per second by the velocity due to twice the head.

7. **Hydraulic Ram.**—The following proportions for hydraulic rams have been found to answer in practice:—

Let h be the height above the pond to which a portion of the water is to be raised;

H, the height of topwater in the pond above the outlet of the waste clack;

L, the length of the supply pipe from the pond to the waste clack;

D, its diameter; then

$$H = \frac{h}{20}; \; L = 2{\cdot}8\,H = 0{\cdot}14\,h; \; D = \frac{H}{10} = \frac{h}{200}.$$

Let Q be the whole supply of water, in cubic feet per second, of which q is lifted to the height h above the pond, and $Q - q$ runs to waste at the depth H below the pond. Then the efficiency of the ram has been found by experience to have the following average value:—

$$\frac{q\,h}{Q\,H} = \frac{2}{3}, \text{ nearly.}$$

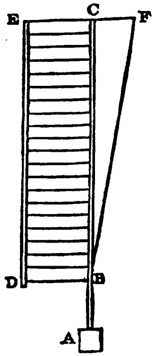


Fig. 106.

8. **Windmills.**—Smeaton's proportions for sails. (See fig. 106.)

$$AB = \frac{1}{6}AC; \; BC = \frac{5}{6}AC; \; BD = CE = \frac{1}{5}AC; \; CF = \frac{2}{15}AC.$$

Angles of weather, or obliquities of the sail to the plane of rotation, at different distances from the axis of the wind-shaft;

Distance in sixths of A B,...	1 (first bar)	2	3	4	5	6 (tip)
Angle of weather,..............	18°	19°	18°	16°	12½°	7°.

Best speed for tips of sails, 2·6 × speed of wind.

Effective power, in foot-lbs. per second $= 0{\cdot}00034\ A\, v^3$; where $A =$ area of circle swept by sails, in square feet, and $v =$ velocity of wind, in feet per second.

Section III.—Rules relating to Propulsion of Vessels.

1. **Resistance of Vessels.**—For relations between speed in feet per second and speed in knots, see pages 102, 114.

Rule I.—Given, the intended greatest speed of a ship in knots; to find the least length of the *after-body* necessary, in order that the resistance may not increase faster than the square of the speed; take *three-eighths* of the square of the speed in knots for the length in feet (Scott Russell's Rule).

To fulfil the same condition, the *fore-body* should not be shorter than the length for the after-body given by the preceding rule, and may with advantage be 1½ times as long.

Rule II.—To find the greatest speed in knots suited to a given length of after-body in feet; take the square root of 2⅔ times that length.

Rule III.—When the speed does not exceed the limit given by Rule II., to find the probable resistance in lbs.; measure the *mean immersed girth* of the ship on her body plan; multiply it by her length on the water-line; then multiply by 1 + 4 (mean square of sines of angles of obliquity of stream-lines). The product is called the *augmented surface*. Then multiply the augmented surface in square feet by the square of the speed in knots, and by a constant co-efficient; the product will be the probable resistance in lbs. (See also page 303).

Co-efficient for clean painted iron vessels, ·01;
" for clean coppered vessels, ·009 to ·008;
" for moderately rough iron vessels, ·011 and upwards.

Rule III. a.—For an approximate value of the resistance in well-designed steamers, with clean painted bottoms; multiply the square of the speed in knots by the square of the cube-root of the displacement in tons. For different types of steamers the resistance ranges from ·8 to 1·5 of that given by the preceding calculation.

Rule IV.—To estimate the *net* or *effective horse-power* expended in propelling the vessel; multiply the resistance by the speed in knots, and divide the product by 326.

RULE IV. A.—To estimate the *gross* or *indicated horse-power* required; divide the same product by 326, and by the combined *efficiency* of engine and propeller. In ordinary cases that efficiency is from ·6 to ·625—average, say ·613; therefore in such cases the preceding product is to be divided by 200.

2. **Thrust of Propellers.**—RULE V.—To calculate the thrust of a propelling instrument (jet, paddle, or screw) in lbs.; multiply together the transverse sectional area, in square feet, of the stream driven astern by the propeller; the speed of that stream, *relatively to the ship*, in knots; the *real slip*, or part of that speed which is impressed on that stream by the propeller, also in knots; and the constant 5·66 for sea-water, or 5·5 for fresh water.

RULE VI.—Given, the product of the velocity of advance, in knots, of a screw propeller as if through a solid (= pitch in knots × revolutions per hour) into the slip of that screw relatively to the water in which it works (also in knots); required the product of speed and slip of the stream from the screw, for use in Rule V. Multiply the first product by $1 - \frac{\cdot 8 \text{ pitch of screw}}{\text{circumference}}$. (This is a good rough approximation when the circumference is between $1\frac{1}{2}$ and $3\frac{1}{2}$ times the pitch.)

REMARK.—The speed of the stream driven astern by feathering paddles is sensibly equal to that of their centres; by radial paddles, to that of their outer edges. The gross power required to drive a radial paddle-wheel is greater than that required to drive a feathering paddle-wheel of equal thrust, in the ratio of

$$\sqrt{\left(\frac{\text{outer radius of wheel}}{\text{height of axis above water}}\right)}, \text{ nearly.}$$

3. **Moment of Sail.**—The *centre of buoyancy* of a ship is the centre of her immersed volume (found by the Rule of page 84, Article 7).

RULE VII.—To find the height of a ship's *metacentre* above her *centre of gravity*. Divide the length of her load water-line into equal intervals, at which measure the *half-breadths* at the load water-line. Cube each of those half-breadths; and regard the cubes as the ordinates of a plane figure having the length of the load water-line as its base. Find the area of that figure by Simpson's Rule (page 64.) Divide two-thirds of that area by the volume of water displaced by the ship. The quotient will be the height of the metacentre above the *centre of buoyancy;* from which subtracting the height of the centre of gravity above the centre of buoyancy, there remains the height required, called the *metacentric height.*

RULE VIII.—To find the *moment of sail* that a ship can bear; multiply together the metacentric height in feet, the displace-

ment in tons, the factor 2240 (to reduce the tons to pounds), and the sine of the intended angle of *steady heel;* the product will be the required moment in foot-lbs.

Ordinary values of sine of angle of steady heel: ships, ·07; schooners and cutters for trade or war, ·105; yachts, ·157.

RULE IX.—To calculate the *moment* of a *given set of sails.* Multiply their area by the estimated intensity of pressure of the wind, and the product by the height of the *centre of effort* of the sails above the *centre of lateral resistance* of the vessel.

REMARKS.—Sails are adapted to a vessel by so adjusting their size and figure that the results of Rule VIII. and Rule IX. are equal. The pressure of wind to which the extent of canvass called "all plain sail" is usually adapted, is about 1 lb. on the square foot.

The *centre of effort* above mentioned is the common centre of magnitude of the sails, found as in pages 83, 84.

The *centre of lateral resistance* is at a depth below the surface of the water nearly equal to half the vessel's draught of water amidships.

The *equivalent triangle* has for its *base* a line which usually extends horizontally from the *clew of the driver* (or aftermost lower corner of the aftermost sail) to a point directly below the *tack of the jib;*—and for its height, three times the height of the centre of effort above its base (called the *base of sail*).

RULE IX. A.—Given, the moment of sail, M, as found by Rule VIII., and the base of sail, b; to find the height, z, of the centre of effort above the base of sail; also the area of sail. Let h be the height of the base of sail above the centre of lateral resistance; then $z = \sqrt{\left(\frac{2}{3}\cdot\frac{M}{b} + \frac{h^2}{4}\right)} - \frac{h}{2}$; and area $= 1\frac{1}{2}\, z\, b$.

Examples of length of base of sail ÷ length of vessel on load water-line. Fore and aft rigged vessels, 1·9 to 1·6; square rigged vessels, 1·6 to 1·35; full-powered steamers, 1·0 to 0·5 (in steamers the base of sail usually has a gap in it over the engines and boilers).

RULE X.—Direct pressure of wind in lbs. on the square foot nearly $= \frac{(\text{velocity of wind in knots})^2}{150}$.

(See *Shipbuilding, Theoretical and Practical,* by Watts, Rankine, Napier, and Barnes.)

PART IX.

HEAT AND THE STEAM ENGINE.

Section I.—Rules relating to the Mechanical Action of Heat, especially through Steam.

1. **Thermodynamics.**—As to measures of temperature, and of quantities of heat, see pages 105, 106.

Rule I.—To find the quantity of heat required to produce a given rise of temperature in a given weight of a given substance; multiply together the rise of temperature, the weight, and the *specific heat* of the substance. (See Table, pages 278, 279.)

Rule II.—To convert quantities of heat into *equivalent quantities of work*:—

	Multiply by
British Fahrenheit-units into foot-lbs., ...	772;
British Centigrade-units into foot-lbs., ...	1,390;
French units into kilogrammetres, ...	424;
British units of evaporation into foot-lbs., ...	745,800;
French units of evaporation into kilogrammetres,	227,300.

The first three numbers are values of the *dynamical equivalent of heat*, often called "Joule's Equivalent," and denoted by J.

Rule III.—To convert temperatures on the ordinary scales into absolute temperatures. (See page 105):—

In Fahrenheit's degrees, ...add	461°·2
In Centigrade degrees, ... „	274·0
In Réaumur's degrees, ... „	219·2

	Fahr.	Cent.	Réau.
Absolute temperature of melting ice, ...	493°·2	274°	219°·2
Atmospheric boiling point of water, ...	673·2	374	299·2

(See Table, pages 280, 281, 282.)

Rule IV.—To find the *efficiency* of a *perfect heat engine*, working between given limits of temperature; divide the difference or *range* between the limits of temperature, by the higher limit of *absolute temperature*.

Remark.—The efficiency thus found is never fully realized by any actual heat-engine, but is approximated to in the course of improvement.

TABLE OF WEIGHT, VOLUME, ELASTICITY, EXPANSION, AND SPECIFIC HEAT.

EXPLANATION OF SYMBOLS.

P_0.—Mean pressure of the atmosphere, in lbs. avoirdupois on the square foot, = 2116·8.

D_0.—Heaviness, or weight of one cubic foot of the substance, in lbs. avoirdupois, under the pressure of one atmosphere, and at the temperature of melting ice, except for water, for which the temperature is 39°·1 Fahrenheit.

V_0.—Volume in cubic feet of one pound avoirdupois of the substance, at the before-mentioned pressure and temperature.

S.G.—Specific gravity, that of water being taken as unity.

E.—Expansion of unity of volume for fluids, or unity of length for solids, at the temperature of melting ice, in rising to the temperature of water boiling under the pressure of one atmosphere.

C.—Specific heat, that of water being taken as unity.

K.—Specific heat in foot-pounds per degree of Fahrenheit. For gases, specific heats at constant volume and constant pressure are distinguished by the symbols C_v, C_p, or K_v, K_p, as the case may be.

GASES.	D_0	V_0	P_0V_0	E	C_v	K_v	C_p	K_p
Air,	0·080728	12·387	26214	·365	0·169	130·3	0·238	183·45
Oxygen,	0·089256	11·204	23710	·367	0·156	120·2	0·218	168·3
Hydrogen,	0·005592	178·83	378819	·366	2·410	1860·6	3·405	2628·7
Steam,	0·05022*	19·913*	42141*	·365*	0·370	286·	0·480	371·
Æther Vapour,	0·2093*	4·777*	10110*	...	...	...	0·481	371·3
Bisulph. Carbon,	0·2137*	4·679*	9902*	...	...	...	0·1575	121·6
Carb. Acid, ideal,	0·12259*	8·157*	17264*	·365*	...	...	...	...
Do., actual,	0·12344	8·101	17145	·370	...	...	0·217	167·
Olefiant Gas,	0·0795	12·58	...	...	...	...	0·369	284·9
Coal Gas, from	0·0323	31·0	...	...	...	...	...	...
Coal Gas, to	0·0404	24·8	...	...	...	...	...	...
Do., Average,	0·0358	27·9	...	...	...	...	...	...
Nitrogen,	0·078411	12·753	26990	...	0·173	133·6	0·244	188·4
Vapour of Mercury,	0·563*	1·7762*	3759*	...	...	...	...	...

* This mark is affixed to results computed for the ideal condition of perfect gas.

LIQUIDS.	D.	S.G.	E.	C.	K.
Water, pure (at 39°·1 Fahrenheit),	62·425	1·000	0·04775	1·000	772.0
„ sea, ordinary,	64·05	1·026	0·05	...	...
Alcohol, pure,	49·38	0·791	0·1112	...	...
„ proof spirit,	57·18	0·916	...	...	...
Æther,	44·70	0·716	...	0·517	399·1
Mercury,	848·75	13·596	0·018153	0·033	25·5
Naphtha,	52·94	0·848	...	...	...
Oil, linseed,	58·68	0·940	0·08	...	...
„ olive,	57·12	0·915	0·08	0·31	239·3
„ whale,	57·62	0·923	...	...	...
„ of turpentine,	54·31	0·870	0·07	·426	328·8
Petroleum,	54·81	0·878	...	...	...
SOLIDS.					
Brickwork and Masonry, about	...	...	...	0·2	154·4
Coal and Coke, average about	...	...	...	0·2415	186·4
Copper,	537 to 556	8·6 to 8·9	·00184	·0951	73·3
Ice,	57·5	·924	...	0·504	389·0
Iron, cast,	444	7·11	·0011	...	...
Iron, wrought,	480	7·69	·0012	·1138	87·8
Lead,	712	11·4	·0029	·0293	22·6
Platinum,	1311 to 1373	21 to 22	·0009	·0314	24·2
Silver,	655	10·5	·002	·0557	43·0
Steel,	490	7·85	·0012	·119	91·8
Tin,	462	7·4	·0022	·0514	39·7
Zinc,	436	7·2	·00294	·0927	71·6

Table of the Elasticity of a Perfect Gas.

EXPLANATION OF SYMBOLS.

T.—Temperature, measured from the ordinary zero.

t.—Absolute temperature, measured from the absolute zero.

P.—Pressure of a perfect gas in pounds avoirdupois on the square foot.

V.—Volume of one pound avoirdupois in cubic feet.

PV.—Product of these quantities at any given temperature.

P_0V_0.—Value of that product for the temperature of melting ice.

Centigrade. T	*t*	Fahrenheit. T	*t*	$\frac{PV}{P_0V_0}$
−30°	244°	−22°	439·2°	0·8905
−25	249	−13	448·2	0·9088
−20	254	− 4	457·2	0·9270
−15	259	+ 5	466·2	0·9453
−10	264	14	475·2	0·9635
− 5	269	23	484·2	0·9818
0	274	32	493·2	1·0000
+ 5	279	41	502·2	1·0182
10	284	50	511·2	1·0365
15	289	59	520·2	1·0547
20	294	68	529·2	1·0730
25	299	77	538·2	1·0912
30	304	86	547·2	1·1095
35	309	95	556·2	1·1277
40	314	104	565·2	1·1460
45	319	113	574·2	1·1643
50	324	122	583·2	1·1825
55	329	131	592·2	1·2007
60	334	140	601·2	1·2190
65	339	149	610·2	1·2373
70	344	158	619·2	1·2555
75	349	167	628·2	1·2738
80	354	176	637·2	1·2920
85	359	185	646·2	1·3103
90	364	194	655·2	1·3285

Centigrade.		Fahrenheit.		
T	t	T	t	$\frac{PV}{P_0V_0}$
95°	369°	203°	664°·2	1·3468
100	374	212	673·2	1·3650
105	379	221	682·2	1·3832
110	384	230	691·2	1·4015
115	389	239	700·2	1·4197
120	394	248	709·2	1·4380
125	399	257	718·2	1·4562
130	404	266	727·2	1·4744
135	409	275	736·2	1·4927
140	414	284	745·2	1·5109
145	419	293	754·2	1·5292
150	424	302	763·2	1·5474
155	429	311	772·2	1·5657
160	434	320	781·2	1·5839
165	439	329	790·2	1·6022
170	444	338	799·2	1·6204
175	449	347	808·2	1·6387
180	454	356	817·2	1·6569
185	459	365	826·2	1·6752
190	464	374	835·2	1·6934
195	469	383	844·2	1·7117
200	474	392	853·2	1·7299
205	479	401	862·2	1·7481
210	484	410	871·2	1·7664
215	489	419	880·2	1·7846
220	494	428	889·2	1·8029
230	504	446	907·2	1·8394
240	514	464	925·2	1·8759
250	524	482	943·2	1·9124
260	534	500	961·2	1·9489
270	544	518	979·2	1·9854
280	554	536	997·2	2·0219
290	564	554	1015·2	2·0584
300	574	572	1033·2	2·0949
310	584	590	1051·2	2·1314
320	594	608	1069·2	2·1679
330	604	626	1087·2	2·2044
340	614	644	1005·2	2·2409
350	624	662	1123·2	2·2774
360	634	680	1141·2	2·3139
370	644	698	1159·2	2·3504
380	654	716	1177·2	2·3869

Centigrade.		Fahrenheit.		$\frac{PV}{P_0V_0}$
T	t	T	t	
390°	664°	734°	1195°·2	2·4234
400	674	752	1213·2	2·4599
410	684	770	1231·2	2·4964
420	694	788	1249·2	2·5329
430	704	806	1267·2	2·5693
440	714	824	1285·2	2·6058
450	724	842	1303·2	2·6423
460	734	860	1321·2	2·6788
470	744	878	1339·2	2·7153
480	754	896	1357·2	2·7518
490	764	914	1375·2	2·7883
500	774	932	1393·2	2·8248
520	794	968	1429·2	2·8978
540	814	1004	1465·2	2·9708
560	834	1040	1501·2	3·0438
580	854	1076	1537·2	3·1168
600	874	1112	1573·2	3·1898
620	894	1148	1609·2	3·2628
640	914	1184	1645·2	3·3358
660	934	1220	1681·2	3·4088
680	954	1256	1717·2	3·4818
700	974	1292	1753·2	3·5547
720	994	1328	1789·2	3·6277
740	1014	1364	1825·2	3·7007
760	1034	1400	1861·2	3·7737
780	1054	1436	1897·2	3·8467
800	1074	1472	1933·2	3·9197
820	1094	1508	1969·2	3·9927
840	1114	1544	2005·2	4·0657
860	1134	1580	2041·2	4·1387
880	1154	1616	2077·2	4·2117
900	1174	1652	2113·2	4·2847
920	1194	1688	2149·2	4·3577
940	1214	1724	2185·2	4·4307
960	1234	1760	2221·2	4·5036
980	1254	1796	2257·2	4·5766
1000	1274	1832	2293·2	4·6496

RULE V.—To find the *total work* in a heat-engine done by a given expenditure of heat; reduce the expenditure of heat to units of work (see Rule II., page 277), and multiply by the efficiency.

REMARK.—A quantity of heat equivalent to the total work thus found disappears; and the remainder of the heat expended is rejected.

RULE VI.—To find the expenditure of heat in a heat-engine required in order to do a given total quantity of work; divide by the efficiency, or multiply by its reciprocal; the product will be the required expenditure of heat expressed in equivalent units of work; which may be reduced to units of heat by dividing by the proper co-efficient, as given in Rule II.

As to *expansion by heat*, see pages 147, 148; also Tables, pages 278 to 282.

RULE VII.—To find the *total heat of evaporation* of an unit of weight of water: the temperature of the feed-water and the boiling point being given. To the latent heat of evaporation of an unit of weight at the atmospheric boiling point (966 British Fahrenheit units, or 537 French units), add 1 for every degree that the feed-water is *below* the atmospheric boiling point, and 0·3 for every degree that the actual boiling point is *above* the atmospheric boiling point.

To calculate the same quantity in *units of evaporation at the atmospheric boiling point*, divide the result of the preceding calculation by 966 for British Measures, or 537 for French Measures. (See Table of Factors of Evaporation, page 284.)

RULE VIII.—To calculate the pressure of steam corresponding to a given boiling point, or the boiling point corresponding to a given pressure. Let p be the pressure (absolute); t, the boiling point, *in absolute temperature* T + 461·2 Fahr.; A,B,C, constants. Then

$$\log p = A - \frac{B}{t} - \frac{C}{t^2};\ \frac{1}{t} = \sqrt{\left(\frac{A - \log p}{C} + \frac{B^2}{4\,C^2}\right)} - \frac{B}{2\,C}.$$

Values of constants for steam, with common logarithms, and pressures in lbs. on the square inch,—

A.	Log B.	Log C.	$\frac{B}{2\,C}$.	$\frac{B^2}{4\,C^2}$.
6·1007	3·43642	5·59873	0·003441	0·00001184

B = 2732; C = 396045.

RULE IX.—Given, the volume of a pound of steam at a given pressure; to calculate the volume of a pound of steam at another pressure. The difference between the logarithms of the volumes is very nearly *sixteen seventeenths* of the difference between the logarithms of the absolute pressures; and the greater volume corresponds to the less pressure.

This rule serves to find volumes of steam corresponding to pressures intermediate between those given in the Table, pages 285 to 288.

Table of Factors of Evaporation.

Boiling Point, T_1. Fahr.	Initial Temperature of feed water, T_2. 32°	50°	68°	86°	104°	122°	140°	158°	176°	194°	212°
212°	1·19	1·17	1·15	1·13	1·11	1·10	1·08	1·06	1·04	1·02	1·00
230	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06	1·04	1·02	1·01
248	1·20	1·18	1·16	1·14	1·13	1·11	1·09	1·07	1·05	1·03	1·01
266	1·21	1·19	1·17	1·15	1·13	1·11	1·09	1·07	1·06	1·04	1·02
284	1·21	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06	1·04	1·02
302	1·22	1·20	1·18	1·16	1·14	1·12	1·11	1·09	1·07	1·05	1·03
320	1·22	1·21	1·19	1·17	1·15	1·13	1·11	1·09	1·07	1·05	1·03
338	1·23	1·21	1·19	1·17	1·15	1·14	1·12	1·10	1·08	1·06	1·04
356	1·23	1·22	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06	1·04
374	1·24	1·22	1·20	1·18	1·17	1·15	1·13	1·11	1·09	1·07	1·05
392	1·24	1·23	1·21	1·19	1·17	1·15	1·13	1·11	1·09	1·07	1·06
410	1·25	1·23	1·22	1·20	1·18	1·16	1·14	1·12	1·10	1·08	1·06
428	1·25	1·24	1·22	1·20	1·18	1·16	1·14	1·12	1·11	1·09	1·07

TABLE OF PROPERTIES OF STEAM OF MAXIMUM DENSITY BY THE POUND AVOIRDUPOIS.

T.	P.	Log. P.	Δ log. P.	p.	V.	Log. V.	—Δ log. V.	U.	Δ U.	H.	h.
32°	12·27	1·0887		0·085	3390	3·5302		0		842872	0
			0·1572				0·1489		15200		
41	17·62	1·2459		0·122	2406	3·3813		15200		844988	6948
			0·1507				0·1427		14850		
50	24·92	1·3966		0·173	1732	3·2386		30050		847103	13896
			0·1446				0·1369		14500		
59	34·77	1·5412		0·241	1264	3·1017		44550		849218	20844
			0·1388				0·1311		14160		
68	47·87	1·6800		0·333	934·6	2·9706		58710		851333	27792
			0·1333				0·1261		13860		
77	65·06	1·8133		0·452	699·0	2·8445		72570		853448	34740
			0·1282				0·1209		13530		
86	87·40	1·9415		0·607	529·2	2·7236		86100		855563	41702
			0·1233				0·1164		13240		
95	116·1	2·0648		0·806	404·8	2·6072		99340		857678	48650
			0·1187				0·1120		12950		
104	152·6	2·1835		1·06	312·8	2·4952		112290		859793	55612
			0·1145				0·1079		12660		
113	198·6	2·2980		1·38	244·0	2·3873		124950		861908	62560
			0·1102				0·1039		12400		
122	256·0	2·4082		1·78	192·0	2·2834		137350		864024	69522
			0·1064				0·1003		12120		
131	327·0	2·5146		2·27	152·4	2·1831		149470		866139	76484
			0·1027				0·0966		11870		

T.	P.	Log. P.	Δ log. P.	p.	V.	Log. V.	—Δ log. V.	U.	Δ U.	H.	h.
140°	414.3	2.6173	0.0992	2.88	122.0	2.0865	0.0933	161340	11620	868254	83459
149	520.6	2.7165	0.0958	3.62	98.45	1.9932	0.0900	172960	11380	870369	90435
158	649.1	2.8123	0.0926	4.51	80.02	1.9032	0.0872	184340	11150	872484	97411
167	803.3	2.9049	0.0897	5.58	65.47	1.8160	0.0843	195490	10920	874600	104387
176	987.6	2.9946	0.0867	6.86	53.92	1.7317	0.0814	206410	10700	876715	111363
185	1206	3.0813	0.0840	8.38	44.70	1.6503	0.0791	217110	10490	878830	118353
194	1463	3.1653	0.0814	10.16	37.26	1.5712	0.0762	227600	10270	880945	125357
203	1765	3.2467	0.0789	12.26	31.26	1.4950	0.0741	237870	10080	883060	132360
212	2116.4	3.3256	0.0765	14.70	26.36	1.4209	0.0718	247950	9860	885175	139363
221	2524	3.4021	0.0741	17.53	22.34	1.3491	0.0697	257810	9670	887290	146380
230	2994	3.4762	0.0721	20.80	19.03	1.2794	0.0677	267480	9500	889405	153412
239	3534	3.5483	0.0700	24.54	16.28	1.2117	0.0656	276980	9310	891520	160429
248	4152	3.6183	0.0678	28.83	14.00	1.1461	0.0637	286290	9120	893635	167460

T.	P.	Log. P.	Δ log. P.	p.	V.	Log. V.	— Δ log. V.	U.	Δ U.	H.	h.
257°	4854	3·6861		33·71	12·09	1·0824		295410		895751	174505
			0·0661				0·0620		8960		
266	5652	3·7522		39·25	10·48	1·0204		304370		897866	181564
			0·0641				0·0602		8790		
275	6551	3·8163		45·49	9·124	0·9602		313160		899981	188637
			0·0624				0·0586		8620		
284	7563	3·8787		52·52	7·973	0·9016		321780		902096	195711
			0·0607				0·0570		8450		
293	8698	3·9394		60·40	6·992	0·8446		330230		904211	202798
			0·0591				0·0555		8290		
302	9966	3·9985		69·21	6·153	0·7891		338520		906327	209885
			0·0575				0·0541		8150		
311	11380	4·0560		79·03	5·433	0·7350		346670		908442	216986
			0·0560				0·0523		8000		
320	12940	4·1120		89·86	4·816	0·6827		354670		910557	224087
			0·0546				0·0513		7840		
329	14680	4·1666		101·9	4·280	0·6314		362510		912672	231216
			0·0531				0·0500		7710		
338	16580	4·2197		115·1	3·814	0·5814		370220		914787	238358
			0·0519				0·0486		7550		
347	18690	4·2716		129·8	3·410	0·5328		377770		916902	245501
			0·0505				0·0475		7430		
356	20990	4·3221		145·8	3·057	0·4853		385200		919017	252658
			0·0493				0·0463		7290		
365	23520	4·3714		163·3	2·748	0·4390		392490		921132	259829
			0·0480				0·0452		7160		

T.	P.	Log. P.	Δ log. P.	p.	V.	Log. V.	—Δ log. V.	U.	Δ U.	H.	h.
374°	26270	4·4194		182·4	2·476	0·3938		399650		923247	267013
			0·0470				0·0443		7020		
383	29270	4·4664		203·3	2·236	0·3495		406670		925362	274198
			0·0458				0·0431		6910		
392	32520	4·5122		225·9	2·025	0·3064		413580		927478	281394
			0.0447				0·0421		6780		
401	36050	4·5569		250·3	1·838	0·2643		420360		929593	288634
			0·0438				0·0411		6660		
410	39870	4·6007		276·9	1·672	0·2232		427020		931708	295874
			0·0426				0·0399		6530		
419	43990	4·6433		305·5	1·525	0·1833		433550		933823	303128
			0·0418				0·0393		6430		
428	48430	4·6851		336·3	1·393	0·1440		439980		935939	310381

EXPLANATION OF SYMBOLS.

T.—Temperature on Fahrenheit's scale, or *boiling point.*

P.—Pressure in pounds avoirdupois on the square foot.

p.—Pressure in pounds on the square inch: Log. $p = \text{Log. } P - 2{\cdot}1584$.

V.—Volume of one pound avoirdupois of steam in cubic feet.

U.—Work in foot-pounds per pound by one pound of steam, admitted into the cylinder at the temperature T°, and expanded *without liquefaction* until its temperature falls to 32° Fahr.

H.—*Total heat,* in foot-pounds of energy, required to raise one pound of water from 32° to T°, and evaporate it at T°.

h.—Heat, in foot-pounds of energy, required to raise the temperature of one pound of water from 32° to T°.

$H - h$ = *Latent heat* of one pound of steam at T°.

RULE IX. A.—(*Founded on Fairbairn and Tate's Rule for the Volume of Steam, but with different constants.*)—To the absolute pressure in lbs. on the square inch, add 0·35; divide 389 by the sum; to the quotient add 0·41; the sum will be the volume of one lb. of steam in cubic feet, nearly, for pressures ranging from $\frac{1}{4}$ atmosphere to 10 atmospheres.

For relations between pressures, volumes, and temperatures of steam, see Plate at end of volume.

RULE IX. B.—To find the weight of steam required to fill a given volume at a given pressure; divide the given volume by the volume of one lb. of steam.

Effect of Salt on Boiling-point.—Each 32d part by weight of salt in water raises the boiling-point 1°·2 Fahr. = 0°·67 Cent. Ordinary sea-water contains one-32d part of salt.

2. **Action of Steam in Cylinder.**—RULE X.—To calculate the *indicated power* of an actual steam-engine from the capacity of cylinder, indicator-diagram, and number of revolutions per minute.

From the indicator-diagram (as explained in page 242, Rules XV. and XVI) determine the *mean effective pressure;* multiply it by the *effective capacity of cylinder* (being the volume swept by the piston per stroke), and by the number of revolutions per minute, for a single-acting engine, or twice that number for a double-acting engine; the product will be the indicated power in *foot-pounds per minute;* which, being divided by 33,000, will give the *indicated horse-power.**

REMARK.—As to the adaptation to each other of the unit of intensity of pressure and the unit of volume swept, see page 239, Remark on Rule IV.

RULE X. A.—*Or otherwise:*—Multiply the *mean effective pressure* by the *area of piston*, for the *load;* then multiply the load by the distance travelled by the piston per minute, for the indicated power in units of work per minute. (In single-acting engines forward strokes alone are to be reckoned in the distance travelled; in double-acting engines both forward and return strokes, whose amount per minute is then called *mean speed of piston.*)

REMARK.—The *effective* or *available power* is usually about 0·8 of the indicated power; that fraction being the *efficiency of the mechanism.*

RULE XI.—In a *proposed steam-engine*, to estimate the ratio in which the initial absolute pressure in the cylinder will be less than the absolute pressure in the boiler. Let v denote the mean velocity

* When indicator-diagrams are taken for scientific purposes, the weather-barometer should be observed, in order that *absolute pressures* may be deduced from the diagram; which of itself shows only *differences* between the pressures of the steam and of the atmosphere. As to conversion of pressures, see pages 103, 115.

of the piston in *feet per second;* $\frac{A}{a}$, the ratio in which the area of the piston is greater than that of the steam-port of the cylinder; t, the *absolute temperature* of the steam in Fahrenheit degrees; then the required ratio is *nearly*,

$$1 - \frac{v^2 A^2}{180\, t\, a^2}.$$

The velocity of the steam in the port, $\frac{vA}{a}$, should not exceed 100 feet per second; and then the ratio becomes $1 - \frac{10000}{180\,t}$ $= 1 - \frac{56}{t}$ nearly for Fahrenheit's scale, or $1 - \frac{31}{t}$ for the Centigrade scale. Let $t = 720°$ Fahr. $= 400°$ Cent.; then the ratio $= 0{\cdot}92$ nearly.

RULE XII.—To calculate approximately the ratio $\left(\frac{p_m}{p_1}\right)$ in which the *mean absolute pressure* in a cylinder will probably be less than the *initial absolute pressure* at a given rate of expansion r. (When r exceeds 2, the accumulation of liquid water in the cylinder must be prevented by jacketing or by superheating; otherwise the economy due to expansion cannot be realized.)

Method 1.—(Nearly exact for dry saturated steam.)

$$\frac{p_m}{p_1} = \frac{17 - 16\, r^{-\frac{1}{16}}}{r}.$$

(The quantity $r^{-\frac{1}{16}}$ may be computed by taking the reciprocal of r (called the *effective cut-off*), and extracting the square root *four* times.)

For results of Method 1, see Table A, page 292; also the right-hand diagram of the plate at the end of the volume.

Method 2.—(Steam moderately moist:—Absolute pressure × volume supposed sensibly constant.)

$$\frac{p_m}{p_1} = \frac{1 + \text{hyp. log. } r}{r}.$$

For hyperbolic logarithms, see page 14. For results of Method 2, see Table B, page 292.

REMARK.—In ordinary practice, the difference between the results of those methods is so small, that the choice between them depends mainly on whether a table of squares or a table of hyperbolic logarithms is at hand.

Method 3.*—(See fig. 107.) Draw a straight line C A B, in which make A B = 4 A C. Draw A D perpendicular to C A B; and about C describe the circular arc B D cutting A D in D. Then in D A take E, so that $\frac{\text{D E}}{\text{D A}}$ shall represent the *effective cut-off* (and consequently $\frac{\text{D A}}{\text{D E}}$ the rate of expansion). At E draw E F parallel to A B. Then $\frac{\text{E F}}{\text{A B}}$ will be the required ratio of mean to initial absolute pressure, nearly.

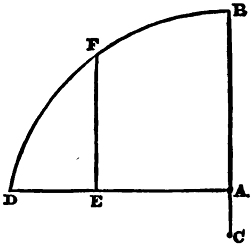


Fig. 107.

The results of Method 3 lie between those of Methods 1 and 2.

Rule XIII.—Given, the initial absolute pressure, the absolute back-pressure, and the rate of expansion; to calculate the *mean effective pressure;* multiply the initial absolute pressure by the ratio found as explained in Rule XII.; the product will be the mean absolute pressure; from which subtracting the *back-pressure,* the remainder will be the required mean effective pressure.

Absolute back-pressure in lbs. on the square inch;

In non-condensing engines, from 15 to 18.
In condensing engines, from 3 to 5.

Rule XIV.—To allow for the effects of *clearance* on the expansion and pressure. Let c be the fraction expressing the ratio borne by the clearance to the effective cylinder-capacity; $\frac{1}{r'}$, the *actual cut-off,* or fraction of the stroke during which the steam is admitted; $\frac{1}{r}$, the *effective cut-off,* or reciprocal of the rate of expansion. Then

$$\frac{1}{r} = \frac{\frac{1}{r'} + c}{1 + c}; \text{ and } r = r' \cdot \frac{1 + c}{1 + c\,r'}.$$

From the real rate of expansion r, as above computed, calculate a value of the mean absolute pressure by Rules XII. and XIII.; let it be denoted by p_m: then the *corrected mean absolute pressure* is as follows:—

Case I. When there is no cushioning; $p'_m = p_m - c\,(p_1 - p_m)$; p_1 being the initial absolute pressure;

Case II. When steam enough is cushioned to fill the clearance at the pressure p_1; $p'_e = \frac{p_e\, r}{r'}$

* First published in the *Engineer* for the 13th April, 1866.

EXPANSIVE WORKING OF STEAM.—TABLE A.—*Dry Saturated Steam.*

r	$\frac{1}{r}$	$\frac{rp_m}{p_1}$	$\frac{p_1}{rp_m}$	$\frac{p_1}{p_m}$	$\frac{p_m}{p_1}$
20	·05	3·73	·268	5·36	·186
13⅓	·075	3·39	·295	3·93	·254
10	·1	3·14	·318	3·18	·314
8	·125	2·97	·337	2·70	·370
6⅔	·15	2·78	·360	2·40	·417
5	·2	2·53	·395	1·98	·506
4	·25	2·33	·429	1·72	·582
3⅓	·3	2·16	·463	1·54	·648
2$\frac{6}{7}$	·35	2·02	·496	1·42	·707
2½	·4	1·89	·529	1·32	·756
2$\frac{2}{9}$	·45	1·78	·562	1·25	·800
2	·5	1·68	·596	1·19	·840
1$\frac{9}{11}$	·55	1·59	·630	1·14	·874
1⅔	·6	1·50	·666	1·11	·900
1$\frac{7}{13}$	·65	1·43	·700	1·08	·926
1$\frac{3}{7}$	·7	1·35	·740	1·06	·945
1⅓	·75	1·28	·778	1·04	·960
1¼	·8	1·22	·819	1·02	·976
1$\frac{3}{17}$	·85	1·16	·861	1·01	·986
1$\frac{1}{9}$	·9	1·11	·903	1·005	·995

TABLE B.—*Moderately Moist Steam.*

20	·05	4·00	·250	5·00	·200
13⅓	·075	3·59	·279	3·72	·269
10	·1	3·30	·303	3·03	·330
8	·125	3·08	·325	2·60	·385
6⅔	·15	2·90	·345	2·30	·435
5	·2	2·61	·383	1·92	·522
4	·25	2·39	·419	1·68	·596
3⅓	·3	2·20	·454	1·51	·661
2$\frac{6}{7}$	·35	2·05	·488	1·39	·717
2½	·4	1·91	·523	1·31	·765
2$\frac{2}{9}$	·45	1·80	·556	1·24	·809
2	·5	1·69	·591	1·18	·846
1$\frac{9}{11}$	·55	1·60	·626	1·14	·878
1⅔	·6	1·51	·662	1·10	·906
1$\frac{7}{13}$	·65	1·43	·699	1·07	·929
1$\frac{3}{7}$	·7	1·36	·737	1·05	·950
1⅓	·75	1·29	·777	1·04	·965
1¼	·8	1·22	·818	1·02	·978
1$\frac{3}{17}$	·85	1·16	·860	1·01	·989
1$\frac{1}{9}$	·9	1·11	·905	1·005	·995

EXPLANATION OF TABLES.—r, rate of expansion; $\frac{1}{r}$, effective cut-off; p_1, initial absolute pressure; p_m, mean absolute pressure.

RULE XV.—To find the *effective cylinder-capacity* required for a proposed steam-engine. To the intended *useful work per minute* add an allowance (say *one-fourth* on an average) for resistance of engine; the sum will be the *indicated work per minute.* Divide, if the engine is single-acting, by the intended number of revolutions, or if double-acting, by twice the intended number of revolutions per minute, for the *indicated work per stroke;* which being divided by the intended mean effective pressure, will give the required effective cylinder-capacity.

As to the units in which it will be expressed, see page 239.

Divide the effective cylinder-capacity by the *length of stroke;* the quotient will be the *area of piston.*

3. **Expenditure of Heat in the Cylinder and Efficiency of the Steam.**—RULE XVI.—To calculate the *absolute pressure of release* (p_2) (that is, the absolute pressure at the end of the expansion);

Case I.—Dry saturated steam,

$$p_2 = p_1 r^{-\frac{17}{16}};$$

or otherwise: in the left-hand diagram of the plate find the volume corresponding to p_1; multiply it by r for the final volume, and find the corresponding pressure from the diagram.

Case II.—Moderately moist steam; divide the initial pressure by the rate of expansion $\left(\text{that is, make } p_2 = \frac{p_1}{r}\right)$.

RULE XVII.—To calculate the intensity of a pressure (p_h), equivalent approximately to the rate at which heat is expended in the cylinder. Find p_m as in Rules XII. and XIII., and p_2 as in Rule XVI.; then

$$\text{In condensing engines, } p_h = p_m + 15\, p_2;$$

$$\text{In non-condensing engines, } p_h = p_m + 14\, p_2;$$

These results are correct to about one per cent.

RULE XVIII.—To calculate the *efficiency of the steam.* Let p_3 be the back pressure, and $p_e = p_m - p_3$ the mean effective pressure, found as in Rule XIII. Then

$$\text{Efficiency of steam} = \frac{p_e}{p_h} = \frac{p_m - p_3}{p_m + 15 \text{ or } 14\, p_2}.$$

RULE XIX.—To find the *expenditure of heat in the cylinder* in a

given time; either multiply the indicated work in that time by the reciprocal of the efficiency, $\frac{p_h}{p_e}$; or multiply the volume swept by the piston in the same time by p_h.

The result is expressed in units of work, which may, if required, be converted into ordinary units of heat, or into units of evaporation, by dividing by the proper co-efficient as given in Rule II., page 277. For practical purposes units of evaporation are the most convenient.

RULE XIX. A.—For the effect of clearance on the expenditure of heat; calculate the expenditure of heat as if there were no clearance; then,—

Case I.—If there is no cushioning, multiply by $1 + c\,r'$.

Case II.—If there is cushioning sufficient to fill the clearance with steam at the absolute pressure p_1; multiply by $r \div r'$. (See Rule XIV.)

REMARKS.—The result of the preceding calculations includes not only the heat required to produce the steam, but the additional heat required to prevent it from condensing to any considerable extent in the cylinder.

The following are rules for obtaining exactly, by the aid of the Table at pages 285 to 288, some of the results to which approximations are given by the preceding rules of this and the previous Article:—

One lb. of steam is supposed to be admitted to the cylinder at the temperature T_1; then expanded, until its temperature falls to T_2, being maintained by the aid of jacketing in the state of dry saturation; and then discharged against a back pressure equal to the final pressure.

The numbers 1 and 2 denote quantities in the Table corresponding to the temperatures 1 and 2 respectively.

RULE A.—Work of one lb. of steam, $U_1 - U_2$.

RULE B.—Expenditure of heat, in units of work, $U_1 - U_2 + H_2 - h$; the value of h being that corresponding to the temperature of the feed-water. Of this heat, $H_1 - h$ is expended in producing the steam, and the remainder in preventing condensation in the cylinder.

4. **Expenditure of Water.**—RULE XX.—To find the *net weight of feed-water* required per stroke; divide the total cylinder-capacity by the volume of one lb. of steam at the pressure of release (p_2), as found by means of Rule IX., page 283, or IX. A., page 289; or of the Table, pages 285 to 288; or of the left-hand diagram in the plate.

RULE XX. A.—For a rough approximation to the *net weight of feed-water* per stroke, correct to 10 per cent., and erring on the safe side; multiply together the absolute pressure of release and cylinder-capacity so as to get the product in foot-lbs., and divide by

50,000. For the approximate *net volume in cubic feet per stroke*, divide the same product by 3,000,000.

Another rough approximation to the net weight of feed-water in a given time is to take the expenditure of heat on the steam (Rule XIX.) in units of evaporation.

RULE XXI.—For the *gross feed-water*, multiply the net feed-water,

If the supply is pure water, by 2;
If ordinary fresh water, by $2\frac{1}{2}$;
If sea-water, and the brine is to be discharged at n times the saltness of sea-water, multiply by $\frac{2n}{n-1}$.

Values of n,	3	$2\frac{1}{2}$	2.
$\frac{\text{Gross}}{\text{net}}$ feed-water,	3	$3\frac{1}{3}$	4.

RULE XXII.—In a condensing engine, to calculate the *net weight* of condensation-water per stroke; from the expenditure of heat, in units of work per stroke, subtract the indicated work per stroke; the remainder will be the *rejected heat*, in units of work per stroke, which is to be divided by 35,000 for British Measures, or 10,600 for French Measures, to give the weight in lbs. or kilos.

For *cubic feet per stroke*, divide the rejected heat in foot-lbs. by 2,200,000.

RULE XXIII.—For the *gross* supply of condensation-water, multiply the net supply by 2.

SECTION II.—RULES RELATING TO FURNACES AND BOILERS.

1. **Fuel.**—RULE I.—To estimate the *theoretical evaporative power*, that is, the *total heat of combustion* of fuel, in units of evaporation (see page 277), per unit of weight of fuel, from the chemical analysis of the fuel. Distinguish the constituents into carbon, hydrogen, oxygen, and refuse, expressing the quantity of each as a fraction of the whole weight analyzed. Let C, H, and O be the fractions for carbon, hydrogen, and oxygen respectively. Then,

$$\text{Theoretical evaporative power} = 15\,C + 64\left(H - \frac{O}{8}\right).$$

RULE II.—*Net weight of air* chemically necessary for the complete combustion of an unit of weight of fuel;

$$12\,C + 36\left(H - \frac{O}{8}\right).$$

In most furnaces some additional air is required to dilute the products of combustion, thus increasing the supply of air required in the ratio of $1\frac{1}{2} : 1$ or $2 : 1$.

Examples of Theoretical Evaporative Powers of Fuel.

Carbon,		15
Hydrogen,		64
Various Hydrocarbons,	from	20 to 22½
Charcoal and Coke,	„	12 to 14
Coal, best qualities:—Anthracite,		15
„ „ Bituminous,	from	14 to 16
„ „ Oxygenous,	about	13¼
„ „ Brown,	„	12
Peat, absolutely dry,	„	10
Wood, do.,	„	7½

Bad qualities of coal from a given coal-field, about ⅔ of the best qualities.

Rule III.—To estimate roughly the *efficiency* of a furnace and boiler (being the ratio of available to total heat).

Case I.—Draught produced by a chimney:—Divide the intended number of square feet of heating surface per lb. of fuel per hour by the same number + 0·5: eleven-twelfths of the quotient will be the probable efficiency of the furnace, nearly. The following are examples:—

	Square feet heating surface per lb. fuel per hour.	Efficiency of Furnace.	Available heat per lb. coal, if total heat is 13¼ units of Evaporation.
Small heating surface,	0·50	0·46	6·21
Ordinary heating surface in tubular boilers,	0·75	0·55	7·43
	1·00	0·61	8·24
	1·25	0·65	8·77
	1·50	0·69	9·31
	2·00	0·73	9·85
Water-tube and cellular boilers,	3·00	0·79	10·66
	6·00	0·84	11·34

The efficiency of a furnace is liable to be diminished by from ·2 to ·5 of its proper value through unskilful firing.

Case II. Draught produced by a blast pipe or by a fan; put 0·3 in the divisor instead of 0·5.

Rule IV.—To estimate the *available heat* of combustion of fuel; multiply the total heat of combustion by the efficiency of the furnace.

Rule V.—To estimate the probable expenditure of fuel in a given time required in a given steam engine.

Estimate the expenditure of heat by Rule XIX. of the preced-

ing section, page 294, and divide it by the available heat of combustion of an unit of weight of the fuel.

2. **Dimensions of Furnaces and Boilers and their Fittings.**—*Area of fire-grate;* in furnaces with chimney draught, from ·1 to ·04 square foot per lb. of fuel burned per hour.

Area of fire-grate; in furnaces with draught forced by blast-pipe or otherwise, from ·04 to ·01 square foot per lb. fuel per hour.

Heating surface; see preceding Article.

Sectional area of flues or tubes from $\frac{1}{7}$ to $\frac{1}{5}$ of area of grate; area of *chimney*, about $\frac{1}{10}$ area of grate.

Capacity of boiler; steam and water space = heating surface × from 3 feet to $1\frac{1}{2}$ foot in stationary cylindrical and flue boilers; from 1 foot to ·5 foot in tubular boilers, stationary or marine; and about ·1 foot in locomotive boilers and water-tube boilers.

Capacity of furnace, flues, and tubes = area of grate × from 6 to 8 feet.

Area of air-holes above level of grate = about $\frac{1}{36}$ area of grate.

Pitch of boiler stays, from centre to centre; in marine boilers, from 12 to 18 inches; in locomotive boilers, 4 or 5 inches; working tension, 3,000 lbs. on the square inch. Working tension on *boiler shells*, from 4,500 to 6,000 lbs. on the square inch. As to strength of flues, see page 211.

Area of safety valve.—Rule.—Multiply the greatest weight of water to be actually evaporated in lbs. per hour by ·006; the product will be the required area in square inches. See p. 303.

Brine refrigerator for marine boilers: surface of tubes should if possible be $\frac{1}{10}$ square foot per lb. of brine blown off per hour (from $\frac{1}{3}$ to $\frac{1}{2}$ of gross feed-water).

Injector.—Sectional area of narrowest part. Rule.—Divide the gross feed-water to be supplied in cubic feet per hour by 800, and by the square root of the pressure of the steam in atmospheres; the quotient will be the required area in square inches. For circular inches, divide by 630 instead of 800.

Section III.—Various Dimensions of Engines.

1. **Condensers—Pumps.**—*Common condenser*, from $\frac{1}{4}$ to $\frac{1}{2}$ capacity of cylinder.

Injection sluice; find the gross volume of condensation-water per minute by Rule XXIII., page 295; divide by 1,620 feet; the quotient will be the area in square feet.

Air-pump, single-acting, for common condenser; from $\frac{1}{5}$ to $\frac{1}{8}$ capacity of cylinder. Valves and passages of such size that speed

of fluids passing through shall not exceed 12 feet per second. Double-acting air-pump may be half the capacity. (See p. 304.)

Feed-pumps depend for their capacity on gross supply of feed-water (see Rule XXI., page 295); and *cold water pumps* on the gross supply of condensation-water. (Rule XXIII., page 295.) *Brine-pumps* for boilers fed with salt water, from $\frac{1}{3}$ to $\frac{1}{2}$ of capacity of feed-pumps.

Surface condenser, from $2\frac{1}{2}$ to 5 square feet surface per indicated horse-power; *air-pump*, if single-acting, $\frac{1}{8}$ capacity of cylinder.

2. **Steam-passages and Valve-ports** to be of such area that velocity of steam shall not exceed 100 feet per second.

3. **Slide-valve Gearing.**—By the *angular advance* of the eccentric is to be understood the angle at which the eccentric radius stands in advance of that position which would bring the slide-valve to mid-stroke when the crank is at its dead-points.

Rule I.—Given, the positions of the crank at the instants of admission and cut-off; to find the proper angular advance of the eccentric, and the proportion of the lap on the induction-side to the half-travel of the slide.*

In fig. 108 let A B and A C be the positions of the crank at the beginning and end of the forward stroke; let the arrow show the direction of rotation; let X x be perpendicular to B C; let A D be the position of the crank at the instant of cut-off, and A E its position at the instant of admission. Draw A F, bisecting the angle E A D; A F will represent the position of the crank at the instant when the slide is at the *forward end* of its stroke; and F A X will be the *angular advance of the eccentric.*

Lay off the distance A F to represent the half-travel; and on A F as a diameter describe the circle A H F G, cutting A D in G and A E in H; then $\frac{\text{A G}}{\text{A F}} = \frac{\text{A H}}{\text{A F}}$ will be the *required ratio of lap at the induction-side to half-travel;* and A G = A H will represent that lap, on the same scale on which A F represents the half-travel.

On the same scale, I K represents the *width of opening of the valve at the beginning of the stroke,* sometimes called the "*lead of the slide.*" Strictly speaking, this is the lead of the induction-edge of the slide only; the lead of the centre of the slide being A K; that is, its distance from its middle position at the beginning of the forward stroke.

* The method used in this and the following rules is that of Professor Dr. Zeuner, of the Swiss Federal Polytechnic School at Zürich, published in his treatise on Slide-valve Gearing, entitled, *Die Schiebersteuerungen.*

Rule II.—Given, the data and results of the preceding rule, and the position, A M, of the crank at the instant of release; to find the ratio of lap on the eduction-side to half-travel, and the position

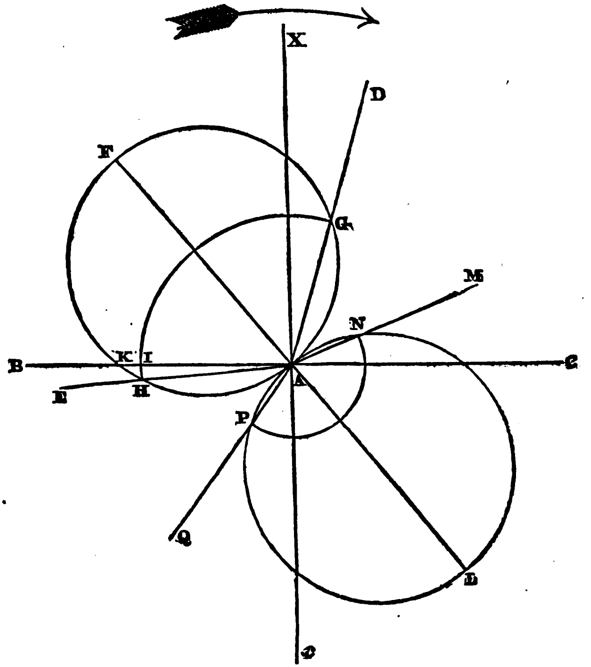


Fig. 108.

of the crank when cushioning begins. Produce F A to L, making A L = A F; on A L as a diameter draw a circle cutting A M in N: then $\frac{\text{A N}}{\text{A L}}$ will be the *required ratio of lap at eduction-side to half-travel.*

About A draw the circular arc N P, cutting the circle A L again in P; join A P; then A P will be the required *position of the crank at the instant when cushioning begins.*

Rule III.—Given, the data and results of Rule I., and the position, A Q, of the crank at the instant of cushioning; to find the ratio of lap at the eduction-side to half-travel, and the position of the crank at the instant of release—produce F A as before; on A L = F A as a diameter draw a circle cutting A Q in P:

$\frac{A\,P}{A\,L}$ will be the *required ratio of lap at the eduction-side to half-travel.*

About A draw the circular arc P N, cutting the circle A L again in N; join A N: A N will be the position of the crank at the instant of release.

Rule IV.—Given, the angular advance of the eccentric, the half-travel of the slide, and the lap at both sides; to find the positions of the crank at the instants of admission, cut-off, release, and cushioning. Draw the straight lines B A C and X A x perpendicular to each other; and take B and C to represent the dead points. Let the arrow denote the direction of rotation. Draw F A L, making the angle F A X = the angular advance of the eccentric; and make A F = A L = half-travel. On A F and A L as diameters, draw circles. About A, with a radius equal to the lap at the induction-side, draw an arc cutting the circle on A F in H and G; also, with a radius equal to the lap at the eduction-side, draw an arc cutting the circle on A L in N and P. Draw the straight lines, A H E, A G D, A N M, A P Q. These will represent respectively the positions of the crank at the instants of *admission, cut-off, release,* and *cushioning.*

Rule V.—For an eccentric to drive a *separate expansion gridiron slide-valve,* make the angular advance 90°; also make width of openings ÷ half-travel of valve = sine of angle made by position of crank when steam is cut-off with position at dead point.

4. **Link-Motion.**—In fig. 109 let A be the axis of the shaft; A B, the forward eccentric radius; A C, the backward eccentric radius;

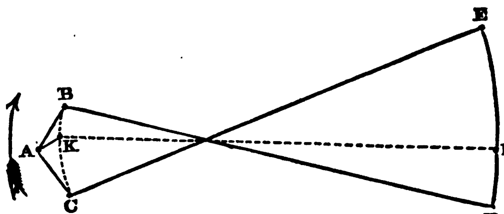


Fig. 109.

B D, the forward, and C E, the backward eccentric rods; D E, the link; F, the slider or stud. Radius of curvature of link = length of rods, or nearly so.

Rule VI.—To find the motion of the slide valve produced by any intermediate position of the stud, such as F.

With a radius bearing the same proportion to *half* the distance B C, that the length of the rods B D bears to that of the link D E,

draw the arc B C.* If the eccentric rods are so placed (as in the figure) that when the eccentrics are inclined towards the link, the rods are crossed, make the arc B C convex towards the axis A. If the eccentric rods are so placed as not to be crossed when the eccentrics are inclined towards the link, make the arc B C concave towards A. In that arc take a point, K, dividing it in the same proportion in which the stud F divides the link D E. Then the motion of the stud, F, will be very nearly the same as if it were directly connected by a rod K F with a crank A K. Consequently, from the *half-travel*, A K, and the angular advance, of that supposed crank, the motions of the slide-valve and their effects may be deduced by Rule IV. of the preceding Article.

5. **Nominal Horse-Power.**—I. *Ordinary Rule for Condensing Engines.*—Multiply the cube root of the stroke in feet by the square of the diameter of the cylinder in inches, and divide by 60.

II. *Admiralty Rule for Screw-Propeller Engines only.*—Multiply the mean velocity of the piston in feet per minute by the square of its diameter in inches, and divide by 6,000.

III. *Rule for Non-Condensing Engines.*—Multiply the cube root of the stroke in feet by the square of the diameter of the cylinder in inches, and divide by 20.

The indicated power of steam engines ranges from *once* to *six times* the nominal power.

* This construction is due to Mr. M'Farlane Gray (see his *Geometry of the Slide Valve.*)

ADDENDA TO PART IX., SECTION I.

Fusion of Solids.—The following are the *melting points* of a few of the more important substances. The last seven are given on the authority of Daniell.

	Fahr.		Fahr.
Mercury,	$-38°$	Bismuth,	493°
Ice,	$+32$	Lead,	630
Alloy—Tin 3, lead 5, bismuth 8, about,	210	Zinc,	773
Sulphur,	228	Silver,	873
Alloy—Tin 4, bismuth 5, lead 1,	246	Brass,	1,869
Alloy—Tin 1, bismuth 1,	286	Copper,	1,996
Alloy—Tin 3, lead 2,	334	Gold,	2,016
Alloy—Tin 2, bismuth 1,	334	Cast-iron,	2,786
Tin,	426	Wrought-iron,	above 3,280

Latent heat of fusion of ice, about 140 British units; of tin, 500.

Flow of Gases.—Let the pressure, bulkiness, and absolute temperature of a gas within a vessel be p_1, v_1, τ_1, and without the vessel, p_2, v_2, τ_2; and let $p_0\,v_0$ be the value of $p\,v$ for the absolute temperature τ_0 of melting ice. (See page 278.) Let γ be the ratio in which the specific heat of the gas is greater at constant pressure than at constant volume;

Let O be the area of an orifice through which the gas escapes from the vessel;

k, a *co-efficient of contraction*, or of *efflux*, so that the *effective* area of the orifice is k O;

u, the maximum velocity which the particles of the gas acquire in escaping, when there is no friction;

W, the weight of the gas which escapes in a second; then,

$$u = \sqrt{\left\{\frac{2\,g\,\gamma}{\gamma-1}\cdot\frac{p_0\,v_0\,\tau_1}{\tau_0}\cdot\left(1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right)\right\}};$$

$$\mathrm{W} = \frac{k\,\mathrm{O}\,u}{v_2} = k\,\mathrm{O}\,u\cdot\frac{\tau_0\,p_1}{p_0\,v_0\,\tau_1}\cdot\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}.$$

Values of γ: air, 1·408; steam-gas, about 1·3.

Values of the co-efficient of efflux k for air, with effective pressures of from ·23 to 1·1 atmosphere (Weisbach):—

	k
Conoidal mouthpieces, of the form of the contracted vein,....................................	0·97 to 0·99
Circular orifices in thin plates,....................	0·563 to 0·788

Outflow of Steam—Rough Approximation.—Let p_1 be the internal and p_2 the external absolute pressure; q, weight of outflow per unit area per second; then when $p_2 = \text{or} \angle \frac{3}{5} p_1$, $q = p_1 \div 70$ nearly; and when $p_2 > \frac{3}{5} p_1$, $q = (p_2 \div 42) \cdot \sqrt{\{(p_1 - p_2) \div \frac{2}{3} p_2\}}$, nearly. Contraction for safety valve openings about 0·6.

Addendum to Part III., page 132.

Levelling by the Barometer.—To correct the difference of level given by the formula, for variations in the force of gravity, divide by the co-efficient of g_1 in the note to page 245.

Addendum to Part VII.

Friction of Leather Collars.—The friction of the leather collar of a hydraulic press plunger is equal to the pressure upon a ring equal in circumference to the plunger, and of a breadth which, according to Mr. William More's experiments, is about $\frac{4}{10}$ of the depth of bearing surface of the collar; and according to the experiments of Mr. Hick and Mr. Luthy, from ·01 inch to ·015 inch, according to the state of lubrication of the collar.

Addendum to Part VIII., page 274.

Additional Resistance of Ship, due to short after-body.—Let v be the speed in knots; l, the proper least length of after-body, in feet $= \frac{3}{8} v^2$; l', the actual length of after-body; S, the area of midship section, in square feet; $\sin^2 \gamma$, the mean of the squares of the sines of the angles of obliquity of the stream-lines of the after-body; then, additional resistance in lbs.—

$$= 5{\cdot}66\, v^2 \sin^2 \gamma \cdot S \sqrt{\left(1 - \frac{l'^2}{l^2}\right)}, \text{ nearly.}$$

Explosive Gas-Engine.—Best proportions of explosive mixture; air, 8 volumes; common coal-gas, 1 volume. Absolute pressure immediately after explosion, $p_1 = 5$ atmospheres $= 10{,}580$ lbs. on the square inch, nearly. Let r = ratio of expansion; p_2 = absolute pressure at end of expansion; p_0 = absolute back pressure; W = indicated work per cubic foot of explosive mixture. Then

$p_2 = p_1 r^{-\frac{7}{5}}$; and

$$\text{W nearly} = \frac{5}{2}(p_1 - p_0) - \frac{7}{2}(r-1)p_2 + (r-1)(p_2 - p_0);$$

the mean effective pressure is $p_e = \frac{W}{r}$.

Approximate formula for final pressure where r is not greater than 7 nor less than 2; p_2 nearly $= 0{\cdot}54\left(\frac{1}{r} + \frac{1}{r^2}\right) - 0{\cdot}025$.

Addendum to Part VI.

Deflection of Springs.—*Straight springs* are to be treated as beams. (See page 221.) For *spiral springs*, made of cylindrical rod or wire, the following are the rules:—

Let r be the mean radius of the spiral spring, measured from the axis to the centre of the wire; n, the number of coils of which it consists; d, the diameter of the wire; C, the co-efficient of rigidity of the material; f, the greatest safe shearing stress upon it; W, any load not exceeding the greatest safe load; v, the corresponding extension or compression; W_1, the greatest safe *steady* load; v_1, the greatest safe extension or compression; then

$$\frac{W}{v} = \frac{C d^4}{64 n r^3};\quad W_1 = \frac{0{\cdot}196 f d^3}{r};\quad v_1 = \frac{12{\cdot}566 n f r^2}{C d}.$$

The greatest safe *sudden* load is $\frac{W_1}{2}$.

The *Resilience* of the spring is given by the formula, $\frac{W_1 v_1}{2} = \frac{2{\cdot}463\, n f^2 r d^2}{C}$.

The values of the co-efficient, C, of transverse elasticity of steel and charcoal iron wire in lbs. on the square inch, range between 10,500,000 and 12,000,000.

By the greatest safe stress must here be understood the greatest stress which is certain not to impair the elasticity of the spring by frequent repetition; say 30,000 lbs. on the square inch.

Addendum to Part IX., page 298.

Resistance of Air-pump of Steam-Engine equivalent to the following additional back-pressure, in lbs. on the square inch of steam piston; good air-pumps, 0·5 to 0·75; bad, 1·0.

X

RESULTS OF EXPERIMENTS ON SOME OF THE PRINCIPAL VARIETIES OF BUILDING STONES BY PROFESSORS DANIEL AND WHEATSTONE.*

Chemical Compositions.

	Sandstones.					Magnesian Limestones.				Oolites.				Limestones.		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
SiO_2,	98.3	96.40	95.1	93.1	49.4	3.6	2.53	0.8	..	..	..	1.20	..	..	10.4	4.7
$CaCO_3$,	1.1	0.36	0.8	2.0	26.5	51.1	54.19	57.5	55.7	93.59	94.52	95.16	92.17	93.4	79.0	79.3
$MgCO_3$,	..	..	..	..	16.1	40.2	41.37	39.4	41.6	2.90	2.50	1.20	4.19	3.8	3.7	5.2
Fe_2O_3, Al_2O_3,	0.6	1.30	2.3	4.4	3.2	1.8	0.30	0.7	0.4	0.80	1.20	0.50	0.90	1.3	2.0	8.3
H_2O and Loss,	..	1.94	1.8	0.5	4.8	3.3	1.61	1.6	2.3	2.71	1.78	1.94	2.83	1.5	4.2	2.5
Bitumen,	..	..	..	..	..	..	..	..	..	Traces	Traces	Traces	Traces	Traces	Traces	Traces
	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.00	100.00	100.00	100.00	100.0	99.3	100.0
Cohesive Powers in Pounds on the Square Inch.																
Experiments on pieces 2 ins. cube,	7887	7106	3979	4974	5116	8313	4335	3908	4335	2343	1491	3908	2556	1776	7177	4050
Specific Gravities.																
Of dry masses,	2.232	2.628	2.229	2.247	2.338	2.316	2.147	2.136	2.138	2.182	1.839	2.145	2.045	2.090	2.481	2.260
Of particles,	2.646	2.993	2.643	2.625	2.756	2.833	2.807	2.840	2.847	2.687	2.675	2.702	2.706	2.627	2.621	2.695
Absorbent Powers when saturated under the Exhausted Receiver of an Air-Pump.																
Weight of water in grains absorbed by pieces 2 ins. cube,	0.143	..	0.156	0.143	0.151	0.182	0.239	0.248	0.249	0.180	0.312	0.206	0.244	0.204	0.053	0.147
Disintegration.																
Weight of Matter in grains disintegrated from pieces 2 ins. cube,	0.6	0.121	10.1	7.9	7.1	1.5	1.9	0.6	1.8	7.1	10.0	2.7	3.3	16.6	9.8	9.5

X

No. 1.—Craigleith, near Edinburgh.
No. 2.—Darley Dale, Stancliffe.
No. 3.—Heddon.
No. 4.—Kenton.
No. 5.—Mansfield, or C. Lindleys.
No. 6.—Bolsovar.
No. 7.—Huddleston.
No. 8.—Roach Abbey.
No. 9.—Park Nook.
No. 10.—Ancaster.
No. 11.—Bath Box Quarry.
No. 12.—Portland, Waycroft Quarry.
No. 13.—Ketton.
No. 14.—Barnack.
No. 15.—Chilmark.
No. 16.—Hans Hill.

* A copy of the foregoing Table, taken from a Blue Book which is now scarce, is due to the courtesy of Professor Tennant, of King's College, London.

INDEX.